RAILWAY BRAKE-GEAR DESIGN METHOD BY MEANS OF PROBABILITY THEORY

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1. Introduction

One of the most important characteristics of the brake gear is the *braking ratio* A, the ratio in percentage of the total brake-shoe force at the maximum brake-cylinder pressure for a given vehicle weight. (See the simplified model in Fig. 1.) Since the weight of a railway car changes as a function of the service load, the braking ratio can be interpreted for different load values. With increasing A, also the effectiveness of brake gear increases but this increase is limited by wheel sliding. So the value of A is bounded from above by the sliding of wheels and from below by the weaker brake action and the increase of stopping distance.

The braking ratio is generally determined by the designer so as to eliminate the sliding of wheels even under unfavourable operating conditions. But if the braking ratio is chosen cautiously (i.e. its value is kept low) an unfavourable increase in stopping distance should be reckoned with. The foregoing point to the difficulty of meeting the contradictory demands in certain cases. Practical design recommendations give no method to determine the optimum.

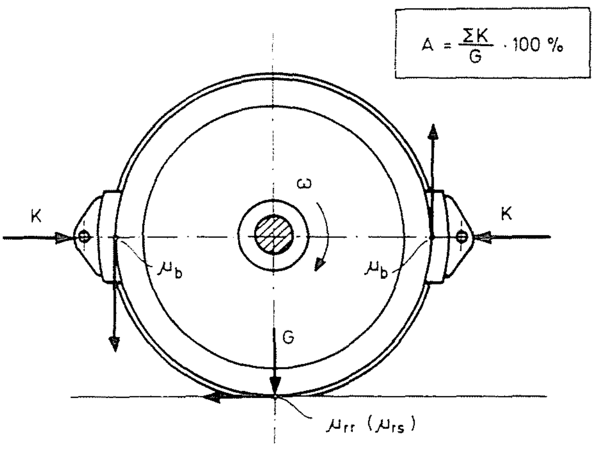

Fig. 1

A probabilistic dimensioning method is required which takes the random character of the friction coefficients, decisive for the braking process, into consideration expected to determine the probabilities of both keeping the stopping distance and of the wheel slide. Such a dimensioning method is underlying the determination of the optimum braking ratio. The calculation method to be described solves the outlined problem under simplifying conditions. It is a new method for the design of railway brakes. This procedure can be refined on the one hand, by involving further characteristics of braking technique, and on the other hand, by taking comprehensive measurement data of the encountered stochastic magnitudes into consideration.

2. Extension of the brake calculation method to the domain of sliding taking the stochasticity of the friction coefficient into consideration

If a railway vehicle is braked with constant brake-shoe force until it stops, then, with a proper approximation, the following three cases can be distinguished, depending on the magnitude of the braking ratio:

1. The wheel rolls until the vehicle stops.
2. The wheel rolls for a while from the beginning of braking, from that point on it slides until the vehicle stops.
3. The wheel slides all the time from beginning of braking until the vehicle stops.

The rolling motion of the wheel is simulated here by pure rolling (without sliding) and the sliding by the full blocking of wheels.

For a given braking ratio to determine the stopping distance, knowledge of three friction coefficients is required:

a) Friction coefficient between the wheel and the brake-shoe: μ_b

b) Friction coefficient between the rail and the wheel in the state of rolling (adhesion coefficient): μ_{rr}

c) Friction coefficient between the rail and the wheel in the state of sliding: μ_{rs}

The properties of these friction coefficients, the effects and variables influencing these phenomena are discussed in [1], [2], [3]. Let us emphasize that friction coefficients in question should be identified as *random variables* on the basis of measurement experiences.

They can be given in the form of:

$$\mu = \frac{a}{v + b} + c + \Delta\mu,$$

where: v is the momentary speed of the vehicle;
a,b,c are constants (See Table I);
$\Delta\mu$ is the deviation of the random variable from its own expected value, a random variable itself.

Table I

Friction coefficient	a	b	c	μ_1	Note
μ_b	3.5*	11.1	—	0.016	Modified formula by Karvacky [1]
μ_{rr}	2.083	12.22	0.13	0.1	Kother's formula [1]
μ_{rs}	0.25	1.4	0.06	0.025	[3]

The variable $\Delta\mu$ has zero expected value and its standard deviation σ is equal to the standard deviation, assumed as constant, of the friction coefficient μ considered as normally distributed, to be justified in the following parts.

For practical calculations the value set of $\Delta\mu$ is discretized as follows:

$$\Delta\mu_i = \frac{i}{m}\mu_1.$$

where: m is the number of equidistant division elements of the interval $[0, \mu_1]$ as shown in Fig. 2;
i is the parameter identifying the end points of division elements. It takes its values from the sequence: $-m, -(m-1), \ldots, -1, 0, 1, \ldots, (m-1), m$;
μ_1 is half-length of the field of scattering $[-3\sigma, 3\sigma]$ of the random variable μ.

The constant parameters of diagrams to be discussed were taken into consideration with the figures in Table I.

Constant a marked with an asterisk includes the value of the brake-shoe force per unit area (brake-shoe pressure): here it comes to 0.7 MPa, belonging in our example to 60% braking ratio.

For an arbitrary braking ratio A the corrected constant a is:

$$a = 3.5\sqrt[3]{\frac{60}{A}}.$$

The friction coefficients used in our calculations are shown in the diagram of Fig. 3, together with the half-length of scattering fields μ_1.

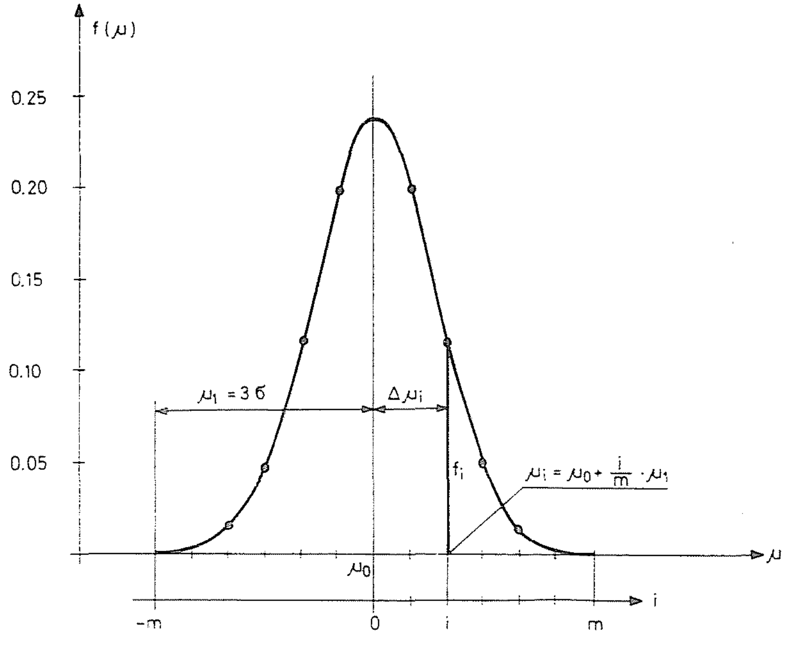

Fig. 2

The stopping distance is known to be composed of two parts: *basic stopping distance and the additional stopping distance*. In the following, only the basic stopping distance will be examined, and calculated by assuming the brake-shoe force (pressure) being constant during the whole braking process. As a first approximation, neither the vehicle resistance nor the effect of rotating masses will be taken into consideration. These neglects are irrelevant to our conclusions but the results will depend on less of variables and so they can be surveyed more clearly.

The relationship for determining the basic stopping distance:

$$S = S_r + S_s = \frac{1}{gA}\int_0^{v_1} \frac{v\,dv}{\mu_b(v)} + \frac{1}{g}\int_{v_1}^{v_{max}} \frac{v\,dv}{\mu_{rs}(v)} \,.$$

In the formula the gravity acceleration is designated by g.

The first term gives the rolling distance S_r, the second term gives the sliding distance S_s. Speed v_1 at the instant of sliding is supplied by solving for v the equation

$$A = \frac{\Sigma K}{G} = \frac{\mu_{rr}(v)}{\mu_b(v)} \,.$$

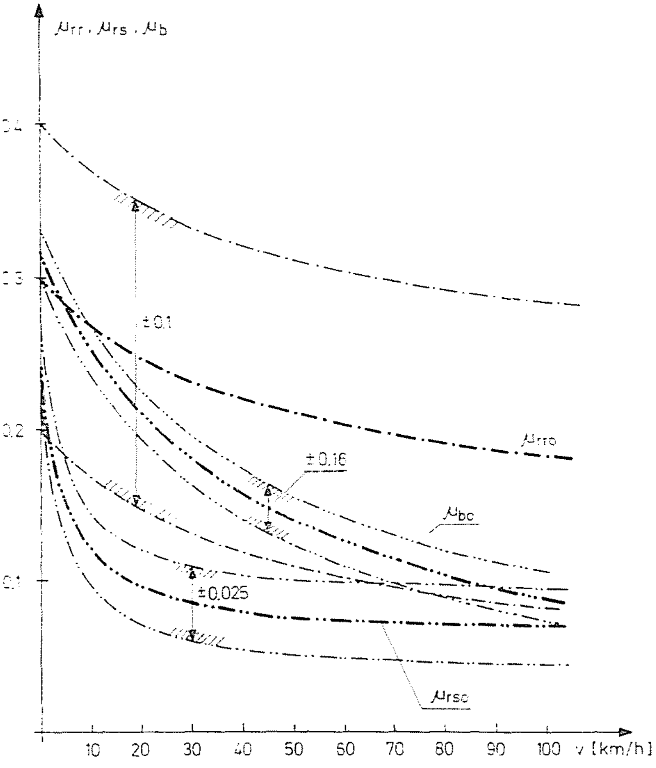

Fig. 3

derived from the condition $\mu_b \Sigma K = \mu_{rr} G$ (see Fig. 1), provided A is constant. This equation can be solved numerically after substituting the relationship for the friction coefficients.

The two terms of the basic stopping distance can be treated computerially identically. E.g. the solution for the first term is:

$$S_r = \frac{1}{g\,A}\left[\left\{\frac{b_b}{c_b}\,v + \frac{1}{c_b^3}\left[\frac{(c_b \cdot v + q_b)^2}{2} - 2q_b(c_b v - q_b) + (q_b^2 - b_b\,c_b\,q_b)\ln\left(c_b v + q_b\right)\right]\right\}\right]_{v=0}^{v=v_1},$$

where: $q_b = a_b + c_b, b_b$,

a_b, b_b, c_b are coefficients in the formula for the friction coefficient μ_b.

These relations with parameter i = constant permit an easy calculation of the stopping distance.

Since, however, the friction coefficients are random variables, there are at least two problems to be cleared:

1. What is the distribution of the three friction coefficients affecting the braking process and whether these distributions are affected by any of the variables of important for braking (e.g. the momentary speed of the vehicle).
2. Is there any correlation between the three friction coefficients as random variables or they are quite independent from each other.

To answer thoroughly both questions, a lengthy investigation is required. According to the evaluation of the experimental results achieved at the Institute of Automotive Engineering of the Technical University, Budapest, friction coefficient μ_b shows a normal (Gaussian) distribution while the standard deviations are not quite equal in different phases of the braking process.

No sufficient statistical information is available on the closer or looser connection between the three friction coefficients. Informatively, the weather factor, decisive for the braking process, affects identically all three friction coefficients. But this problem still awaits to be cleared.

As a first approximation to this problem, calculations assume normal distribution of each of the three friction coefficients of constant standard deviation for each coefficient and a strict functional relationship between them.

The procedure involves the following particulars: Half-length of scatter fields μ_1 corresponding to 3σ of all the three friction coefficients were divided into m parts. After chosing parameter i, the random characteristics were calculated with values belonging to i of the density functions of the three friction coefficients.

Namely, plotting the realized value triads of the random variables μ_b, μ_{rr}, μ_{rs} on axes of a spatial orthogonal coordinate system, in a general case, the end points of the vectors determined by the coordinate triads form a discrete set of points around the vector of expected values μ_{0b}, μ_{0rr} and μ_{0rs} (Fig. 4). But applying the above-mentioned simplifying conditions, the points are aligned on the straight line e in dash-and-dot line. In terms of probability theory: the three dimensional distribution is taken into consideration as concentrated on the straight line e as a limit case.

Applying the described method to calculate the stopping distance, using a parameter $i =$ constant, it is found to vary vs. braking ratio according to the curve in continuous line in Fig. 5. On the stopping distance diagram three characteristic points can be marked out.

The sliding of wheels occurs at point A (just at the moment of stopping), while for braking ratios to the left from point A the total stopping distance is covered by pure rolling (without slide). For braking ratios between A and B the wheel is still rolling at the beginning of braking but it is sliding at the end of braking. The stopping distance between A and B can be divided into

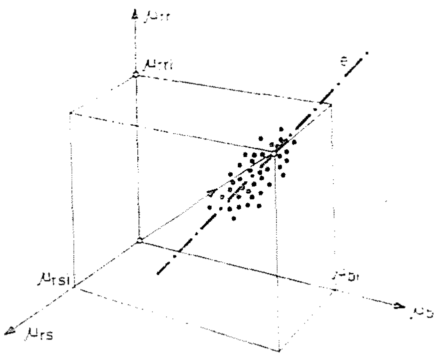

Fig. 4

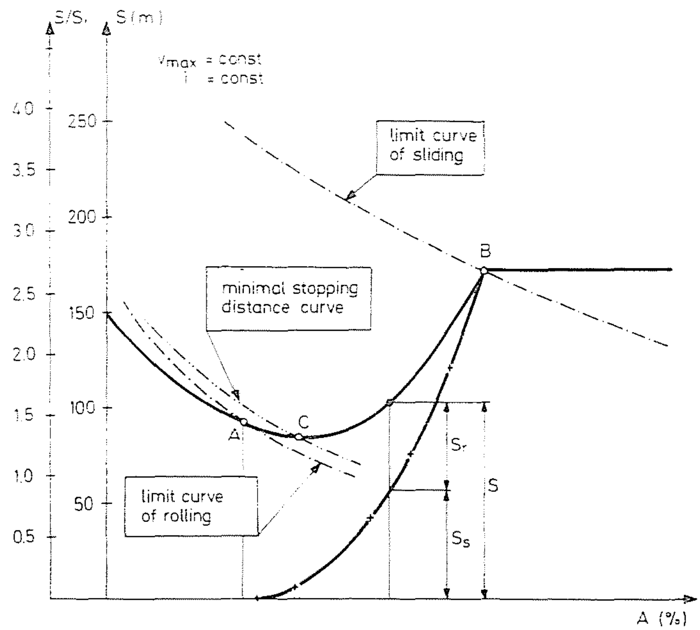

Fig. 5

two sections, covered by rolling and sliding. The stopping distance diagram has a minimum at C. Up to C, the stopping distance decreases with increasing design braking ratio, it increases between B and C, and beyond point B it is constant. The curve connecting points A for different i values is the *limit curve of rolling*, the curve connecting points B is the *limit curve of the total sliding*, while the curve connecting points C represents the *curve of the minimum stopping distances*.

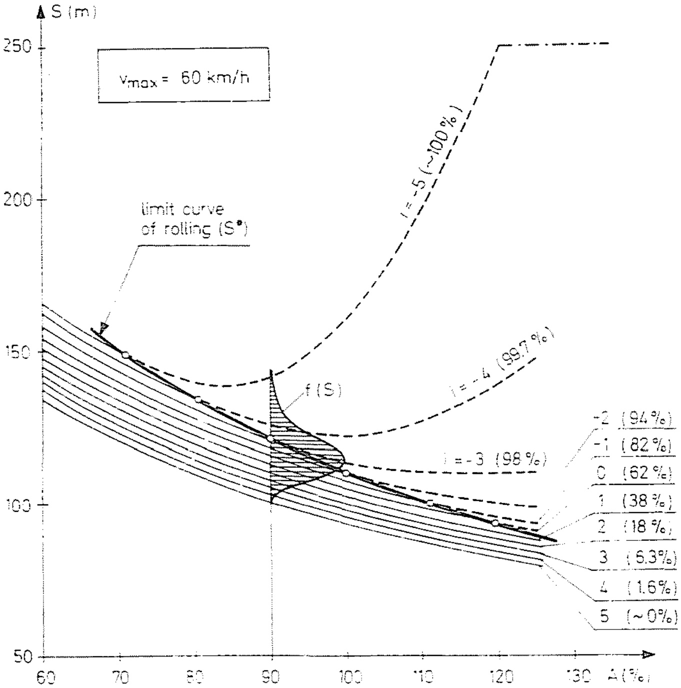

Fig. 6

Calculating stopping distances with friction coefficient constants in Table I. with different parameters i and assuming $v_{max} = 60$ km/h as initial speed yields the set of curves in Fig. 6. Taking a fixed braking ratio A, the density function $f(S)$ and the distribution function $F(S)$ of the stopping distance as a random variable, as well as its statistical parameters can be determined (Fig. 7).

S^* on the limit curve of rolling is the stopping distance for the chosen braking ratio, where the wheels just start sliding at the moment of stopping. For this stopping distance, the *probability of sliding* can be marked on the distribution curve: $R(S^*) = 1 - F(S^*)$. And specifying a stopping distance S_0, the *probability* $F(S_0)$ *of keeping this fixed stopping distance* can be determined at S_0 of the same distribution curve. Both the $R(S^*)$ and $F(S_0)$ values can be calculated as a function of the design braking ratio, to yield the diagram in Fig. 8. The parameter of curves $F(S_0)$ is the specified stopping distance. The two dominant properties of this set of curves are the following:

1. With increasing design braking ratio the probability of keeping the specified stopping distance is first increasing, then, owing to the increasingly more adverse effect of sliding, it is decreasing. At the

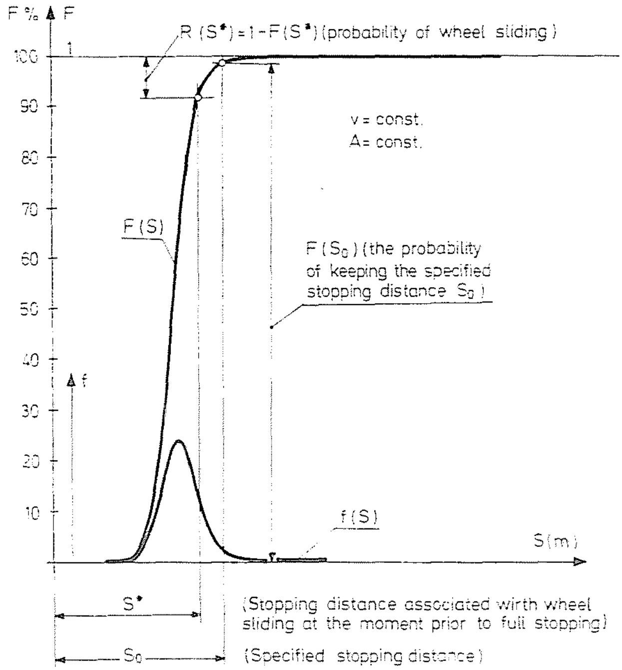
F % F
R (S*) = 1 − F (S*) (probability of wheel sliding)
1
100
90
80
70
60
50
40
30
20
10
f
v = const.
A = const.
F (S)
F (S0) (the probability of keeping the specified stopping distance S0)
f (S)
S (m)
S*
S0
(Stopping distance associated wirth wheel sliding at the moment prior to full stopping)
(Specified stopping distance)

Fig. 7

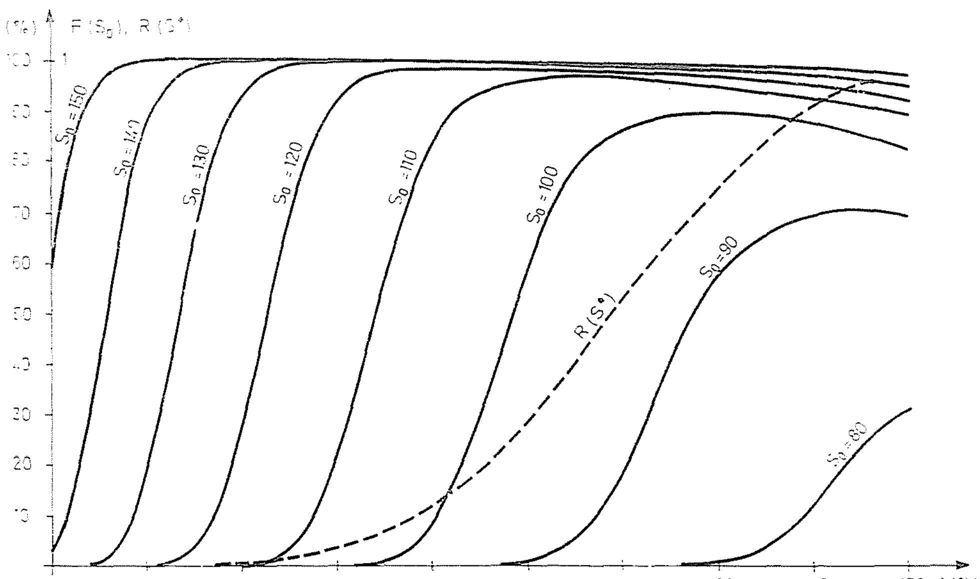
(%) F (S0), R (S*)
1
70
60
30
20
S0 = 150
S0 = 140
S0 = 130
S0 = 120
S0 = 110
S0 = 100
S0 = 90
S0 = 80
R (S*)
60
70
80
90
100
110
120
130
150 A (%)

Fig. 8

vicinity of curve peaks — in a rather wide interval of braking ratios — the probability value little changes. But the constancy intervals decrease with shorter stopping distances.

2. The specified stopping distances belong to two categories, namely those in a certain interval of braking ratios the specified stopping distance can be kept practically at 100% probability (referred to the $\pm 3\sigma$ range of the reference normal distribution) this limit is about 120 m in Fig. 7; and those where the probability of keeping the stopping distance is below 100% for any A.

3. Brake design method based on the probability of sliding and of keeping the specified stopping distance

Assuming a given specified stopping distance S_0, three cases of the relative location of curves $R(S^*)$ and $F(S_0)$ in Fig. 7 can be distinguished (Figs 9/1, 9/2, 9/3).

1. For a practically zero probability of sliding the specified stopping distance can be kept at practically 100% probability. This case is that of low-speed vehicles, with no problem in brake engineering. The design braking ratio can be anywhere within the interval in thick

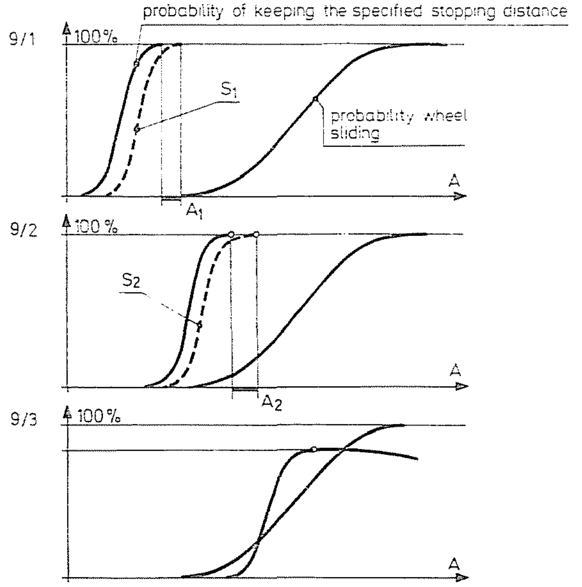

Fig. 9

line, in limit case it is A_1 involving stopping distance S_1. *S_1 is called the design upper limit stopping distance.*

2. The specified stopping distance can still be kept at practically 100% probability, but already sliding has a probability shown in the figure in the braking section just prior to stopping. In this case, likelihood and admissibility of service problems (damages) have to be considered. There are three possible ways of solution:
   a) Application of more complex brake-gear (e.g. rapid brake or anti-skid device) shifting the curves $R(S^*)$ and $F(S_0)$ to a more favourable position.
   b) Reduction of A to reduce also the probability of sliding. But in this case the specified stopping distance cannot be kept at 100% probability.
   c) Specifying a longer basic stopping distance. In the limit case the braking ratio corresponding to point A_2 is obtained, which involves a stopping distance S_2 which can still be kept practically at 100% probability. *Stopping distance S_2 is called the design lower limit stopping distance.*

3. The specified stopping distance cannot be kept at 100% probability and also sliding has a considerable probability. Of course, this case is to be avoided from brake operation aspect, so in this case more complex, high-power brakes should be used. This is the brake-design problem of high-speed vehicles.

The design limit stopping distances S_1 and S_2 can be considered as the extension of the notion of the theoretical limit stopping distance [1] to the field of brake-gear design. But the two design limit stopping distances depend on much more variables than does the original limit stopping distance. Their calculation permits to outline the scope of brake engineering possibilities available in the given case.

The values of S_1 and S_2 calculated under simplifying conditions are shown in Fig. 10 vs. the initial vehicle speed where the curve of the limit stopping distance [1] is plotted in dash-and-dot line completed with its scatter field $\pm 3\sigma$.

At last let us note that often the value of the design braking ratio considered as favourable should be chosen from a determined interval. In practical cases the probability of keeping the specified stopping distance increases with increasing braking ratios, but at the same time the risk of sliding increases. An aspect of setting the limits to the favourable braking ratio is the existence of a critical value of sliding speed v_1 above that important damage (e.g. wheel flattening to be repaired by turning) arises. It is also to be considered what reserves are included in the specified stopping distance for the case of unex-

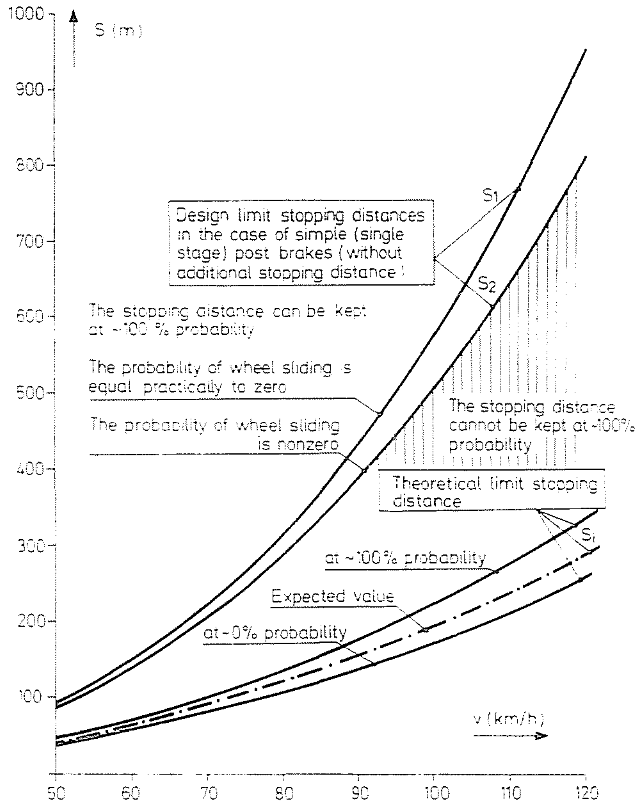

Fig. 10

pected events. Setting a range of favourable parameters is conditioned by a close co-operation between the designer and the upkeeper, and by careful consideration of brake engineering phenomena.

Summary

When designing the brake-gear of railway vehicles the design braking ratio at the maximum brake cylinder pressure must be determined. With increasing braking ratio also the effectiveness of brake-gear increases but this increase is limited by wheel sliding. The harmonization of the contradictory demands is complicated by the stochasticity of the friction coefficients. This study approaches the solution of this problem by means of the methods of the probability theory. Setting out from the probability distributions of the friction coefficients, the probability of keeping the specified stopping distance, and that of the wheel sliding can be determined as a function of the braking ratio. On the basis of the relative position of the two probability curves yielded, the practical value of the braking ratio can be set out. The design procedure outlined can be improved for the area of more difficult break-gear systems, but a deeper digging out of the statistical properties of the characteristic friction coefficients — first of all by means of measurements — is required.

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