EQUIVALENCE CLASSES AND OPTIMIZATION OF VEHICLE SWINGING SYSTEMS

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Introduction

Vehicles will be treated as swinging systems. In this case any particular swinging system a_1 can be considered as an element of the set V of all vehicle swinging systems.

Let ϱ be an equivalence relation defined on the pairs of elements of V. As it is well known, every equivalence relation generates a division of the elements of V into disjoint classes (Fig. 1).

Two vehicle swinging systems, say a_1 and a_2 are equivalent if and only if any pair of input and output belonging to a_1 belongs also to a_2, and conversely (Fig. 2).

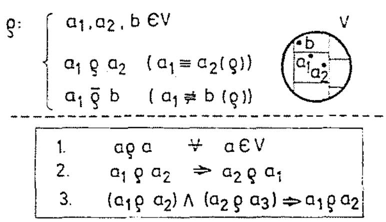

Fig. 1

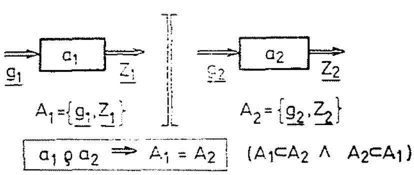

Fig. 2

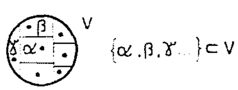

Fig. 3

From every equivalence class due to the equivalence relation it is sufficient to pick out one representative element making up a set $\{\alpha, \beta, \gamma, \dots\} \subset V$ (Fig. 3).

1. Basic assumptions

The following assumptions will be made on the investigated vehicle swinging systems:

I. The car body is rigid.

II. The springs and shock absorbers have nonlinear characteristics.

III. The vehicle is excited only by the stochastic road unevennesses.

For the sake of simplicity only plane models will be considered neglecting transversal oscillations and studying only vertical ones (Figs 4 and 5).

The systems of differential equations describing such systems are either well known from the literature or can be set up without particular difficulties.

The applicability of such models for vehicles having not too long frames has been justified by measurements by ROBSON, J. D., DODDS, C. J., MITSCHKE, M., ILOSVAY, L. etc. but exclusively for particular vehicle types i.e. nonlinear systems with given numerical characteristics and parameters ([1], [2], [3]).

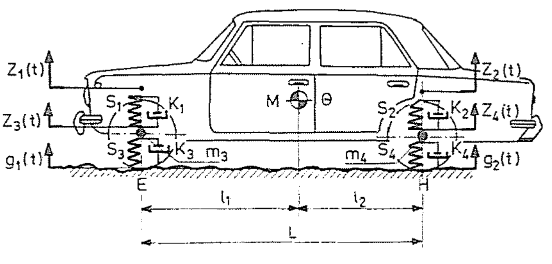

Fig. 4

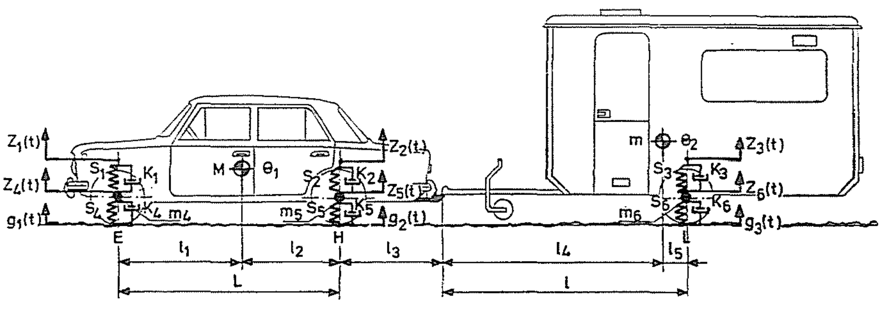

Fig. 5

2. The transformations and the structure of systems

Essentially, our equivalence investigation is an expedient transformation of the nonlinear differential equation system describing the vehicle oscillations. Thereby equivalent systems are described by (numerically) identical sets of differential equations. However, the formal identity of the sets of differential equations of two vehicle swinging systems having two or more inputs is only a necessary but not always sufficient condition of equivalence. Namely, if the sets of input functions of the two systems have no common element then no equivalence between the systems (as defined in the Introduction) can be shown. This is the case of two vehicle swinging systems equivalent up to their sets of differential equations but of different gauges. (This is the case of quasi-equivalence and note that if the inputs of the two systems comprise identical excitation then the systems will satisfy the strict conditions of the definition of equivalence.) Consequently, in the case of vehicles with two or more axles a sufficient condition of the equivalence is the gauge identity between the two systems.

In this analysis, each equivalence class will comprise systems equivalent up to their sets of differential equations; we choose representative elements from them, making up a representative system to be studied in the optimization procedure. In most practical cases the elements of the system have a constant parameter μ and freely chosen parameters $[\varphi, \psi]$ where:

μ — matrix of generalized mass proportion factors,

$[\varphi, \psi]$ — vector of nonlinear spring and shock absorber characteristics referring to unit generalized masses.

Thus, the problem is to determine the optimal value of $[\varphi, \psi]$ for a given value of μ. (There are, however, favourable cases where μ is not fixed either.)

It should be mentioned that this method suits transformation of existing systems with excellent properties to new systems with different dimensions and masses but with theoretically identical vibrational properties.

The coordinates of the swinging system are advisably chosen according to Fig. 4. Namely then it is easy to see if the front and rear parts of our swinging system can be decomposed into two subsystems with two degrees of freedom vibrating independently of each other.

Similar advantages are offered by choosing the coordinates as shown in Fig. 5 where:

S_i — nonlinear spring characteristics;

K_i — nonlinear shock absorber characteristics;

M, m — car body masses;

m_i — axle masses;

Θ_i — moments of inertia of car bodies about their centroids.

The system of differential equations describing the vertical vibrations of the single vehicle in Fig. 4 is:

$$\begin{bmatrix} m_1 & m_{12} & 0 & 0 \\ m_{21} & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \begin{bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \\ \ddot{Z}_3 \\ \ddot{Z}_4 \end{bmatrix} + \begin{bmatrix} K_1\{\dot{Z}_1 - \dot{Z}_3\} \\ K_2\{\dot{Z}_2 - \dot{Z}_4\} \\ K_3\{\dot{Z}_3 - \dot{g}_1\} - K_1\{\dot{Z}_1 - \dot{Z}_3\} \\ K_4\{\dot{Z}_4 - \dot{g}_2\} - K_2\{\dot{Z}_2 - \dot{Z}_4\} \end{bmatrix} + \tag{1}$$

$$+ \begin{bmatrix} S_1\{Z_1 - Z_3\} \\ S_2\{Z_2 - Z_4\} \\ S_3\{Z_3 - g_1\} - S_1\{Z_1 - Z_3\} \\ S_4\{Z_4 - g_2\} - S_2\{Z_2 - Z_4\} \end{bmatrix} = 0$$

where

$$\vartheta^2 = \frac{\Theta}{M}; \qquad m_1 = M\frac{l_2^2 + \vartheta^2}{L^2}; \qquad m_2 = M\frac{l_1^2 + \vartheta^2}{L^2};$$

$$m_{12} = m_{21} = M\frac{l_1 l_2 - \vartheta^2}{L^2}.$$

The condition that the system decomposes to two independently vibrating subsystems is

$$m_{12} = m_{21} = 0 \Longleftrightarrow l_1 l_2 = \vartheta^2$$

The system of differential equations of the vehicle-trailer pair in Fig. 5 is:

$$\begin{bmatrix} m_1 & m_{12} & m_{13} & 0 & 0 & 0 \\ m_{21} & m_2 & m_{23} & 0 & 0 & 0 \\ m_{31} & m_{32} & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix} \begin{bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \\ \ddot{Z}_3 \\ \ddot{Z}_4 \\ \ddot{Z}_5 \\ \ddot{Z}_6 \end{bmatrix} + \begin{bmatrix} K_1\{\dot{Z}_1 - \dot{Z}_4\} \\ K_2\{\dot{Z}_2 - \dot{Z}_5\} \\ K_3\{\dot{Z}_3 - \dot{Z}_6\} \\ K_4\{\dot{Z}_4 - \dot{g}_1\} - K_1\{\dot{Z}_1 - \dot{Z}_4\} \\ K_5\{\dot{Z}_5 - \dot{g}_2\} - K_2\{\dot{Z}_2 - \dot{Z}_5\} \\ K_6\{\dot{Z}_6 - \dot{g}_3\} - K_3\{\dot{Z}_3 - \dot{Z}_6\} \end{bmatrix} + \tag{2}$$

$$+ \begin{bmatrix} S_1\{Z_1 - Z_4\} \\ S_2\{Z_2 - Z_5\} \\ S_3\{Z_3 - Z_6\} \\ S_4\{Z_4 - g_1\} - S_1\{Z_1 - Z_4\} \\ S_5\{Z_5 - g_2\} - S_2\{Z_2 - Z_5\} \\ S_6\{Z_6 - g_3\} - S_3\{Z_3 - Z_6\} \end{bmatrix} = 0$$

where

$$a = \frac{l_3}{L}; \quad m_1 = \frac{1}{L^2}(Ml_2^2 + \Theta_1) + \frac{a^2}{l^2}(ml_5^2 + \Theta_2);$$

$$m_2 = \frac{1}{L^2}(Ml_1^2 + \Theta_1) + \frac{(1+a)^2}{l^2}(ml_5^2 + \Theta_2);$$

$$m_3 = \frac{1}{l^2}(ml_4^2 + \Theta_2); \quad m_{12} = m_{21} = \frac{1}{L^2}(Ml_1 l_2 - \Theta_1) - \frac{a(1+a)}{l^2}(ml_5^2 + \Theta_2);$$

$$m_{13} = m_{31} = \frac{a}{l^2}(\Theta_2 - ml_4 l_5); \quad m_{23} = m_{32} = \frac{1+a}{l^2}(ml_4 l_5 - \Theta_2).$$

The condition that the system decomposes to three vibrating subsystems with two degrees of freedom is:

$$\begin{cases} m_{12} = m_{21} = 0 \\ m_{13} = m_{31} = 0 \\ m_{23} = m_{32} = 0. \end{cases} \tag{3}$$

Hence the decomposition has the necessary condition $l_4 l_5 = \vartheta_2^2$. A sufficient condition is that the parameters of the swinging system satisfy

$$l_3 = -\frac{L}{2} \pm \sqrt{\left(\frac{L}{2}\right)^2 + \frac{l_1 l_2 - \vartheta_1^2}{l_5} \cdot \frac{lM}{m}} \tag{4}$$

where l_3 is the distance between the hinge and the rear axle of the vehicle [4].

Replace the third and the fourth equations of system (1) by sums of the first and third and of the second and fourth equations, respectively. Again, replace the fourth, fifth and sixth equations of system (2) by sums of the first and fourth, the second and fifth, and finally, the third and sixth equations, respectively. Dividing now the i-th row of both systems by the main diagonal element m_i of the mass matrix of reduced masses leads to the representative systems (5) and (6) defined by spring and damping characteristics φ_i and ψ_i and reduced mass proportion factors μ_{ij} referring to unit reduced masses:

$$\begin{bmatrix} 1 & \mu_{12} & 0 & 0 \\ \mu_{21} & 1 & 0 & 0 \\ \mu_{31} & \mu_{32} & 1 & 0 \\ \mu_{41} & \mu_{42} & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \\ \ddot{Z}_3 \\ \ddot{Z}_4 \end{bmatrix} + \begin{bmatrix} \psi_1(\dot{Z}_1 - \dot{Z}_3) \\ \psi_2(\dot{Z}_2 - \dot{Z}_4) \\ \psi_3(\dot{Z}_3 - \dot{g}_1) \\ \psi_4(\dot{Z}_4 - \dot{g}_2) \end{bmatrix} + \begin{bmatrix} \varphi_1(Z_1 - Z_3) \\ \varphi_2(Z_2 - Z_4) \\ \varphi_3(Z_3 - g_1) \\ \varphi_4(Z_4 - g_2) \end{bmatrix} = 0 \tag{5}$$

where

$$\mu_{ij} = \frac{m_{ij}}{m_i}; \quad \psi_i(\ ) = \frac{K_i(\ )}{m_i}; \quad \varphi_i(\ ) = \frac{S_i(\ )}{m_i}; \quad \begin{matrix}(i = 1, 2, 3, 4)\\ (j = 1, 2, 3, 4)\end{matrix}$$

$$\mu_{32} = \frac{m_{32}}{m_3} = \frac{m_{12}}{m_3} = \frac{m_1}{m_3} \cdot \frac{m_{12}}{m_1} = \underline{\mu_{31} \cdot \mu_{12}}$$

$$\mu_{41} = \frac{m_{41}}{m_4} = \frac{m_{21}}{m_4} = \frac{m_2}{m_4} \cdot \frac{m_{21}}{m_2} = \underline{\mu_{42} \cdot \mu_{21}}$$

$$\left[\begin{array}{ccc|ccc} 1 & \mu_{12} & \mu_{13} & 0 & 0 & 0 \\ \mu_{21} & 1 & \mu_{23} & 0 & 0 & 0 \\ \mu_{31} & \mu_{32} & 1 & 0 & 0 & 0 \\ \hline \mu_{41} & \mu_{42} & \mu_{43} & 1 & 0 & 0 \\ \mu_{51} & \mu_{52} & \mu_{53} & 0 & 1 & 0 \\ \mu_{61} & \mu_{62} & \mu_{63} & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \\ \ddot{Z}_3 \\ \ddot{Z}_4 \\ \ddot{Z}_5 \\ \ddot{Z}_6 \end{bmatrix} + \begin{bmatrix} \Psi_1(\dot{Z}_1 - \dot{Z}_4) \\ \Psi_2(\dot{Z}_2 - \dot{Z}_5) \\ \Psi_3(\dot{Z}_3 - \dot{Z}_6) \\ \Psi_4(\dot{Z}_4 - \dot{g}_1) \\ \Psi_5(\dot{Z}_5 - \dot{g}_2) \\ \Psi_6(\dot{Z}_6 - \dot{g}_3) \end{bmatrix} + \begin{bmatrix} \varphi_1(Z_1 - Z_4) \\ \varphi_2(Z_2 - Z_5) \\ \varphi_3(Z_3 - Z_6) \\ \varphi_4(Z_4 - g_1) \\ \varphi_5(Z_5 - g_2) \\ \varphi_6(Z_6 - g_3) \end{bmatrix} = \underline{0} \tag{6}$$

where

$$\mu_{ij} = \frac{m_{ij}}{m_i}; \quad \psi_i(\ ) = \frac{K_i(\ )}{m_i}; \quad \varphi_i(\ ) = \frac{S_i(\ )}{m_i} \quad \begin{matrix}(i = 1, 2, 3, \ldots, 6);\\ (j = 1, 2, 3, \ldots, 6)\end{matrix}$$

$$\mu_{42} = \mu_{41} \cdot \mu_{12}, \ \mu_{43} = \mu_{41} \cdot \mu_{13}; \ \mu_{51} = \mu_{52} \cdot \mu_{21}, \ \mu_{53} = \mu_{52} \cdot \mu_{23};$$

$$\mu_{61} = \mu_{63} \cdot \mu_{31}, \ \mu_{62} = \mu_{63} \cdot \mu_{32}.$$

Consider the lower left submatrix of the partitioned reduced mass proportion matrix μ of systems (5) and (6). The non-diagonal elements of this matrix (framed in dashed lines) can be obtained as the product of the element of the upper left submatrix on the same place and the diagonal element in the same row of the lower left submatrix.

All in all, the considered systems with four and six degrees of freedom are determined by 12 and 21 independent parameters, respectively (reduced mass proportions and nonlinear characteristics referring to unit reduced masses).

Functioning of the systems is easier to understand by analyzing them decomposed to suitable subsystems. For the sake of simplicity, let the spring and shock absorber characteristics be linear.

Using Laplace transforms on the systems of differential equations permits to determine the transfer functions between each input and output. In occurrence of the deduced characteristics, the relations between the subsystems presenting the structure of the system are as shown in Fig. 6.

The possibility of decomposing the swinging vehicle systems to independently vibrating subsystems of two degrees of freedom and reckoning with vehicle seats and passengers as biological vibrating systems justifies the analy-

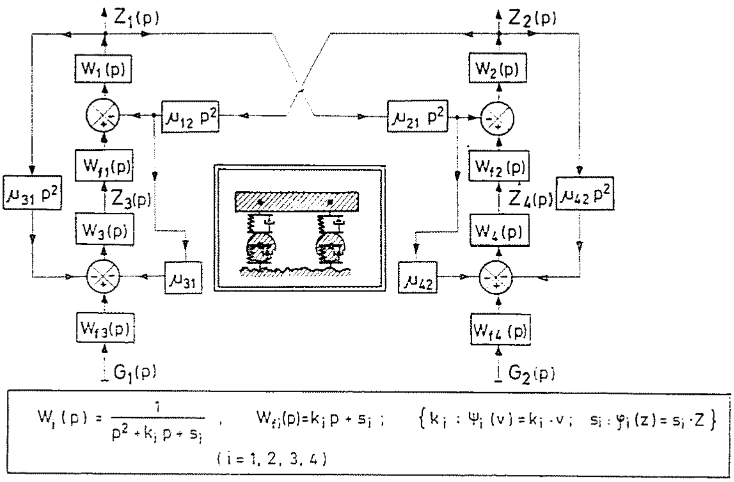

Fig. 6

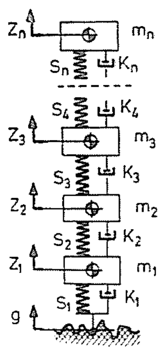

Fig. 7

sis of so-called chain models of vehicle vibrating systems [5]. Fig. 7 shows a chain model of n degrees of freedom; its transformed system of differential equations is:

$$\mu\, \ddot{Z}(t) + \psi(\dot{Z}(t), \dot{g}(t)) + \varphi(Z(t), g(t)) = 0 \qquad (7)$$

where

$$\mu = \begin{bmatrix} 1 & \mu_{12}, & \mu_{12}\mu_{23}, & \mu_{12}\mu_{23}\mu_{34}, & \ldots, & \mu_{12}\mu_{23}\mu_{34}\ldots\mu_{(n-1),n} \\ 0 & 1 & \mu_{23}, & \mu_{23}\mu_{34}, & \ldots, & \mu_{23}\mu_{34}\ldots\mu_{(n-1),n} \\ 0 & 0 & 1 & \mu_{34}, & \ldots, & \mu_{34}\ldots\mu_{(n-1),n} \\ 0 & 0 & 0 & 1, & \ldots, & \mu_{45}\ldots\mu_{(n-1),n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0, & \ldots, & 1 \end{bmatrix}$$

$$\psi = \begin{bmatrix}\psi_1\\ \psi_2\\ \vdots\\ \psi_n\end{bmatrix}; \quad \varphi = \begin{bmatrix}\varphi_1\\ \varphi_2\\ \vdots\\ \varphi_n\end{bmatrix};$$

$$\begin{cases} \psi_1(\dot{Z}_1 - \dot{g}) = \dfrac{K_1(\dot{Z}_1 - \dot{g})}{m_1}; \; \varphi_1(Z_1 - g) = \dfrac{S_1(Z_1 - g)}{m_1} \\ \psi_i(\dot{Z}_i - \dot{Z}_{i-1}) = \dfrac{K_i(\dot{Z}_i - \dot{Z}_{i-1})}{m_i}; \; \varphi_i(Z_i - Z_{i-1}) = \dfrac{S_i(Z_i - Z_{i-1})}{m_i} \\ \mu_{i-1,\,i} = \dfrac{m_i}{m_{i-1}}; \quad (i = 2, 3, \ldots, n) \end{cases}$$

A link chain model with n degrees of freedom is seen to have $3n - 1$ independent parameteres.

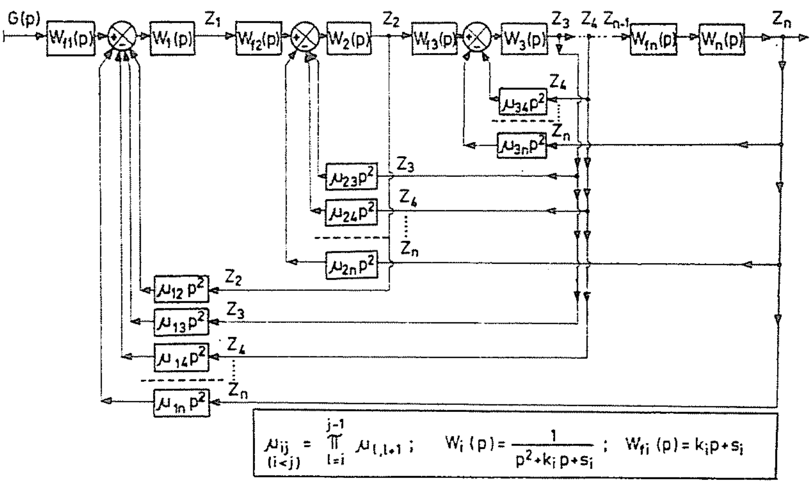

Fig. 8

Fig. 8 shows the subsystems, their transfer functions and couplings in the case of a linear system.

3. The optimization

The optimization procedure will be illustrated on a plane model with 4 degrees of freedom. The original nonlinear spring and shock absorbed characteristics of the plane model are shown in Fig. 9 (full line). Fig. 10 shows the masses and geometrical dimensions of the model.

The example refers to a system such that $l_1 l_2 \neq \vartheta^2$. In the optimization, however, the couplings between the vibrating systems above the front and the rear axle are neglected.

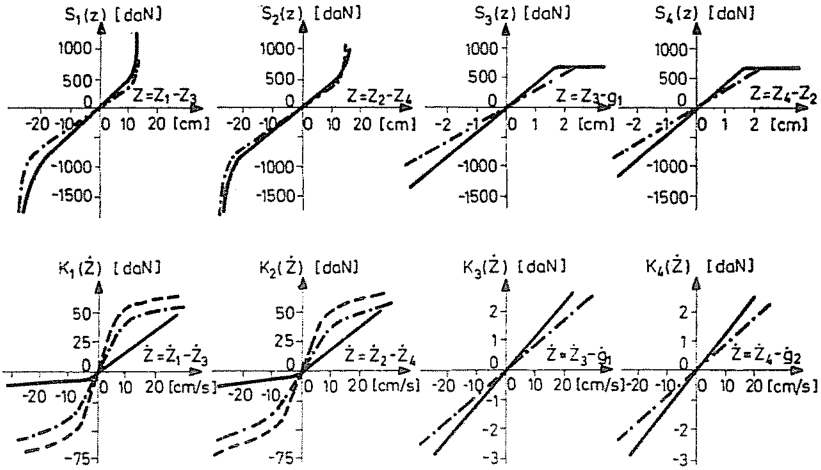
S1(z) [daN]
Z=Z1-Z3
S2(z) [daN]
Z=Z2-Z4
S3(z) [daN]
Z=Z3-g1
S4(z) [daN]
Z=Z4-Z2
[cm]
K1(Ż) [daN]
Ż=Ż1-Ż3
K2(Ż) [daN]
Ż=Ż2-Ż4
K3(Ż) [daN]
Ż=Ż3-ġ1
K4(Ż) [daN]
Ż=Ż4-ġ2
[cm/s]

Fig. 9

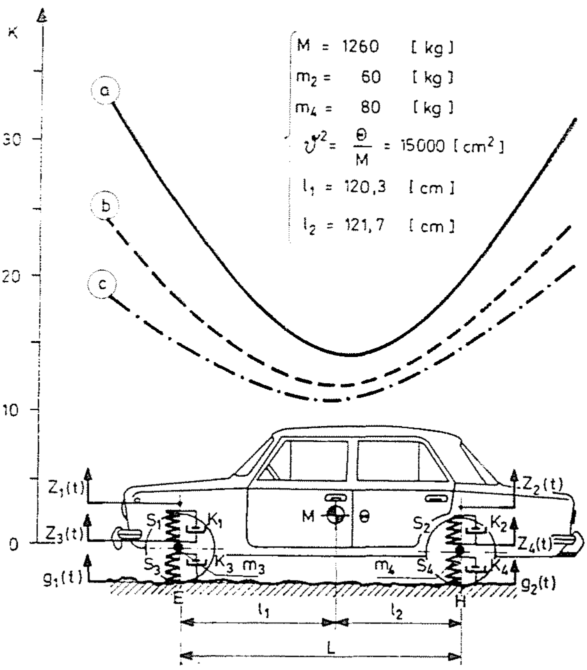
K
M = 1260 [kg]
m2 = 60 [kg]
m4 = 80 [kg]
ϑ² = Θ/M = 15000 [cm²]
l1 = 120,3 [cm]
l2 = 121,7 [cm]
a
b
c
Z1(t)
Z3(t)
g1(t)
Z2(t)
Z4(t)
g2(t)
S1
K1
S3
K3
m3
M
Θ
S2
K2
m4
S4
K4
E
H
l1
l2
L

Fig. 10

The results show the vibration characteristics to be much improved by replacing the spring and shock absorber characteristics of the original system by the optimized ones.

As for the mathematical details, the problem is to optimize two swinging systems with two degrees of freedom, made according to the following steps [6]:

(1) A road section spectral density function $S_g(v, \omega)$ is chosen, fitting best to the expected stresses [7]. Beside the angular frequency ω[rad/s] this function depends also on the vehicle speed v[m/s].

(2) Amplitude transfer characteristics $|W_z(\mu, k_1, k_2, s_1, s_2, i\omega)|$ of importance for these investigations are determined.

(3) Optimization is made for different speed values v by minimizing objective functions for linear combinations of variances of output signals most important for evaluating the vibrations:

$$\boxed{F(k_1, k_2, s_1, s_2) = \lambda_1 D_{\ddot{Z}_2} + \lambda_2 \frac{D_{Z_1-g}}{Z_{\mathrm{STAT}}} + \lambda_3 \cdot k_2 D^2_{\dot{Z}_2-\dot{Z}_1} \to \mathrm{Min}\,!}$$

$$(0 \le k_1, k_2;\ 0 < s_a \le s_1 \le s_b;\ 0 < s_A \le s_2 \le s_B;\ \lambda_1, \lambda_2, \lambda_3 \ge 0) \tag{8}$$

$$\Longrightarrow \begin{cases} k_{1\mathrm{OPT}}(v),\ k_{2\mathrm{OPT}}(v),\ s_{1\mathrm{OPT}}(v),\ s_{2\mathrm{OPT}}(v) \\ D_{Z_1-Z_2,\,\mathrm{OPT}}(v),\ D_{Z_1-g,\,\mathrm{OPT}}(v),\ D_{\dot{Z}_2-\dot{Z}_1,\,\mathrm{OPT}}(v),\ D_{\dot{Z}_1-\dot{g},\,\mathrm{OPT}}(v) \end{cases}$$

where

$$D_{\ddot{Z}_2} = \sqrt{\frac{1}{\pi}\int_0^\infty |W_{\ddot{Z}_2}(\mu, k_1, k_2, s_1, s_2, i\,\omega)|^2 S_g(v, \omega)\, d\omega}$$

$$D_{Z_1-g} = \sqrt{\frac{1}{\pi}\int_0^\infty |W_{Z_1-g}(\mu, k_1, k_2, s_1, s_2, i\omega)|^2 S_g(v, \omega)\, d\omega} \tag{9}$$

$$D^2_{\dot{Z}_2-\dot{Z}_1} = \frac{1}{\pi}\int_0^\infty |W_{\dot{Z}_2-\dot{Z}_1}(\mu, k_1, k_2, s_1, s_2, i\omega)|^2 S_g(v, \omega)\, d\omega.$$

(4) The optimum nonlinear characteristics are determined by generalizing the statistical linearization method by BOOTON, R. C. and KAZAKOV, I. E. [8], [9]: nonlinear characteristics φ_i, ψ_i, are to be found, which, statistically linearized at various speeds, best approximate the linear optimum parameters computed in step (3) [10].

$\psi_1, \psi_2, \varphi_1, \varphi_2$:

$$\left\{\begin{aligned}
&\int_0^{V_{max}} \left[k_{1OPT}(v) - \int_{-\infty}^{\infty} x f_1\left(D_{\dot{z}_1-\dot{g}_1, OPT}(v), x\right) \cdot \psi_1(a, x)\, dx\right]^2 dv \rightarrow \text{Min !}\\
&\int_0^{V_{max}} \left[k_{2OPT}(v) - \int_{-\infty}^{\infty} x f_2\left(D_{\dot{z}_2-\dot{z}_1, OPT}(v), x\right) \cdot \psi_2(b, x)\, dx\right]^2 dv \rightarrow \text{Min !}\\
&\int_0^{V_{max}} \left[s_{1\,OPT}(v) - \int_{-\infty}^{\infty} x f_3\left(D_{z_1-g, OPT}(v), x\right) \cdot \varphi_1(c, x)\, dx\right]^2 dv \rightarrow \text{Min !}\\
&\int_0^{V_{max}} \left[s_{2\,OPT}(v) - \int_{-\infty}^{\infty} x f_4\left(D_{z_2-z_1, OPT}(v), x\right) \cdot \varphi_2(d, x)\, dx\right]^2 dv \rightarrow \text{Min !}\,.
\end{aligned}\right. \tag{10}$$

The functions f_j $(j = 1, 2, 3, 4)$ in (10) are density functions of the input signals (depending on speed v) of the nonlinear characteristics ψ_i, φ_i $(i = 1, 2)$ divided by the variances of these input signals.

In (10), nonlinear characteristics ψ_i, and φ_i are sought for in some defined function form. Our calculations showed the most suitable function forms to be:

$$\psi_1(a, x) = a_1 \operatorname{sign}(x) \cdot [1 - \exp(-a_2 |x|)]; \quad a = [a_1, a_2]$$

$$\psi_2(b, x) = b_1 \operatorname{sign}(x) \cdot [1 - \exp(-b_2 |x|)]; \quad b = [b_1, b_2]$$

$$\varphi_1(c, x) = \sum_{j=1}^{n} c_j x^j; \quad c = [c_1, c_2, \ldots, c_n]$$

$$\varphi_2(d, x) = \sum_{j=1}^{n} d_j x^j; \quad d = [d_1, d_2, \ldots, d_n].$$

In final account Eqs (10) serve for the determination of vectors a, b, c, d.

$$\boxed{a, b, c, d = ?}$$

4. Results of the optimization

In Fig. 9, nonlinear characteristics of the original system **A** have been plotted in full line, optimized damping characteristics of the system **B** obtained by optimizing only the shock absorbers in dash line, and optimized spring and damping characteristics of system **C** in dotted line. An analysis of nonlinear characteristics points out a slight softening of springs and a considerable increase in damping due to optimization.

Fig. 10 shows swinging comfort factors K approximately proportional to the variance of vertical accelerations at various points of the car body for driving speed $v = 50$ [km/h] on an asphalt road. The comfort factor im-

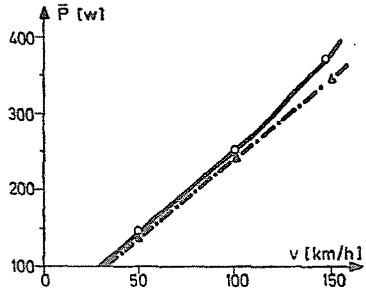

Fig. 11

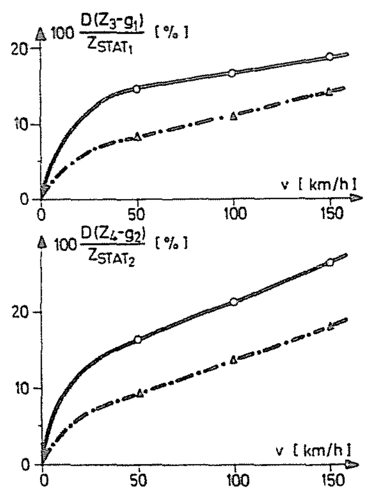

Fig. 12

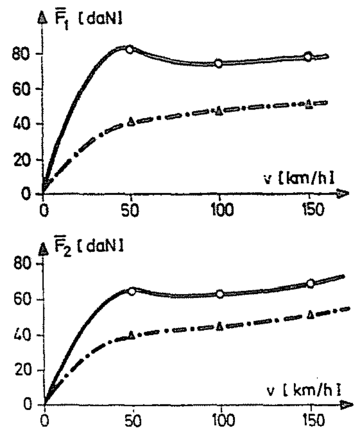

Fig. 13

proved by 20 to 25% upon optimizing the dampings; combined spring and damping optimization resulted in an improvement of 30 to 45%.

Fig. 11 shows the specific power losses (in W) versus driving speed of the swinging systems. The optimized system shows an average power loss by 5—10% lower than the original system.

The stability index characterizing the road section tracking ability of the wheel versus the driving speed has been plotted in Fig. 12 by referring the variance of the relative displacement between the axle and the road profile to the static sinking of the geometrical centre of the wheel. (Upper diagram refers to the front axle and the lower one to the rear axle.)

The optimization improved the stability indices in front by 25—50% and in the rear by 30—45%.

Finally, Fig. 13 shows the effective means of the specific dynamic stresses in the bearing springs. A non-negligible result of the optimization is an improvement of the average dynamic spring loads by 35—50% in front and by 25—40% in rear, compared to the original system.

Summary

Systems equivalent up to their systems of differential equations have been studied. Plane models with four and six degrees of freedom have been found to have 12 and 21 independent parameters, resp.; link models with n degrees of freedom to have $3n-1$ independent parameters.

Optimization referred to linear systems for various speeds. An inverse statistical linearization method has been applied to find optimum nonlinear characteristics which, statistically linearized for each driving speed, provide the best quadratic approximation of the optimum characteristics.

The results of the optimization are shown on a plane model of four degrees of freedom corresponding to a medium category passenger car. Optimization of the springs and shock absorbers of this vehicle showed a swinging comfort increase by 30—45%, a 5—10% diminution of power losses through absorbers, an improvement by 25—50% (in terms of the stability index defined in this paper) of the road section tracking capability of the wheels and finally a 25 to 50% decrease of dynamic stresses in the bearing springs.

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