DIGITAL SIMULATION OF WIND SPEED FIELD

By

E. ABD EL FATTAH

Department of Mathematics, Transport Engineering Faculty, Technical University Budapest

Received October 25, 1980

Presented by Prof. Dr. G. SZÁSZ

Introduction

Wind speed not only depends on the height above the ground level, but also on the time. Wind speed at any level within the height of buildings can be assumed to be stationary and ergodic random process. In the analysis of wind forces acting upon tall buildings given in paper [7] the author simulated the generalized forces of wind by using Fast Fourier Transform. In this paper a developed method is introduced for simulating wind speed field by using TWO-DIMENSIONAL FAST FOURIER TRANSFORM technique. The simulation of wind speed field presented here saves time of computation and gives flexible data for the structural analysis. A numerical example is presented to indicate the time history of simulated wind speed wave samples.

Spectra of wind speed

The experimental studies on wind speed fields proved that the spectral density function of horizontal wind speed is independent on the height above the ground level within heights of buildings. The SDF given by DAVENPORT [1] is considered in this work. This spectrum was obtained from the results of about 70 spectra of the horizontal component in strong wind. It is formulated in normalized form which gives flexibility in simulation of wind speed field. Eq. 1 represents this spectrum

$$\frac{f_1 \cdot S(f_1)}{k\, V_{10}^2} = \frac{4X^2}{(1 + X^2)^{4/3}} \tag{1}$$

where $X = 1200\, f_1/V_{10}$ cycles/m,
k = drag coefficient,
V_{10} = mean wind speed at 10 m above the ground level,
f_1 = frequency cycles/sec.

Fig. 1 shows the normalized logarithmic spectra of horizontal wind speed.

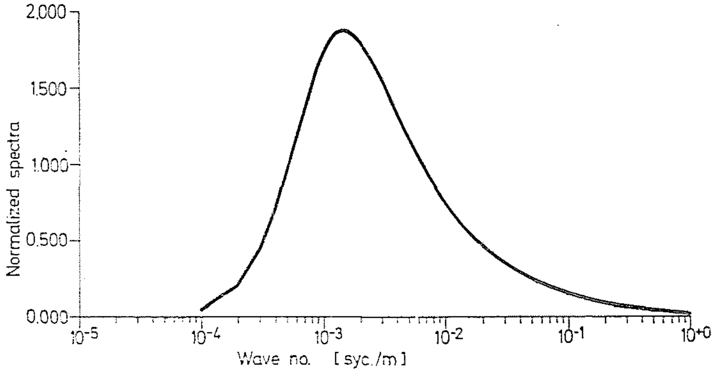

Fig. 1. Normalized Logarithmic Spectra of Horizontal Wind Speed

The variance of the variable random wind speed v is given by

$$\sigma_v^2 = \int_0^\infty S(f_1)\, df_1 = 6 \cdot k V_{10}^2 \tag{2}$$

The coherence is defined as the absolute square of the cross-correlation spectrum.

The square root of wind speed coherence which gives the moduls of the cross-correlation spectrum between any two points lying on the wind speed waves front is represented by Eq. 3.

$$|R_{12}(f_1)| = \exp\left(-\,\alpha \frac{\Delta z \cdot f_1}{V_{10}}\right) \tag{3}$$

where $\alpha \approx 7.7$ and Δz is the vertical separation between the considered two wind waves.

Thus the cross-spectral density of the horizontal wind speed fluctuations can be represented by

$$S(f_1, \Delta z) = \frac{8k\, V_{10}^2\, X^2}{f_1(1+X^2)^{4/3}} \exp\left(-\,\alpha \frac{\Delta z \cdot f_1}{V_{10}}\right). \tag{4}$$

By taking the Fourier Transform of the two sides of the above equation with respect to Δz it yields the two dimensional spectrum of wind speed field

$$S(n_1, n_2) = \frac{\alpha k X^2}{\pi^3 (1 + X^2)^{4/3} \cdot (\alpha n_1^2 + n_2^2)} \tag{5}$$

where $$n_1 = f_1 / V_{10},$$
$$n_2 = f_2 / V_{10}$$

and f_2 is the wave division frequency.

Simulation of wind waves

The wind speed field can be simulated by using the following formula

$$v(t, z) = 2\sqrt{2}\,\pi V_{10} \sum_{i=1}^{N_1} \sum_{l=1}^{N_2} A(n_1, n_2) \cos(2\pi V_{10} n_1 t + 2\pi V_{10} n_2 z + \Phi_{il}) \quad (6)$$

where

$$A(n_1, n_2) = [S(n_1, n_2)\, \Delta n_1\, \Delta n_2]^{1/2},$$

$$n_1 = (i - 1)\, \Delta n_1,$$

$$n_2 = (l - 1)\, \Delta n_2,$$

Φ_{il} are the realized values of independent random phase angles uniformly distributed between 0 and 2π. The intervals Δn_1 and Δn_2 are obtained by $\Delta n_1 = 2n_{1u}/N_1$ and $\Delta n_2 = 2n_{2u}/N_2$ where n_{1u} and n_{2u} are the ultimate values of n_1 and n_2, respectively. Extension of the right hand side of Eq. 6 results in

$$v(t, z) = 2\sqrt{2}\,\pi V_{10} \sum_{i=1}^{N_2} \sum_{l=1}^{N_2} A(n_1, n_2) \{\cos(2\pi V_{10} n_1 t) \cos(2\pi V_{10} n_2 z + \Phi_{il}) -$$

$$- \sin(2\pi V_{10} n_1 t) \sin(2\pi V_{10} n_2 z + \Phi_{il})\}$$

$$v(t, z) = 2\sqrt{2}\,\pi V_{10} \sum_{i=1}^{N_1} [B_{iz} \cos(2\pi V_{10} n_1 t) - D_{iz} \sin(2\pi V_{10} n_1 t)] \quad (7)$$

where

$$B_{iz} = \sum_{l=1}^{N_2} A(n_1, n_2) \cos(2\pi V_{10} n_2 z + \Phi_{il}) \quad (8)$$

$$D_{iz} = \sum_{l=1}^{N_2} A(n_1, n_2) \sin(2\pi V_{10} n_2 z + \Phi_{il}) \quad (9)$$

Eq. 7 can be written in a complex form

$$v(t, z) = 2\sqrt{2}\,\pi V_{10} Re\left[\sum_{i=1}^{N_1} (B_{iz} + jD_{iz}) \exp(-j\, 2\pi V_{10} n_1 t)\right] \quad (10)$$

where *Re* means real part

or

$$v(m, z) = 2\sqrt{2}\,\pi V_{10} Re\{\sum_{i=1}^{N_1} (B_{iz} + jD_{iz}) \exp[-j\, 2\pi (m - 1)(i - 1)/N_1\} \quad (11)$$

where

$$t = (m - 1)T,\ m = 1, 2, \ldots, N_1;$$

T = small time interval.

Eq. 11 can be put in the following form

$$v(m, z) = 2\sqrt{2}\,\pi \frac{V_{10}}{T} Re\{F \cdot T \cdot (B_{iz} + j\,D_{iz})\} \tag{12}$$

which means that the wind speed waves can be obtained by taking the Fourier transform of the complex value $(B_{iz} + jD_{iz})$. The real part of the resulting value is considered only. Fast Fourier transform must be used to save time of computation. Moreover, the whole wind speed field can be obtained by taking the two dimensional Fast Fourier Transform.

The one dimensional FFT program given in reference [6] has been developed into two dimensional Transform, because the one dimensional FFT program is not fast enough. The developed FFT FORTRAN PROGRAM is given in the Appendix.

The mathematical representation of the two dimensional FFT given in the appendix is

$$[v(m, z)] = 2\sqrt{2}\,\pi \frac{V_{10}}{T} Re\{\text{F. F. T. [COM]}\} \tag{13}$$

where [COM] is a complex matrix whose general element is $(B_{iz} + jD_{iz})$, where

$$i = 1, 2, \ldots, N_1,$$

$$m = 1, 2, \ldots, N_2.$$

The values of $v(m, z)$ for all values of m and z are generated, where z is chosen at floor levels of the considered structure under analysis of wind forces.

Numerical example

The method explained above was used for simulating wind speed field of 16 waves at every 3.6 m of height. The following data were used in this simulation

$V_{10} = 20$ m/sec $k = 0.013$

$N_1 = 512$ $N_2 = 256$

$n_{1u} = 0.16$ $n_{2u} = 0.002$ $T = 0.12625$ sec

The root mean square values of the simulated waves were as shown in Table I.

In case of N_1 tends to ∞ the simulated waves should be Gaussian and the variances should be of the form: $6\,k\,V_{10}^2$ as given by Eq. 2.

Fig. 2 shows the first simulated wave, while Fig. 3 shows the 16th simulated wave.

Table 1

Root Mean Square Values of Simulated Wind Speed

Wave order	1	2	3	4	5	6	7	8
r.m.s.	6.46	6.40	6.31	6.20	6.12	6.05	5.98	5.88
Wave order	9	10	11	12	13	14	15	16
r.m.s.	5.74	5.58	5.42	5.30	5.23	5.19	5.17	5.15

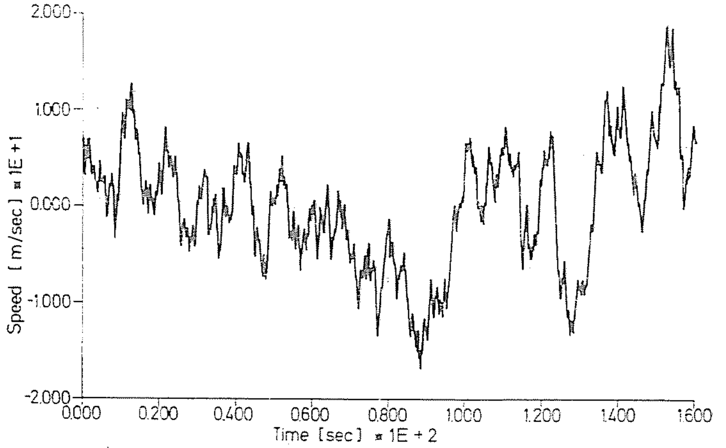

Fig. 2. Time History of First Wind Speed Wave

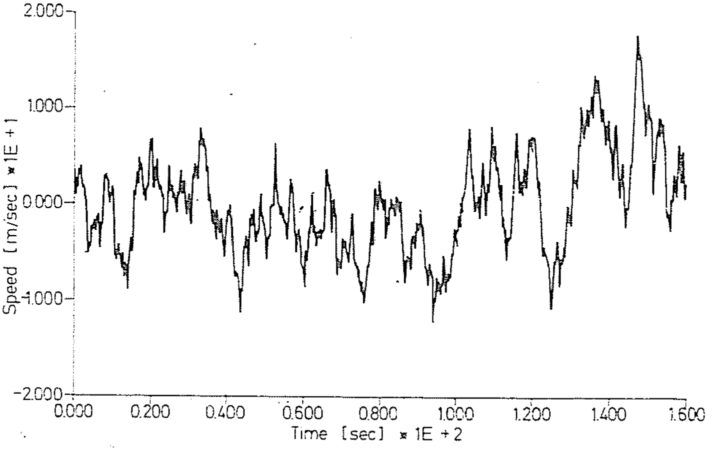

Fig. 3. Time History of Last Wind Speed Wave

Conclusion

A complete method of digital simulation of wind speed field is presented. The method is very fast compared to the previous methods. FORTRAN PROGRAM for TWO-DIMENSIONAL FAST FOURIER TRANSFORM WAS ARRANGED FOR solving the problem. The simulated waves can be checked and statistically tested ensuring the accuracy of calculations.

Acknowledgement

The author is greatly indebted to Prof. Dr. Károly SEITZ who encouraged him to make this effort. Thanks are also due to Prof. Dr. Gábor SZÁSZ, Head of the Mathematical Department, Technical University Budapest for his help and advice.

Summary

The stochastic wind speed field can be obtained by the following steps:

1. The two-dimensional spectrum of wind speed field are obtained by taking the Fourier transform of its cross-spectral density function.
2. Calculation of the elements of the complex matrix [COM] defined by Eq. 8 and Eq. 9.
3. Application of the given two-dimensional Fast Fourier Transform FORTRAN PROGRAM on the obtained matrix.
4. The real parts of the transformed matrix give the required wind speed field.

References

1. DAVENPORT, A. G.: The Spectrum of Horizontal Gustiness Near the Ground in High Winds Quarterly Journal of Royal Meteorological Society, London, England, *87*, 194 (1961).
2. HARRIS, R. I.: Measurements of Wind Structure at Heights up to 598 Feet above Ground level. Symposium on Wind Effects on Buildings, and Structures, Loughborough University of Technology, England Apr., 1968, pp. 1.1—1.35.
3. ROBSON, J. D.: An Introduction to Random Vibration. Elsevier Publishing Company (1963).
4. KOLTAI, M.: Rajzgépi Software az R 32 Számítógépen. Budapesti Műszaki Egyetem Számítóközpont.
5. JAMES, M. L.—SMITH, G. M.—WOLFORD, J. C.: Applied Numerical Methods for Digital Computation with FORTRAN. Dept. of Eng. Mechanics, the University of Nebraska, 1967.
6. OTNES, R. K.—ENOCHSON, L.: Applied Time Series Analysis. John Wiley and Sons., 1978.
7. VAICAITIS, R.—SHINOZUKA, M.—TAKENO, M.: Response Analysis of Tall Buildings to Wind Loading. Journal of the Structural Division, ASCE, March 1975.
8. SHINOZUKA, M.—JAN, C. M.: Digital simulation of Random Process and its Applications. Journal of Sound and Vibration, *25*, 111 (1972).

El Akabawi Abd EL FATTAH H-1521 Budapest

Appendix

```
      SUBROUTINE FFTRAN(SIGN, T, V, NPOW, KK)
      DIMENSION V(1024, 17), CS(2), MSK(13)
      COMPLEX V, CXCS, HOLD, XA
      EQUIVALENCE (CXCS, CS)
      NMAX=2**NPOW
      ZZ=6.283185306*SIGN/FLOAT(NMAX)
      DELTA=T
      IF (SIGN) 10, 10, 5
    5 DELTA=1./(T*FLOAT(NMAX))
   10 MSK(1)=NMAX/2
      DO 15 I=2, NPOW
   15 MSK(I)=MSK(I-1)/2
      NN=NMAX
      MM=2
      DO 45 LAYER=1, NPOW
      NN=NN/2
      NW=0
      DO 40 I=1, MM, 2
      II=NN*I
      W=FLOAT(NW)*ZZ
      CS(1)=COS(W)
      CS(2)=SIN(W)
      DO 20 J=1, NN
      II=II+1
      IJ=II-NN
      DO 20 K=1, KK
      XA=CXCS*V(II, K)
      V(II, K)=V(IJ, K)-XA
   20 V(IJ, K)=V(IJ, K)+XA
      DO 25 LOC=2, NPOW
      LL=NW-MSK(LOC)
      IF (LL) 30, 35, 25
   25 NW=LL
   30 NW=MSK(LOC)+NW
      GO TO 40
   35 NW=MSK(LOC+1)
   40 CONTINUE
   45 MM=MM*2
      NW=0
      DO 80 I=1, NMAX
      NW1=NW+1
      DO 56 K=1, KK
      HOLD=V(NW1, K)
      IF (NW1-I) 60, 55, 50
   50 V(NW1, K)=V(I, K)*DELTA
   55 V(I, K)=HOLD*DELTA
   56 CONTINUE
   60 DO 65 LOC=1, NPOW
      LL=NW-MSK(LOC)
      IF (LL) 70, 75, 65
   65 NW=LL
   70 NW=MSK(LOC)+NW
      GO TO 80
   75 NW=MSK(LOC+1)
   80 CONTINUE
      RETURN
      END
```