A SIMPLE STOCHASTIC MODEL FOR OPTIMAL SHORT-RUN CAPITAL INVESTMENT POLICY OF SOCIALIST ENTERPRISES

By

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Introduction

Because of the gradual decrease in disposable sources, the problem of continuous costs depending on output — in other words optimization of output — is of special interest today in the socialist national economy. Sequence, selectivity and reliability play important roles in the course of optimization.

Therefore, research of their roles with appropriate means cannot be omitted in decision-making strategies. The increasing capital intensity of ensuring production output, the limited financial resources at disposal. and last but not least, the stochastic regularities present in the ever-changing external and domestic market needs give grounds for the experts relying on economic planning methods that are suitable for the consideration of these circumstances, during the elaboration of the concepts of capital investment.

In our present economic situation the improvement of the foreign trade balance is of primary importance. This has either direct or indirect effect upon the activity of our production enterprises, thus in uncertain market conditions. economic planning should be built into the production price level.

In our study such a substantial, preliminary simple procedure of decesion-making is described, with the application of which the optimal ratios of connected short-run capital investments can be better defined for production systems, than before in which reliability features describable with probability distribution with regard to meeting market demands are available. The problem of decision-making is the formation of such an optimal strategy that fixes the possible maximum profit of the production system for a given period or, in our case, that fixes the average growth-rate of the profit. The decision-making strategy to be presented is applicable not only in the planning operation of production systems in the strictest sense of the word, but in every other enterprise as well which is based on socialist ownership (e.g. prime export enterprises, economic ventures[1]), in which firms holding joint interest can make decisions under usually unstable market conditions.

1. The formulation of simple decision-making in connection with the short-run capital investment of "not entirely reliable" production systems

A production system — according to the market demands on a given merchandise in a unit of time (monthly, quarterly, yearly etc.) — manufactures products. The manufactured quantity of goods has to be delivered to the "market" by the end of the given unit of time.

The production system will be regarded as a "not entirely reliable" service system from the point of view of manufacturing the products — because of the limits of production capacity, disturbance of production, problems of stockpiling etc. — i.e. capable of meeting market demands only up to a certain extent.

It is supposed that the statistical information characterizing the ability of the production system in meeting demands is available.

With respect to the performance of order, the behaviour of the market can be investigated in two ways. *In the general case*, when the behaviour shown by the market is "distinguished"; in this case a_{rs} times the cost of production, as a function of the ordered r and the s goods actually delivered, is paid to the production system. It is assumed that a_{rs} is either less or greater than 1. The interpretation of $a_{rs} < 1$ is that the production system, as a consequence of its inadequate performance, is "fined" by the market, i.e. the production system has to pay a penalty for non-performance.

In the special case when the behaviour shown by the market is "harsher": in this hypothetical situation it is supposed that only a perfect satisfaction of market demands, i.e. a performance in accordance with specific demands and times of delivery, is considered to be acceptable by the market (e.g. the manufactured goods cannot be used in the next period). So in this case, if the production system cannot meet the demands of a given period, it will lose the investments for the manufacture of the goods. According to this, the behaviour of the market is similar to that of the "impatient customer", as it fills its unsatisfied needs from another production system.

If the needs of the market are satisfied by the production system, i.e. if the delivery of the ordered r quantity is totally fulfilled, then a sum of a_r

[1] The Inter-Invest Foreign Trade Development Deposit Company in Hungary is a good example of this.

times the invested cost of production is paid to the production system. (In this case, a_r is a number, dependent on the amount of r, that can obviously be considered to be $a_r > 1$).

The following problem which is going to be called "simple decision-making", is in connection with the utilization of production capacity relating of given market demands.

In every unit of time a λ_r ratio (that is dependable only on the extent of the order) of the capital at disposal is invested by the production system in the manufacture of the quantity of goods ordered by the market. With what sort of strategy does one have to determine this λ_r ratio (of capital investment), assigned to production and to the utilization of production capacity by the production system, in such a way that the growth rate of its profit should be maximal? Thus in the formulation of optimal decision-making strategy it is taken for granted that the conversion of production capacities (manpower as well) into products can be realized in the direction of the increasing efficiency of the socialist national economy.

2. The determination of the ratio of optimal capital investment and the appropriate decision-making strategy

In modelling the optimal ratio of the capital investment depending on the service reliability of the production system and the decision-making strategy determining it, which actually determine a multi-phase stochastic decision-making process, the following conditions were taken into consideration.

2.1 It is supposed that the stochastic parameters characterizing the claim supply reliability of the production system, based on statistical researches and estimates by experts, are available. The above-mentioned parameters are conditional probabilities, that provide the probability that the production system delivers a given s unit of goods, if an r unit is ordered by the market. In this case the values of a and r can vary from the 0 order of goods to the maximum order of n, when it is useful to give r and s in such discreet values that are dependent on the characteristic features of the product. A $n \times n$ stochastic matrix is determined by these conditional probabilities, where the general element of the matrix $p(s|r)$ is the probability of the situation, when an r quantity of goods is ordered by the market and an s quantity is delivered by the production system.

2.2 In the course of the first decision-making in connection with the utilization of production capacity, the initial capital at the disposal of the production system is V_0.

2.3 A sum of $a_{rs} > 0$ times the invested cost is paid to the production system by the market, as a function of the ordered r and the delivered s unit of goods. According to the performance of the production system, it is either "recompensated" or "punished" by the market, so in compliance with this a_{rs} is either smaller or larger than 1.

For the sake of simplicity it is supposed that $a_{rs} \neq 1$, but it should be mentioned that with a slight modification the calculations can be extended even to this case.

2.4 The survey of the behaviour of the "market- and production system" scheme, namely the process of a stochastic type of decision-making is going to be investigated during discreet time units (multiphase decision-making process).

2.5 It is supposed that in each time unit any r amount of goods can be ordered with independent positive probability by the market, that is $p_r > 0$, where the meaning of p_r is the probability that an r amount is ordered.

2.6 It is assumed that only the (1-ε)th of the whole capital can be invested into the production by the production system. In this case ε is any small positive number, representing the amount of the risk "basis".

2.7 The ratio of the accumulated capital to be invested by the production system in the manufacture of the ordered goods is made to be dependent only on the r value of the order.

We would like to determine the optimal ratio of the capital to be invested and the decision-making strategy in such a way that *the average growth-rate of the profit should be maximal.* Thus, because of the exponential nature of the growth of the profit, the following logarithmic function of rate

$$G = \lim_{N \to \infty} \frac{1}{N} \log \frac{V_N}{V_0} \tag{1}$$

is considered to be the function of our goal, where N is the number of decision-making periods, while V_N is the whole capital of the production system in the N^{th} decision-making period.

In practical (short-term) cases instead of the former theoretical function of goal the following maximization of capital growth-rate is given

$$\frac{1}{N} \log \frac{V_N}{V_0} . \tag{1/a}$$

It can be shown that the function of goal modelled this way is equivalent to the maximization of the expected value of the short-term profit [5].

If the above conditions and the "distinguished" market behaviour (general case) apply, the following can be stated:

Thesis 1.

The optimal ratio of the capital short-run invested in the production of goods and the appropriate optimal decision strategy are determined by the following:

$$\lambda_r = \begin{cases} 0, & \text{if} \quad \sum_{s=1}^{n} p(s/r) \cdot a_{rs} < 1 \\ 1-\varepsilon, & \text{if} \quad \sum_{s=1}^{n} p(s/r) \cdot a_{rs} > 1 \quad \text{and} \\ & \sum_{s=1}^{n} \frac{p_{rs} \cdot (a_{rs} - 1)}{1 + (1-\varepsilon)(a_{rs} - 1)} > 0. \end{cases} \tag{2}$$

In cases different from the above the optimal value of λ_r is determined by the non-negative roots of the equation:

$$\sum_{s=1}^{n} \frac{p_{rs}(a_{rs} - 1)}{1 + \lambda_r(a_{rs} - 1)} = 0 .$$

Proof:

From what we have assumed it follows by simple consideration that the aggregate capital of the production system after the N^{th} order can be determined from the following:

$$V_N = \prod_{r,s} (1 - \lambda_r + \lambda_r \cdot a_{rs})^{n_{rs}} \cdot V_0 . \tag{3}$$

Here n_{rs} stands for the co-occurrence of order, r, and delivery, s, under the time unit, N (i.e. the decision period). Let us write the goal function as

$$G = \lim_{N\to\infty} \frac{1}{N} \log \frac{V_N}{V_0} = \lim_{N\to\infty} \sum_{r,s} \frac{n_{rs}}{N} \log (1 - \lambda_r + \lambda_r \cdot a_{rs}). \tag{4}$$

As
$$\lim_{N\to\infty} \frac{n_{rs}}{n_r} \frac{n_r}{N} = \lim_{N\to\infty} \frac{n_{rs}}{N} = p(s/r) \cdot p_r = p_{rs},$$

because

$$\frac{n_{rs}}{n_r} \to p(s/r)$$

$$\frac{n_r}{N} \to p_r \quad \text{if} \quad N \to \infty$$

the goal function can be written

$$G = \sum_{r,s} p_{rs} \log (1 - \lambda_r + \lambda_r \cdot a_{rs}) \tag{5}$$

where p_{rs} means the bivariate probability distribution (of co-occurrence of order, r, and delivery, s).

Through the results obtained in (5) for the goal function, the decision problem described in point 2. is reduced to solving (6) which is a simple convex programming problem:

$$0 < \lambda_r < 1 - \varepsilon \tag{6}$$

$$\max_{\lambda_r} G.$$

Let us apply the Kuhn—Tucker saddle point theorem to this problem in the form of an equation system. Then the minimum requirement which satisfies, namely that a λ_r is optimal in the decision problem in point 2. must be equivalent with λ_r as the solution to the following system of equation:

$$v_r - \sum_{s=1}^{n} \frac{p_{rs}(a_{rs}-1)}{1+\lambda_r(a_{rs}-1)} - x_r = 0 \tag{7.1}$$

$$v_r - y_r = 0 \tag{7.2}$$

$$\lambda_r - \delta_r - 1 + \varepsilon = 0 \tag{7.3}$$

$$\lambda_r \cdot x_r = 0 \tag{7.4}$$

$$\lambda_r \cdot y_r = 0 \tag{7.5}$$

$$\lambda_r > 0,\ \delta_r > 0,\ v_r > 0,\ x_r > 0,\ y_r > 0, \tag{7.6}$$

where

δ_r, v_r, x_r és y_r are

variables.

After eliminating these variables, the following equation containing one unknown for λ_r is arrived at through simple calculation:

$$(1-\varepsilon-\lambda_r) \cdot \lambda_r \cdot \left(\sum_s \frac{p_{rs}}{A_{rs}+r} \right) = 0 \tag{8}$$

where

$$A_{rs} = \frac{1}{a_{rs}-1}.$$

As a solution, one of the following applies:

a) $\lambda_r = 1 - \varepsilon$

b) $\lambda_r = 0$

c) $\sum_s \frac{p_{rs}}{A_{rs}+\lambda_r} = 0.$

In any one of the above equations only those values for λ_r can be considered where the conditions of (7.6) were satisfied by the variables v_r, x_r, y_r.

Let us look at each case in turn:

a) For $\lambda_r = 1 - \varepsilon$

the following solution system is obtained:

$$v_r = \sum_{s=1}^{n} \frac{p_{rs}}{A_{rs} + 1 - \varepsilon} > 0, \quad y_r = \sum_{s=1}^{n} \frac{p_{rs}}{A_{rs} + 1 - \varepsilon} > 0.$$

From this it can be seen that the $\lambda_r = 1 - \varepsilon$ solution is optimal if. and only if

$$\sum_{s=1}^{n} \frac{p_{rs}}{A_{rs} + 1 - \varepsilon} = \sum_{s=1}^{n} \frac{p_{rs}(a_{rs} - 1)}{1 + (1 - \varepsilon) \cdot (a_{rs} - 1)} > 0 \tag{9}$$

condition is fulfilled.

b) For $\lambda_r = 0$,

then

$$v_r = 0, \; y_r = 0$$

$$\delta_r = 1 - \varepsilon > 0, \; x_r = -\sum_{s=1}^{n} p_{rs}(a_{rs} - 1) > 0.$$

As only this last inequality means a restriction, let us examine it further. As

$$\sum_{s=1}^{n} p_{rs}(a_{rs} - 1) = \sum_{s=1}^{n} p_{rs} \cdot a_{rs} - \sum_{s=1}^{n} p_{rs} = \sum_{s=1}^{n} p_{rs} \cdot a_{rs} - p_r.$$

so

$$p_r > \sum_{s=1}^{n} p_{rs} \cdot a_{rs} \tag{10}$$

is the sufficient condition for the optimality of $\lambda_r = 0$.

c) For the case of

$$\sum_{s=1}^{n} \frac{p_{rs}}{A_{rs} + \lambda_r} = 0$$

the optimum value of λ_r is given by the non-negative roots of this equation. Then also

$$v_r = 0, y_r = 0, x_r = 0$$

and

$$\delta_r = 1 - \varepsilon - \lambda_r \geqq 0.$$

So the optimum value of λ_r is given by the positive roots of

$$\sum_{s=1}^{n} \frac{p_{rs}(a_{rs} - 1)}{1 + \lambda_r(a_{rs} - 1)} = 0. \tag{11}$$

These roots also meet the inequality

$$\lambda_r < 1 - \varepsilon. \tag{12}$$

Considering the results of cases a) and b), the condition for all of this is the fulfillment of the two inequalities in the above thesis.

If we look at the goal function (1/a), which expresses the short-term practical case. then with the a priori information $\hat{n}_{rs}$ and $\hat{n}_r$ (which can be taken as accurate estimations the probability of the fulfillment, s, in the case of order quantity, r). the reliable characteristics of the production is

$$p(s/r) = \frac{\hat{n}_{rs} + n_{rs}}{\hat{n}_r + n_r},$$

$$p_r = \frac{\hat{n}_r + n_r}{\sum_r \hat{n}_r + N}$$

where

$$\sum_s \hat{n}_{rs} = \hat{n}_r, \quad \sum_s n_{rs} = n_r$$

can be described by the a posteriori subjective probabilities[1] obtained through the Bayes (deductive) method [5]. If the a priori information on the reliability of production, is not at our disposal we must take the following empirical probabilities (relative frequences):

$$p(s/r) = \frac{n_{rs}}{n_r}, \quad p_r = \frac{n_r}{N}.$$

In the following we shall use the term "probability" without indicating whether it is subjective or objective.

We are now going to deal with the problems of capital investment in a Production System and Market model, which can be considered ideally typical and derived from a general model. It also serves to illustrate practical cases.

Here the specific behaviour of the market can be derived simply from the general case through the following:

$$a_{rs} = \begin{cases} 0, & \text{if } r \neq s \\ a_r, & \text{if } r = s. \end{cases} \tag{13}$$

Thesis 2.

The optimum ratio of the short-run capital investment (in the production system), and the adequate decision strategy can be defined by the following equations:

[1] The ratio of $\sum_s n_r$ and N reflects the importance of the a priori and observed information relative to each other.

$$\lambda_r = \begin{cases} 1-\varepsilon, & \text{if } a_r p(r/r) - 1 \geqq (1-\varepsilon)\cdot(a_r-1) \\ 0, & \text{if } a_r p(r/r) \leqq 1 \\ \dfrac{a_r p(r/r)-1}{a_r-1} & \text{if } a_r p(r/r) > 1 \quad \text{and} \\ & a_r p(r/r) - 1 < (1-\varepsilon)\cdot(a_r-1). \end{cases} \tag{14}$$

The maximum average growth rate can be calculated by using the following equations:

$$G_{\max} = \sum_{r=1}^{n} (n-1)\log\varepsilon + \log(a_r + \varepsilon - a_r\,\varepsilon), \text{ if } \lambda_r = 1-\varepsilon \quad \text{condition a)} \tag{15}$$

$$G_{\max} = 0, \text{ if } \lambda_r = 0, \quad \text{condition b)}$$

$$G_{\max} = \sum_{r=1}^{n} p_r \log^2 \frac{a_r\,[1-p_r(r/r)]}{a_r-1} + \sum_{r=1}^{n} p_r\,p(r/r)\log^2\frac{p(r/r)\,(a_r-1)}{1-p(r/r)}, \text{ if}$$

$$\lambda_r = \frac{a_r p(r/r)-1}{a_r-1} \quad \text{condition c).}$$

Remark

If $a_{rs} = a_r$ for $r = s$ and $a_{rs} = b_r$ for $r \neq s (0 < b_r < 1)$ then the optimal ratio of short-run capital investment is $0 < \lambda_r < 1-\varepsilon$):

$$\lambda_r = \frac{a_r p(r/r) - b_r q(r/r) - 1}{(a_r-1)(1-b_r)}$$

where $q(r/r) + p(r/r) = 1$.

For the sake of simplicity of economical interpretation we discuss the case $b_r = 0$.

Proof

Let us examine the formula described in Thesis 1. when (13) is valid.

a) if $\lambda_r = 1-\varepsilon$

$$0 < \sum_{s=1}^{n} \frac{p_{rs}(a_{rs}-1)}{1+(1-\varepsilon)(a_{rs}-1)} = \frac{p_{rr}(a_r-1)}{1+(1-\varepsilon)(a_r-1)} -$$

$$- \sum_{\substack{s=1\\ s\neq r}}^{n} \frac{p_{rs}}{1-1+\varepsilon} = \frac{p_{rr}(a_r-1)}{1+(1-\varepsilon)(a_r-1)} - \frac{p_{rh}}{\varepsilon},$$

with the error probability

$$p_{rh} = \sum_{\substack{s=1 \\ s\neq 1}}^{n} p_{rs}.$$

Considering the case $p_{rh} \neq 0$, then

$$\frac{p_{rr}}{p_{rh}} = \frac{p_{rr}}{p_r - p_{rr}} = \frac{p(r/r)}{1 - p(r/r)}$$

and

$$1 < \frac{\varepsilon \dfrac{p(r/r)}{1 - p(r/r)} - (a_r - 1)}{1 + (1 - \varepsilon)(a_r - 1)} .$$

By reduction we arrive at the inequality in the thesis.

$$a_r p(r/r) - 1 > (1 - \varepsilon) \cdot (a_r - 1). \tag{14a}$$

The relationship where $\lambda_r = 0$ can be changed into

$$p_r > \sum_{s=1}^{n} p_{rs} \cdot a_{rs} = p_{rr} \cdot a_r$$

that is

$$p(r/r) \cdot a_r < 1. \tag{14b}$$

When the (14b) condition is fulfilled, $\lambda_r = 0$ is optimal. The following equation is to be solved:

$$0 = \sum_{s=1}^{n} \frac{p_{rs}(a_{rs} - 1)}{1 + \lambda_r(a_{rs} - 1)} = \frac{p_{rr}(a_r - 1)}{1 + \lambda_r(a_r - 1)} - \sum_{\substack{s=1 \\ s\neq 1}}^{n} \frac{p_{rs}}{1 - \lambda_r} =$$
$$= \frac{p_{rr}(a_r - 1)}{1 + \lambda_r(a_r - 1)} - \frac{p_{rh}}{1 - \lambda_r} .$$

Expressing λ_r, we have

$$\lambda_r = \frac{p_{rr} + p_{rh} - p_{rr} \cdot a_r}{p_{rr} + p_{rh} - a_r(p_{rr} + p_{rh})} = \frac{a_r p(r/r) - 1}{a_r - 1} . \tag{16}$$

The determinative inequality is

$$1 - \varepsilon > \lambda_r = \frac{a_r p(r/r) - 1}{a_r - 1} \tag{14c}$$

from which the condition in the thesis can be derived by reduction, if

$$a_r - 1 > 0.$$

For $G_{\max}$ optimal values can be simply derived through substitution of the λ_r values into the expression given.

3. Economic interpretation of results from the decision model

In this section the economic interpretation of the optimal investment ratios will be described, along the determining decision strategies.

The importance of the connections under examination lies in the fact that they make clear how market price conditions and the reliability of the production system affect optimal short-run investments (for instance production enlargements) in the socialist economy.

General case:

Let us adjust the connections in (2) to the economic interpretation of the formula for the general case.

$$\sum_{s=1}^{n} p(s/r)\, a_{rs} - 1 < 0 \tag{17}$$

$$\sum_{s=1}^{n} p(s/r) \frac{a_{rs}}{1+\lambda_r(a_{rs}-1)} = \sum_{s=1}^{n} p(s/r) \frac{1}{1+\lambda_r(a_{rs}-1)} . \tag{18}$$

The interpretation is as follows. The numerator on the left is the increase in the amount of capital after maximum capital investment and the denominator is the real increase in capital, derived by taking into account the restriction of investment. The numerator on the right is the original capital, and the denominator is the increase in capital after taking the restriction into account.

The optimum ratio can be determined by the equation of the expected values of the two quotients, where the order is r.

In other words, the ratio of capital to be invested into production is optimal if, and only if the equation (17) is valid for the ratio of the capital invested. Namely, the conditional mean of the proportion of profit ratio for the full (theoretical) capital investment and the profit ratio for the actual (real) capital investment is equal to the conditional mean of the proportion of profit ratio for zero "capital investment" and the profit ratio for the actual (real) capital investment (capital's "increase" in the case of zero "investment"). On the basis of this the most important interpretation is the following (for the case $0 < \lambda_r < 1 - \varepsilon$):

The choice of ratio λ_r is optimal if and only if the average ratio of loss originating from total capital investment, e.g. $\lambda_r = 1$ is equal to the average ratio of loss originating from zero capital investment, e.g. $\lambda_r = 0$.

Now, the relationship in (17) has to be described. It can be easily seen that this expresses the conditional mean of the profit quotient gained from a maximum order with an order r. If this is less than *0*, viz. no profit can be expected, then the decision strategy says that it is not worth investing. (Of course, situations can arise where investment is needed in spite of this.) If the conditional mean of profit is greater than 0, that is profit is expected, then the optimal proportion is given by one of the two cases above.

Special case:

Economic interpretation of the connections between the decision strategy and the optimum ratio of capital invested in a special case is, in fact, quite simple. These can be understood heuristically.

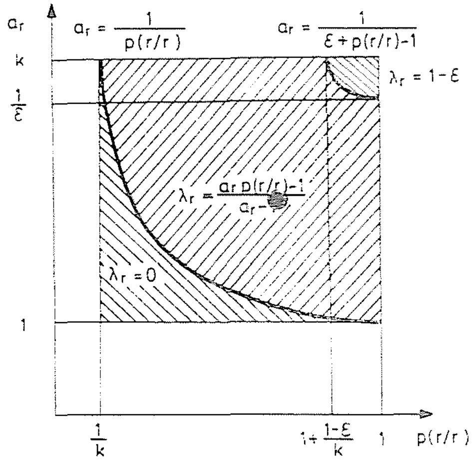

Fig. 1

According to decision strategy, it is not worth investing in production if the conditional mean of profit ratio is less than or equal to 0, namely, no profit is expected. If profit is expected (this value is grater than 0) but if, because of investment restriction, the maximum profit achievable is smaller than the profit with full capital investment, the optimum to be invested in production is the ratio of the expected value of the real profit and the maximum achievable profit, when there is full capital investment.

If the above decision condition is fulfilled contrary-wise, then it is worth investing all available capital into production. (This can be interpreted as the above condition is met only in the case of high production reliability.)

If the values for r and s are fixed when examining the decision strategy, then this same strategy determines the decision field, as illustrated in Fig. 1. (To make the demonstration more flexible, the restriction is included.)

In this case the optimal ratio of short-run capital investment (utilization of capacity) belonging to the given system reliability can be readily defined graphically with the help of these decision fields.

If a_r is given, depending on r order, then there will be three variables: so it can be illustrated by a decision just as has been done with the decision field.

If the value of a_r is fixed and is known for the production system, the choice of the optimal ratio of capital investment is illustrated by Fig. 2. The degree of reliability of the system must be taken into account.

Economic interpretation of the results obtained through these functions cannot be given by using the same methods as when the conditions and ratios for optimum capital investment were interpreted. To examine these connections, interpretation of the equations will be carried out in a manner differing from the previous methods in using the information theory. To do so, we need to construct a process of information theory adequate to the above decision process.

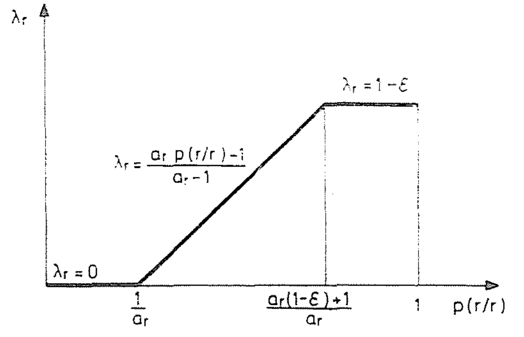

Fig. 2

4. Information theory interpretation of cost-oriented stochastic decision process in the simple Market and Production System model

With an information theory examination of the decision process, first we have to make clear the relationship between the flow of information and the flow of material on the one side, and the cost effect on the other.

We make the stochastic matrix containing conditional probabilities characteristic of the production system equal to the matrix in the information theory usage. In our examination, the j^{th} element of the row i^{th} of the noise matrix characteristic of the reliability of the production system is the conditional probability that the production system delivered goods of quantity j if the order was i.

According to this, in our model, the reliability of the production system is equivalent to a noisy channel without memory.

When the market demands are considered and the input signal x is the quantity of goods ordered by the market, and the output signal y is the information on the quantity of goods delivered by the production system. Thus by the information theory the reliability of the production system can be characterized by using the concept of channel (Fig. 3).

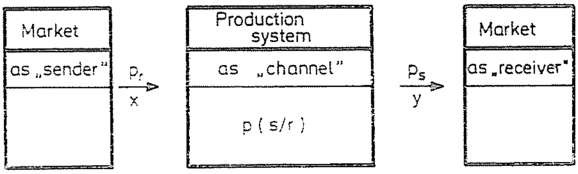

Fig. 3

It can be easily seen, that from the noisy matrix conditional probabilities of satisfied order with quantity r can be obtained

$$P(y = r/x = r) \tag{19}$$

and of unsatisfied order with quantity r,

$$P_{rH} = P(y \neq r/x = r) = \sum_{k=1}^{r-1} P(y = k/x = r), \tag{20}$$

e.g. P_{rH} is a conditional probability for the case when the production system does not totally satisfy the order of market with the quantity r of goods.

The probability that the production system does not meet the demands of the market in a given time period can be defined with the help of (20) through the theorem of total probability.

$$P_H = \sum_{r=1}^{n} P_r P_{rH}, \tag{21}$$

where P_H is the probability that the production system does not fully meet the demands of the market.

Let us examine the error probabilities, both conditionally and fully transferred, when applied to the examination of information transfer through noise channels. The conditional error probability means the probability, if the transfer (decoding) is in error when the r^{th} information is brought forward:

$$\sum_{\substack{k=1 \\ r \neq k}}^{n} P(y = k/x = r). \tag{22}$$

If the probability of the occurrence of the r^{th} information is p_r, then the error probability of the decoding is

$$\sum_{r=1}^{n} p_r \sum_{\substack{k=1 \\ r \neq k}}^{n} P(y = k/x = r). \tag{23}$$

The formal equality of (20), (21), (22) and (23) is evident.

Considering all this, the following can be stated: in case where the strategy condition (14c) is fulfilled, the maximum average increase growth rate of profit (total capital) can be obtained through

$$G_{\max} = H(a_r, p_r, p_{rH}) - H(\eta/x) \tag{24}$$

where

$$H(a_r \cdot p_r, p_{rH}) = \sum_{r=1}^{n} p_r \log a_r - \sum_{r=1}^{n} p_r p_{rH} \log (a_r - 1) \tag{25}$$

and $H(\eta/x)$ stands for the uncertainty of the system regarding the meeting of demand — expressed in conditional entropy — when the market order is known.

The η random variable can be defined as

$$\eta = \begin{cases} 1, & \text{if } x = y \\ 0, & \text{if } x \neq y. \end{cases} \tag{26}$$

and

$$H(\eta/x) = \sum_{r=1}^{n} p_r[(1 - p_{rH}) \log (1 - p_{rH}) + p_{rH} \log p_{rH}]. \tag{27}$$

If the optimal values for λ_r of (14) are substituted into (14c), the maximum value of the goal function will be:

$$G_{\max} = \sum_{r=1}^{n} p_r \log a_r + \sum_{r=1}^{n} p_r \log p_{rH} - \sum_{r=1}^{n} p_r \log (a_r - 1) +$$
$$+ \sum_{r=1}^{n} p_r p(r/r) \cdot \log p(r/r) - \sum_{r=1}^{n} p_r(1 - p_{rH}) \cdot \log (1 - p(r/r)) +$$
$$+ \sum_{r=1}^{n} p_r p(r/r) \cdot \log (a_r - 1).$$

By reduction and simple calculation we arrive at:

$$\sum_{r=1}^{n} p_r[\log a - \log (a_r - 1) + (1 - p(r/r)_r) \log (a_r - 1)] +$$
$$+ \sum_{r=1}^{n} p_r[(1 - p_{rH}) \log (1 - p_{rH}) + p_{rH} \log p_{rH}].$$

By using the definition of conditional entropy characteristic of the reliability of meeting the demands and by reduction the following equation is obtained:

$$\sum_{r=1}^{n} p_r[\log a_r - p_{rH} \log (a_r - 1)] - H(\eta/x) = H(a_r, p_r, p_{rH}) - H(\eta/x).$$

If a_r is independent of the quantity ordered, that is $a_r = a$ independent of r, (24) is modified into:

$$G_{\max} = H(a, p_H) - H(\eta/x), \tag{28}$$

where

$$H(a, p_H) = \log a - p_H \log (a - 1)$$

In the special case where a equals 2. or in other words. the market pays double of the amount for effective meeting of demand (the system operates on the basis of "double or quits"), the following can be arrived at from the previous function:

$$G_{max} = 1 - H(\eta/x). \tag{29}$$

We have been able to construct a model from measurements taken from information theory, viz. conditional entropy and error probability of decoding. It is interesting in the proved connections that we have constructed a model for the maximum average growth rate of capital in the production system. It can be seen that the reliability of the "service" system — as measured by entropy — presents itself as a cost factor; more precisely, the average growth rate of profit is characterized unambigously by system reliability as measured by conditional entropy.

If the order is of determining value r, we can obtain one of the equations in Kelly's simplified model, that is

$$G_{max} = 1 + (1 - p_{rH}) \log (1 - p_{rH}) + p_{rH} \log p_r = C, \tag{30}$$

where the above expression is equal to the capacity C of a binary symmetrical noise channel. (Though the general relationships given by Kelly are very interesting, they cannot be used in our study because they refer to the stochastic type multi-phased decision process of specific game theory.)

Outline

With the knowledge of the statistical characteristics that show uncertainty in the production of particular products, the model described in our paper can be applied further to other products. Here the optimal task is to assign capital at hand into the production of these particular products. Taking into consideration the statistical characteristics of reliability, this assignment is done in such a way that the average rate of profit should be maximal.

This problem leads to a more complicated problem in mathematics, the solution of which requires further research.

* Kelly's original problem is as follows: a gambler bets on the result of a game of chance for which he has already been given information. The probability that the information is true is p, and that it is false is 1-p.

If the gambler's original capital is V_0, his task is to maximize the average growth rate of profit with the best possible ratio of his capital. As he is not sure that the information given is completely reliable, at each bet he lays only a certain ratio of his capital. Then it can be proved that the optimum ratio of capital for the investment is $2p$-1 and the maximum average growth rate of the capital will be C by formula (30).

Bellman, Kalaba and, later, Murphy have applied the theory of dynamic programming to the Kelly model, and they have outlined the possibility of creating a model for decision processing with adaptive algorithms. These procedures can be applied to the field that we have examined.

Thus, the decision problem operated on by stochastic learning algorithms seems to be a promising development of the simple model given. An example would be the effect of the exertion of the increase in profit, in unknown market circumstances, in the case of increasing the reliability of the production system.

Summary

The paper deals with the creation of a simple stochastic model for short-run capital investment decisions of socialist enterprises. The connection between delivery's reliability and profit as well as the optimal capital growth process of socialist enterprises are investigated.

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