

# PRELIMINARY LOAD ANALYSIS OF COMMERCIAL VEHICLES

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## I. Introduction

Our previous studies outlined the main impacts on static and dynamic stresses in commercial vehicles (e.g. autobuses, trucks) [1, 2]. Simplifying assumptions were made to establish two-dimensional distribution function of stresses. Since practical damage theories require the knowledge of stress level intersection numbers, this paper will be dealt with their expected values.

## 2. Recapitulation of earlier test results

Equation of motion of a linear system of discrete mass points and rigid bodies is described by the differential equation [3]:

$$\mathbf{M} \ddot{\mathbf{y}} + \mathbf{K} \dot{\mathbf{y}} + \mathbf{S} \mathbf{y} = \mathbf{G} \mathbf{f}_v^h(t) + \mathbf{D} \dot{\mathbf{f}}_v^h(t)$$

where  $\mathbf{M}$  — mass matrix comprising point-like masses and adequately transformed principal moments of inertia;

$\mathbf{K}$  — quadratic symmetric matrix comprising dampings acting on mass points (rigid bodies);

$\mathbf{S}$  — stiffness matrix;

$\mathbf{D}$  — damping matrix applied on the road surface as constraint coordinates due to vehicle components (tyres);

$\mathbf{G}$  — stiffness matrix applied on the road surface as constraint coordinates due to vehicle components (tyres);

$\mathbf{y}$  — vector of coordinates of the vertical displacement of discrete mass points (rotation of rigid bodies around the centroid);

$\mathbf{f}_v^h(t)$  — function of excitation by speed  $v$  of a road type  $h$  vs. time.

In the general case where the vehicle is affected by a four-input excitation of random type for the vehicle wheels (Fig. 1), it is rather difficult to determine the spectral density matrix of process  $\mathbf{f}_v^h(t)$  because of the generally unknown cross correlations between road profiles passed by the left and the

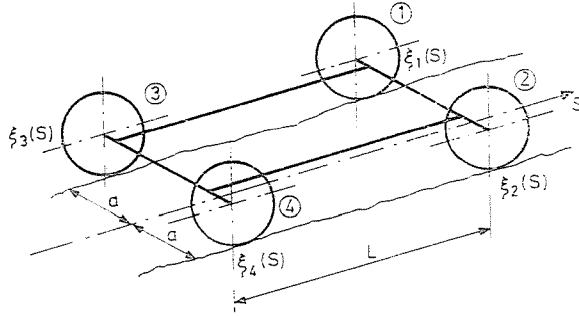


Fig. 1

right side of the vehicle. Robson [4] suggested the simplifying assumption of road surface isotropy and the further simplification of fully identical realization of right and left wheels (permitting the road to be considered as orthotropic). Now, process  $f_p^h(t)$  becomes, with symbols in Fig. 2:

$$f_p^h(t)^T = \left[ \xi_v^h(t), \xi_v^h(t), \xi_v^h\left(t - \frac{L}{v}\right), \xi_v^h\left(t - \frac{L}{v}\right) \right]^T =$$

$$= [\xi^h(v \cdot t), \xi^h(v \cdot t), \xi^h(v \cdot t - L), \xi^h(v \cdot t - L)]$$

where  $L$  — wheel base:

$\xi_v^h(t)$  — value of the road unevenness at  $s$  (transformed to time function at constant speed  $v$ ).

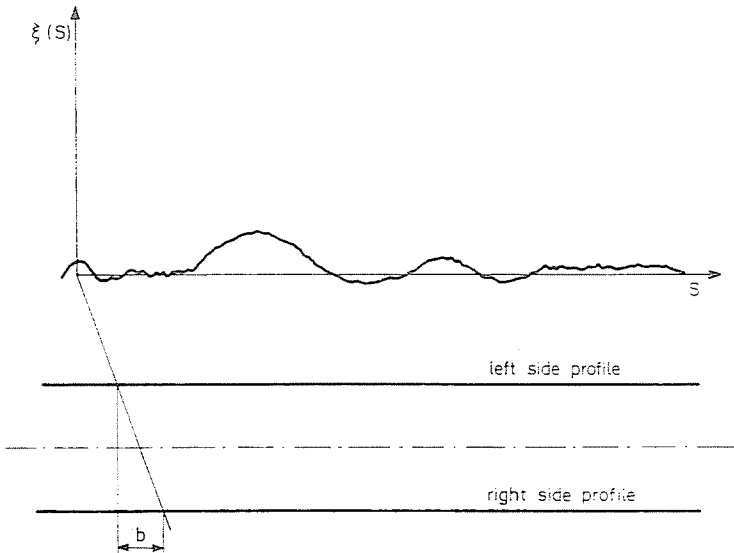


Fig. 2

Now, spectral density matrix  $\Phi_v^h(\nu)$  of process  $f_v^h(t)$  becomes:

$$\Phi_v^h(\nu) = \frac{1}{v} \cdot \Phi^h\left(\frac{\nu}{v}\right) \begin{bmatrix} 1 & 1 & e^{-i\nu\frac{L}{v}} & e^{-i\nu\frac{L}{v}} \\ 1 & 1 & e^{-i\nu\frac{L}{v}} & e^{-i\nu\frac{L}{v}} \\ e^{i\nu\frac{L}{v}} & e^{i\nu\frac{L}{v}} & 1 & 1 \\ e^{i\nu\frac{L}{v}} & e^{i\nu\frac{L}{v}} & 1 & 1 \end{bmatrix}$$

where  $\Phi_v^h(\nu)$  is the power density spectrum of the Gaussian process  $\xi^h(s)$  of zero expected value, and  $\nu = 2\pi f$  is the circular frequency. A somewhat complex but still easily treated spectral density matrix results from the assumption of a "road of clinotropic realization". Clinotropy of a road results from the relative shift of realizations assumed to be equal between right and left sides. In this case — for a shift "b" — vector process  $f_v^h$  becomes:

$$\begin{aligned} f_v^h(t)^T &= \left[ \xi_v^h(t), \xi_v^h\left(t - \frac{b}{v}\right), \xi_v^h\left(t - \frac{L}{v}\right), \xi_v^h\left(t - \frac{L}{v} - \frac{b}{v}\right) \right] = \\ &= [\xi^h(v \cdot t), \xi^h(v \cdot t - b), \xi^h(v \cdot t - L), \xi^h(v \cdot t - L - b)] \end{aligned}$$

and the spectral density matrix:

$$\Phi_v^h = \frac{1}{v} \Phi^h\left(\frac{\nu}{v}\right) \begin{bmatrix} 1 & e^{-i\nu\frac{b}{v}} & e^{-i\nu\frac{L}{v}} & e^{-i\nu\frac{L+b}{v}} \\ e^{i\nu\frac{b}{v}} & 1 & e^{-i\nu\frac{L-b}{v}} & e^{-i\nu\frac{L}{v}} \\ e^{i\nu\frac{L}{v}} & e^{i\nu\frac{L-b}{v}} & 1 & e^{-i\nu\frac{b}{v}} \\ e^{i\nu\frac{L+b}{v}} & e^{i\nu\frac{L}{v}} & e^{i\nu\frac{b}{v}} & 1 \end{bmatrix}$$

Taking them into consideration, spectrum  $\Phi_{v,z}^{h,y}(\nu)$  of the steady-state Gaussian solution of zero expected value of the original equation of motion under arbitrary — but fixed — conditions of load  $z$ , speed  $v$  and fixed road category, is obtained from:

$$\begin{aligned} \Phi_{v,z}^{h,y}(\nu) &= [\mathbf{S} - \mathbf{M}\nu^2 + \mathbf{K}i\nu]^{-1}(\mathbf{G} + \mathbf{D}i\nu) \Phi_v^h(\nu) \times \\ &\times (\mathbf{G} + \mathbf{D}i\nu)^* [\mathbf{S} - \mathbf{M}\nu^2 + \mathbf{K}i\nu]^{-1*} \end{aligned}$$

where the matrix in brackets has always an inverted, \* and is the symbol of conjugated transposition.

Displacement  $y(t)$  being a steady-state Gaussian process of zero expected value, also process

$$\mathbf{F}(t)^T = [F_1(t), F_2(t), \dots, F_n(t)] = \mathbf{L}\mathbf{M}\tilde{y}(t)$$

describing the dynamic stress at different tested places will be a steady-state Gaussian process of zero expected value, with a spectrum of the form [5]:

$$\Phi_{v,z}^{h,F}(v) = v^4 \mathbf{L} \mathbf{M} \Phi_{v,z}^{h,y}(v) \mathbf{M}^T \mathbf{L}^T$$

where  $\mathbf{L}$  is the matrix of stresses due to unit mass forces.

Although the response spectrum permits to calculate the stress distribution function, from the aspects of service life design and of fatigue test programs, it is more expedient to establish the distribution function of separate stress level intersection numbers. Spectrum of component  $F_d(t)$  of vector  $\mathbf{F}(t)$  will be element  $(d, d)$  of matrix  $\Phi_{v,z}^{h,F}(v)$ , denoted for the sake of simplicity by  $\varphi_d(v)$ . Then, according to [6]:

$$\tilde{N}_{h,v,z}^{d,u} = \frac{1}{\pi} \left[ \frac{\int_0^\infty v^2 \varphi_d(v) dv}{\int_0^\infty \varphi_d(v) dv} \right]^{\frac{1}{2}} \times \exp \left[ -\frac{u^2}{2 \int_0^\infty \varphi_d(v) dv} \right]$$

where  $\tilde{N}_{h,v,z}^{d,u}$  — expected number of intersections of level  $u$  by process  $F_d(t)$  during unit time, yielding the number of level inter-sections per unit road as:

$$N_{h,v,z}^{d,u} = \frac{1}{v} \tilde{N}_{h,v,z}^{d,u}.$$

### 3. Determination of the number of dynamic load level intersections in a variable mode of operation

#### 3.1. Examination of different modes of operation

Opposite private cars, measurement possibilities for commercial vehicles (autobus, truck) have their economy limits, imposing to design on the basis of information either obtained in the operation of earlier, similar vehicles, or available independent of the vehicle to be designed (e.g. statistics on national road network, loads and driving speeds) [7].

Be  $H_h$  ( $h = 1, 2, \dots, H$ ) the possible road categories of an arbitrary high but finite number. Of course, only "standard" (concrete, asphalt, macadam etc.) roads are of practical importance (Fig. 3).

The operating vehicle is in various static load states, approximated by the following assumption: let variable mass matrix  $\mathbf{M}$  of the vehicle decomposed to

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{M}_i$$

where

- $M_0$  — diagonal matrix corresponding to unloaded state;  
 $M_i = z \cdot A$  — diagonal matrix corresponding to the load, to be calculated from load state  $z$  and diagonal matrix  $A$  typical of the system (an assumption involving proportional load distribution according to coordinates).

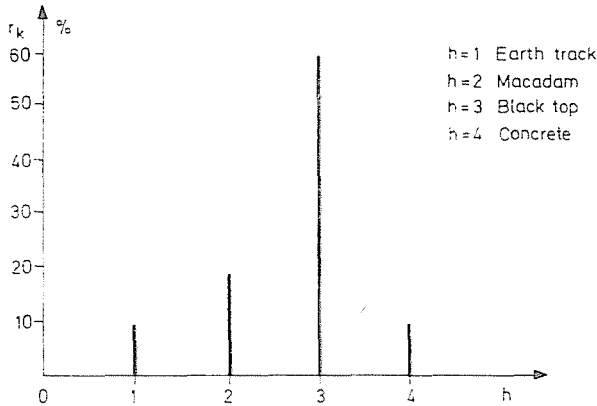


Fig. 3

Be  $l$  ( $l = 1, 2, \dots, n$ ) the possible vehicle operation modes. In the actual case,  $n = 3$  will be restricted to and distinguished.

$m_1 =$  interurban;

$m_2 =$  urban;

$m_3 =$  hill climb

being the fundamental modes of operation.

(By the way, processes of e.g. loading, unloading, etc. could be considered as separate modes of operation, not to be considered here.)

Different modes of operation involving basically different relations between road, speed and load realizations have to be expounded before deductions.

#### a) *Interurban mode of operation*

Much of the travel of a vehicle in interurban traffic can be decomposed into lengths passed at different constant speeds. Speed changes affect unimportant travel percentages and their effect soon decays. Thus, these transient effects are negligible.

Decomposing the vehicle travel into homogeneous lengths  $S$  belonging throughout to the same road category  $h = h(S)$ , these can be stated to involve

— conditionally independent realizations of road profile  $\xi(s)$  and speed  $v(s)$  for given  $S$  and  $h(S)$ , a self-intended assumption since the driver selects the speed in conformity to the road category without sensing its concrete realization. (This concept is different in its principle from examinational considerations by Jánosdeák [8]);

— Load state realization  $z(s)$  is independent of realizations of road profile  $\xi(s)$  and  $v(s)$ , an assumption valid only in case of a sufficient motor power. Accordingly, the combined distribution function of load and speed (in a mode of operation  $l = 1$ ) becomes:

$$F_{v,z}^{l,h}(v, z) = F_v^{l,h}(v, z) = F_v^{l,h}(v) \cdot F_z^{l,h}(z)$$

with relative frequencies of load state and speed as seen in Figs 4 and 5.

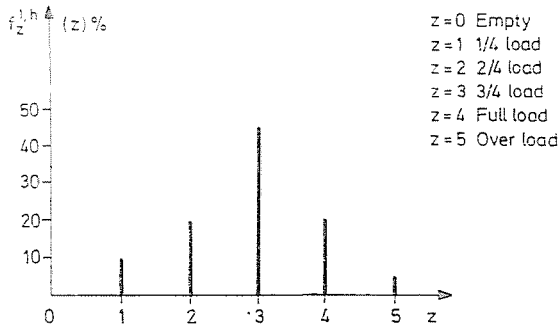


Fig. 4

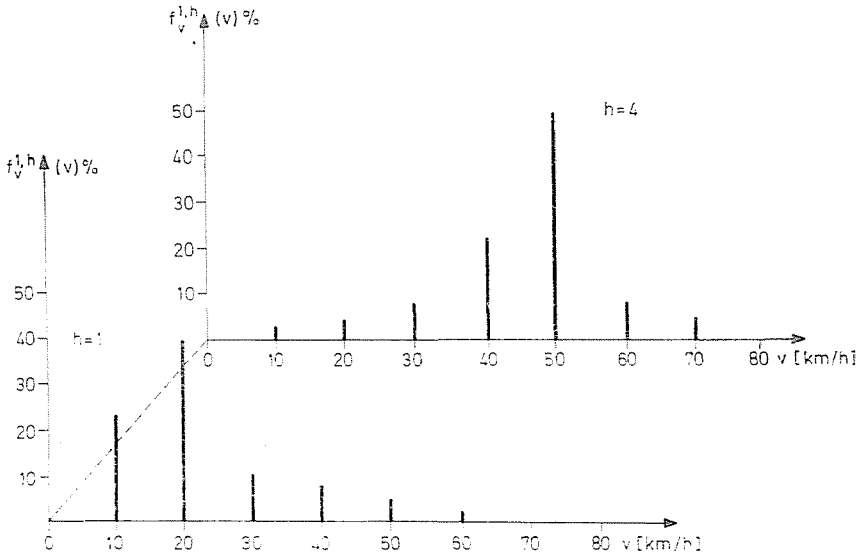


Fig. 5

### b) Urban mode of operation

In urban traffic, the traffic rhythm — or, for an autobus, the timetable stops — compel the driver to traffic speeds with specified upper limits or below, irrespective of the pavement quality. Speed selection is also independent of the subjective feeling of the driver, since the traffic prevails over other (biomechanical etc.) effects. At the same time, frequent changes of traffic conditions (traffic lights, pedestrian crossings etc.) impose vehicle speed variations in the greatest part of the road. As a conclusions,

- speed realization  $v(s)$  is independent of the road profile realization  $\xi(s)$ , and its distribution can be considered as identical for all road categories (Fig. 6):
- load state  $z(s)$  is anyway independent of realizations  $v(s)$  and  $\xi(s)$ . Its distribution function is  $F_z^{2,h}(z)$ .

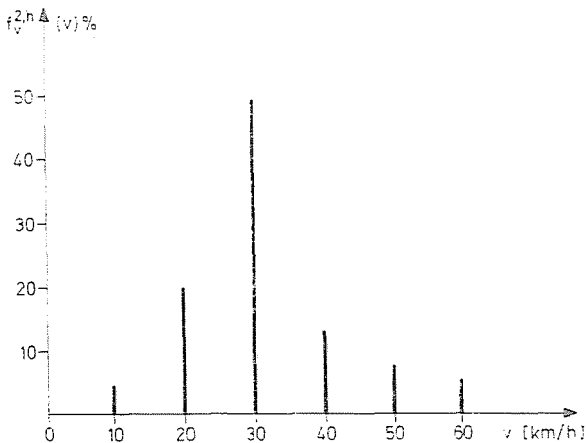


Fig. 6

### c) Hill climbing mode of operation

In hill climbing, consecutive gradients, slopes, curves force the driver to much lower speeds than that corresponding to the road category or the vehicle comfort. This is sometimes enhanced by the limited motor power, again imposing lower speeds in certain load states.

These statements lead to the conclusion that the speed is essentially determined by the road geometry and the load state, irrespective of the road profile:

- load process  $z(s)$  is perfectly independent of the process  $\xi(s)$  rather than of the process  $v(s)$ . Their combined distribution is of the type seen in Fig. 7. Actually, however, no data on the complex distribution function

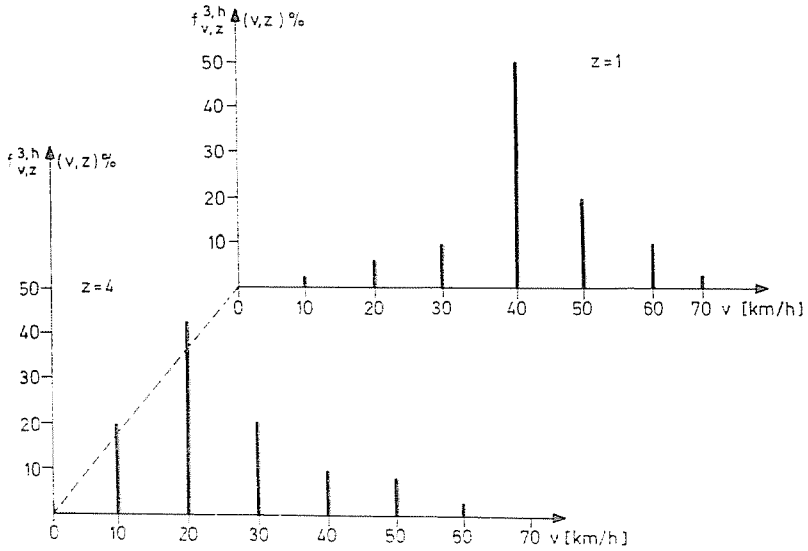


Fig. 7

of load and speed are available, therefore our calculations will assume — as a first approximation — independence, hence illimited motor power:

$$F_{v,z}^{3,h}(v,z) = F_v^{3,h}(v) \cdot F_z^{3,h}(z).$$

### 3.2 Determination of the number of stress level intersections

Further analyses will involve the following notations:

Be  $S_k$  ( $k = 0, 1, 2, \dots$ ) consecutive maximum road lengths with constant mode of operation  $l_k$  and road category  $h_k$ . To simplify notations, in the following, the pair of numbers  $(l, h)$  will be identified by number  $A = (l-1) \cdot H + h$  ( $A = 1, 2, \dots, 3H = H^*$ ). Accordingly, value  $A_k = (l_k - 1) \cdot H + h_k$  will be assigned to interval  $S_k$ .

Let us introduce the vehicle state process  $A(S)$ ,  $S \geq 0$  such that:

$$\begin{aligned} \zeta &= 0, \quad \zeta_k = S_0 + S_1 + \dots + S_{k-1} \text{ for } k \geq 1; \\ A(S) &= A_k \text{ for } \zeta_{k-1} \leq S < \zeta_k, \quad k \geq 1. \end{aligned}$$

The following assumptions will be made: Irrespective of its history, once the vehicle has got in state  $i$ , it passes to state  $j$  at a probability  $P_{ij}$  ( $1 \leq i, j \leq H^*$ ) so that the time passed by the vehicle in state  $i$  is a random variable of distribution  $F_{ij}(x)$ , or formulated:



$$(1) \quad \left\{ \begin{array}{l} P\{A(\zeta_{k+1}) = j \mid A(\zeta_k) = i, A(\zeta_{k-1}) = i_{k-1}, \dots, A(\zeta_0) = i_0\} = \\ = P[A(\zeta_{k+1}) = j \mid A(\zeta_k) = i] = P_{ij} \\ \text{and} \\ P\{S_k < x \mid A(\zeta_{k+1}) = j, A(\zeta_k) = i_k\} = F_{ij}(x) \end{array} \right.$$

for any  $k \geq 0$  and  $1 \leq i, j, i_0, \dots, i_{K-1} \leq H^*$ .

Be the starting state  $A(\zeta_0) = A(o)$  of a distribution:

$$(2a) \quad \left\{ \begin{array}{l} P[A(o) = i] = a_i; \quad i = 1, 2, \dots, H^*, \\ a_i \geq 0; \quad \sum_{i=1}^{H^*} a_i = 1 \end{array} \right.$$

and be

$$(2b) \quad P\{S_0 < x \mid A(o) = i, A(\zeta_1) = j\} = F_{ij}^0(x) \leq F_{ij}(x)$$

Process  $A(S)$ ,  $S \geq 0$  defined by (1) and (2) is used to be called a semi-Markovian process, and the process  $A(\zeta_k)$ ,  $k \geq 0$  an embedded Markov chain.

Some other assumptions, of no practical restriction, will be made on the model:

(3) — Semi-Markovian process  $A(S)$ ,  $S \geq 0$  is a regular one (in fact, with no restriction, the assumption can be made that there is a constant  $c > 0$  such that for all  $k \geq 0$ ,  $S_k \geq c$  is met at a probability of one.

$$(4) \quad \mu_{ij} = \int_0^{\infty} F_{ij}(x) dx < \infty; \quad 1 \leq i, j \leq H^*.$$

(5) — Embedded Markov chain  $A(\zeta_k)$ ,  $k \geq 0$  is irreducible.

(6) — Road profile, speed and load state realizations over road length  $S_k$ , only statistically determined by  $l_k$  and  $h_k$ , are independent of the  $S_k$  value and of former realizations (described for each mode of operation for fixed  $l_k$  and  $h_k$ ).

Let us denote consecutive road lengths with  $A(\zeta_k) = i$ ,  $A(\zeta_{k+1}) = j$  in the set  $S_k$  ( $k = 1, 2$ ) by  $S_{\alpha}^{ij}$ , ( $\alpha = 1, 2, \dots$ ), of them a number  $\psi_{ij}(S)$  is within interval  $(0, S)$ .

Condition (1) involves  $S_{\alpha}^{ij}$  to be a set of random variables of a distribution  $F_{ij}(x)$ .

Let us denote the number of  $u$ -level intersections along an (infinite) interval  $S_{\alpha}^{ij}$  by  $Q_{\alpha}^{ij}(u)$ , and the expected number of  $u$ -level intersections over unit road length by  $N_i^u$ ,  $1 \leq i \leq H^*$ , provided the vehicle travels in state  $i$ .  $S_{\alpha}^{ij}$  being a set of independent random variables of the same distribution, (6) causes also  $Q_{\alpha}^{ij}(u)$  to be a set of independent random variables of identical distribution.

Obviously, conditions (4) and (6) cause equality

$$EQ_{\alpha}^{ij}(u) = \mu_{ij} N_i^u$$

to hold.

Because of conditions (1), (3) to (5), for any initial condition (2) the rule of high numbers applied on Markovian and semi-Markovian processes causes the affirmations

$$\lim_{S \rightarrow \infty} \frac{\psi_{ij}(S)}{S} = \psi_{ij} < 1, \infty$$

to hold at a probability of 1.

Hence:

$$\lim_{S \rightarrow \infty} \frac{1}{S} \sum_{z=1}^{v_{ij}(S)} S_z^{ij} = \lim_{S \rightarrow \infty} \frac{\psi_{ij}(S)}{S} \cdot \frac{1}{\psi_{ij}(S)} \sum_{z=1}^{v_{ij}(S)} S_z^{ij} = \psi_{ij} \mu_{ij}$$

thus, relative frequency of road lengths passed in state  $i$ :

$$\lim_{S \rightarrow \infty} \sum_{j=1}^{H^*} \sum_{z=1}^{v_{ij}(S)} S_z^{ij} = \sum_{j=1}^{H^*} \psi_{ij} \mu_{ij}.$$

Be this relative frequency denoted by  $q_i$  (or  $q_{l,h}$  for  $l$  and  $h$  corresponding to state  $i$ ), that is:

$$q_i = \sum_{j=1}^{H^*} \psi_{ij} \mu_{ij}.$$

leading to:

$$\begin{aligned} \lim_{S \rightarrow \infty} \frac{1}{S} \sum_{j=1}^{H^*} \sum_{z=1}^{v_{ij}(S)} Q_z^{ij}(u) &= \sum_{j=1}^{H^*} \lim_{S \rightarrow \infty} \frac{\psi_{ij}(S)}{S} \cdot \frac{1}{\psi_{ij}(S)} \cdot \sum_{z=1}^{v_{ij}(S)} Q_z^{ij}(u) = \\ &= \sum_{j=1}^{H^*} \psi_{ij} \cdot \mu_{ij} N_i^u = q_i \cdot N_i^u \end{aligned}$$

and, taking also the varying mode of operation into consideration:

$$N^u = \lim_{S \rightarrow \infty} \frac{1}{S} \sum_{i=1}^{H^*} \sum_{j=1}^{H^*} \sum_{z=1}^{v_{ij}(S)} Q_z^{ij}(u) = \sum_{i=1}^{H^*} q_i N_i^u.$$

Back again at the original meaning of  $i$  (with corresponding  $l$  and  $h$  values):

$$N^u = \sum_{l=1}^3 \sum_{h=1}^H q_{l,h} N_{l,h}^u.$$

Let us consider now the determination of the expected number  $N_{l,h}^u$  ( $N_i^u$ ) of intersections of level  $u$  along unit road length for each mode of operation and road category.

Vehicle travel being somewhat similar in interurban and hill climb traffic, these cases will be discussed together. Be  $l = 1$  or  $l = 3$  and  $1 \leq h \leq H$  by arbitrary, to be fixed in the following. Let us take an interval  $(0, S)$  passed by the vehicle all along in state  $(l, h)$  for road, speed and load state realizations  $\xi(s)$ ,  $v(s)$  and  $z(s)$ , respectively. In conformity with assumptions made in

connection with modes of operation  $l = 1$  and  $l = 3$ , path  $S$  can be decomposed — with a slight neglect — to part intervals  $S_1^*$ ,  $S_2^*$ ,  $\dots$ ,  $S_{n^*}^*$  (of random length and number) where speed and load state take constant values  $v_\beta^*$ ,  $z_\beta^*$  ( $\beta = 1, 2, \dots, n^*$ ) and where the vehicle is in steady motion.

Be the numbers of intersections of level  $u$  in intervals  $(0, S)$  and  $S_\beta^*$  denoted by  $Q^*(u)$  and  $Q_\beta^*(u)$ ,  $\beta = 1, \dots, n^*$ , respectively. Obviously:

$$Q^*(u) = \sum_{\beta=1}^{n^*} Q_\beta^*(u).$$

Relying on statements on the modes of operation  $l = 1, 3$ :

$$\begin{aligned} N_{l,h}^u &= E \frac{Q^*(u)}{S} = \frac{1}{S} E \left\{ E \left[ \sum_{\beta=1}^{n^*} Q_\beta^*(u) \mid S_\beta^*, v_\beta^*, z_\beta^* \right] \right\} = \\ &= \frac{1}{S} E \left\{ \sum_{\beta=1}^{n^*} S_\beta^* N_{l,h}^u(v_\beta^*, z_\beta^*) \right\} = \frac{1}{S} E \int_0^S N_{l,h}^u(v(s), z(s)) ds = \\ &= \frac{1}{S} \int_0^S E \{ N_{l,h}^u(v(s), z(s)) \} ds = \\ &= \frac{1}{S} \int_0^S \left\{ \int_0^\infty \int_0^\infty N_{l,h}^u(x, y) \cdot F_v^{l,h}(dx) \cdot F_z^{l,h}(dy) \right\} ds = \\ &= \int_0^\infty \int_0^\infty N_{l,h}^u(x, y) \cdot F_v^{l,h}(dx) \cdot F_z^{l,h}(dy). \end{aligned}$$

Let us determine now  $N_{2,h}^u$ . Be  $1 \leq h \leq H$  an arbitrary road category to be fixed below. The vehicle is assumed to travel throughout in state 2,  $h$ , an assumption involved first in the determination of the expected number of level intersections  $\tilde{N}_{2,h}^u$  in unit time to obtain  $N_{2,h}^u$ . Let the vehicle travel along road profile realization  $\xi(s)$  in load state realization  $z(s)$  at speed realization  $\tilde{v}(t)$  in the function of time (again assumed to be steady in a restricted meaning).

Composition of the time scale will ignore stoppage times (reducing the complete stoppage interval to a point).

In conformity with the assumptions, the three processes are mutually independent, and steady in a restricted sense. For a travel  $s(t)$  laid back in time  $t$ , obviously  $s(t) = \int_0^t \tilde{v}(x) dx$ . Clearly,  $s(t)$  is a process of steady increment.

Let us consider now the road profile defined by:

$$\tilde{\xi}(t) = \xi(s(t)).$$

Now, in conformity with items 1. and 2. in [9], also process  $\tilde{\xi}(t)$  will be a steady one, and since  $\tilde{\xi}(s)$  is a Gaussian process of zero expected value, also  $\tilde{\xi}(t)$  will be such.

Its covariance function is:

$$R_{\tilde{\xi}}(\tau) = \int_0^{\infty} R^h(x) F_{s(\tau)}(dx)$$

where  $F_{s(\tau)}(x)$  is the distribution function of random variable  $s(\tau)$ . The spectrum of process  $\tilde{\xi}(t)$  is given by:

$$\Phi_{\tilde{\xi}}(v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iv\tau} \left\{ \int_0^{\infty} R^h(x) F_{s(\tau)}(dx) \right\} d\tau.$$

In knowledge of the spectrum of process  $\tilde{\xi}(t)$ , the spectrum of excitation process  $\tilde{f}^h(t)$  or of the process of dynamic stress  $F(t) = \mathbf{LM}y(t)$  — assuming a road of orthotropic realization — for a load state  $z$ , is given by:

$$\begin{aligned} \Phi_z^{h,F}(v) &= \mathbf{LM} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iv\tau} \left\{ \int_0^{\infty} R^h(x) F_{s(\tau)}(dx) \right\} d\tau \times \\ &\times v^4 [\mathbf{S} - (\mathbf{M}_0 + z\mathbf{A})v^2 + \mathbf{K}iv]^{-1} (\mathbf{G} + \mathbf{D}iv) \begin{bmatrix} 1 & 1 & e^{-ivL} & e^{-ivL} \\ 1 & 1 & e^{-ivL} & e^{-ivL} \\ e^{ivL} & e^{ivL} & 1 & 1 \\ e^{ivL} & e^{ivL} & 1 & 1 \end{bmatrix} \times \\ &\times (\mathbf{G} + \mathbf{D}iv)^* [\mathbf{S} - (\mathbf{M}_0 + z\mathbf{A})v^2 + \mathbf{K}iv]^{-1*} \end{aligned}$$

with notations as before. Relationships by S. Rice permit to calculate the expected number  $\tilde{N}_{2,h}^u(z)$  of intersections of level  $u$  in unit time, for a fixed load state  $z$ .

Be the  $u$ -level intersection numbers over a length  $S$ , and during time  $t(s)$ , denoted by  $Q(s)$  and  $Q^*(t(s))$ , respectively. Obviously,  $Q(s) = Q^*(t(s))$ . Then the expected number of  $u$ -level intersections over unit road length (for fixed  $z$ ) becomes [10]:

$$N_{2,h}^u(z) = E \frac{Q(s)}{S} = E \frac{Q^*(t(s))}{S} = \tilde{N}_{2,h}^u \cdot \frac{1}{S} E \int_0^S \frac{1}{v(x)} dx$$

where

$$\frac{1}{S} E \int_0^S \frac{1}{v(x)} dx = \int_0^{\infty} \frac{1}{x} F_v(dx)$$

reciprocal of mean velocity  $\bar{v}$  being.

Ignoring mass change due to fuel consumption, load-state process  $z(s)$  will be length-wise constant, hence, making use of the independence and statements on modes of operation  $l = 1, 3$ , the expected number of  $u$ -level intersections in unit road length becomes:

$$N_{2,h}^u = \frac{1}{v} \int_0^{\infty} \tilde{N}_{l,h}^u(x) F_z^{2,h}(dx)$$

### Summary

The expected number of intersections of the stress level  $u$  has been established for varying modes of operation.

Its knowledge permits to compose the full load complex, hence, applying an adequate damage theory, the service life may be predicted.

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