FORTRAN PROGRAM FOR MODES OF VIBRATION OF STRUCTURES

By

E. Abd El Fattah

Department of Mathematics, Technical University, Budapest

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1. Introduction

In high buildings subject to undeterministic or stochastic lateral loads such as wind loads, earthquakes and blasts, it is essential to determine the modes of vibration and the corresponding shapes and frequencies. The classical method is to solve a polynomial of n roots, where n is the number of degrees of freedom of the structure in the horizontal direction. This method is very labour- and time-consuming and leads to relatively high percent of error in the numerical operation by the computer.

The iteration procedure or the power method for determining the modes of vibration is the shortest one giving only the required numbers of modes and not all the unimportant ones. In many structures it is sufficient to use only the first three or four modes.

Thus in this paper a complete FORTRAN PROGRAM which gives the required number of modes is presented. The program depends on using the flexibility matrix of the structure to obtain the lowest modes.

For each mode the program gives the shape of the vibrating structure and the corresponding frequency.

The program was tested and proved to be accurate. The level of accuracy can be controlled by a given numerical value in the program. A numerical example is given here for explanation. It is important to mention that the results of the given FORTRAN program are very useful in the modal analysis of structures for determining the response under variable loads.

2. Motion equations of free vibration of structures

Consider that the masses of the structure are concentrated at the floor levels and that they are denoted by $m_1, \ldots, m_n$ where n is the number of stories.

Since the fundamental and the nearest to it modes are the most important ones it is essential to use the flexibility matrix of the frame structure to obtain only the first three or four lowest modes.

The forces affecting the horizontal motion are the inertia forces in addition to the internal forces.

If the lateral deflection of the plane frame at floor levels are denoted by $y_1, y_2, \ldots, y_n$, these deflections are given by the following free motion equations.

$$\begin{aligned} y_1 &= a_{11}(m_1 w^2 y_1) + a_{12}(m_2 w^2 y_2) + \ldots + a_{1n}(m_n w^2 y_n) \\ y_2 &= a_{21}(m_1 w^2 y_1) + a_{22}(m_2 w^2 y_2) + \ldots + a_{2n}(m_n w^2 y_n) \\ &\vdots \\ y_n &= a_{n1}(m_1 w^2 y_1) + \ldots \qquad a_{nn}(m_n w^2 y_n) \end{aligned}$$

In matrix form they are

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} = w^2 \begin{bmatrix} a_{11} & m_1 \ldots & \ldots & a_{1n} & m_n \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ a_{n1} & m_1 \ldots & \ldots & a_{nn} & m_n \end{bmatrix} \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}.$$

The smallest frequency and the associated mode shape can be obtained by the iteration procedure assuming a shape

$$y_1 = y_2 = \quad = y_n = 1.0$$

By using these values in the right-hand side a new improved mode is got in the left-hand side. Using the new values in the the right-hand side and by repeating the above procedure, a correct shape of the fundamental mode and its frequency is obtained.

The second mode can be obtained by using the orthogonality condition between the first two different modes and by substituting the values of the fundamental mode we obtain an equation in $y_1, y_2, \ldots, y_n$. Solving this equation with the set of motion equations we obtain a new set of equations equal to $(n - 1)$ equations.

These equations are treated by the iteration procedure in order to get the shape of the second mode and its frequency. The third mode can also be obtained by the same described steps and so on. The important orthogonality conditions are represented by a set of equations in this form.

$$m_1 y_1 y_1' + m_2 y_2 y_2' + \ldots m_n y_n y_n' = 0$$

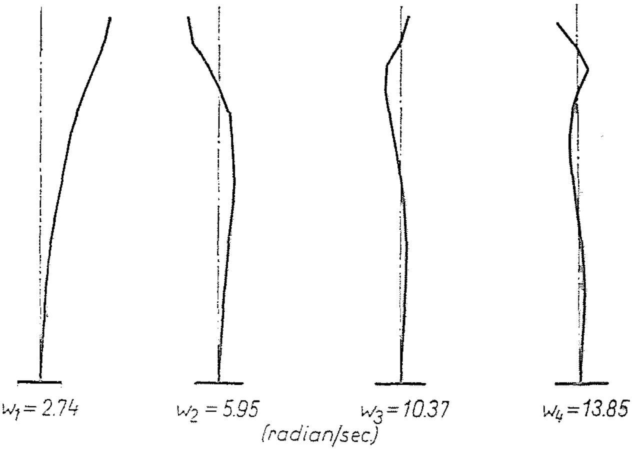

Fig. 1. The first modes of vibration of the given example

where $\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \begin{bmatrix} y'_1 \\ \cdot \\ \cdot \\ \cdot \\ y'_n \end{bmatrix}$ are any two different mode vectors.

3. The fortran program

The described steps were arranged in a FORTRAN program. The input data consist of the flexibility matrix and the masses of the floors.

The output data are vectors of the required lowest modes of vibration and the corresponding frequencies. The program consists of three segments; the main program and two subroutines. One subroutine is applied for the iteration procedure and the other for the solution of linear equation systems given by the orthogonality condition. The Gaussian method was used in this subroutine.

The FORTRAN program can be seen at the end of this paper.

4. Numerical example

A structure of 16 stories is given by its floor masses and its flexibility matrix for the lateral displacements. The vector of masses is

$$2 \cdot x \begin{bmatrix} 40.90 \\ 33.70 \\ 33.70 \\ 33.70 \\ 28.70 \\ 28.70 \\ 28.70 \\ 28.70 \\ 28.70 \\ 28.70 \\ 28.70 \\ 28.70 \\ 28.70 \\ 28.70 \\ 28.70 \\ 20.40 \end{bmatrix} .$$

The elements of the flexibility matrix $a_{11}, a_{12}, \ldots a_{21}, a_{22}, \ldots a_{nn}$ were fed into the computer, as well as the masses to apply the iteration procedure by the given program.

The output was as follows:

SMALLEST EIGENVALUE IS 0.75306244E 01
SMALLEST EIGENVALUE IS 0.35431046E 02
SMALLEST EIGENVALUE IS 0.10745300E 03
SMALLEST EIGENVALUE IS 0.19164816E 03
THE ASSOCIATED EIGENVECTOR COMPONENTS ARE

0.100000E 01	0.100000E 01	0.100000E 01	0.100000E 01
0.224411E 01	0.222349E 01	0.217321E 01	0.211891E 01
0.370399E 01	0.361479E 01	0.340087E 01	0.317562E 01
0.536269E 01	0.511576E 01	0.453785E 01	0.395182E 01
0.748947E 01	0.689039E 01	0.553963E 01	0.424833E 01
0.100573E 02	0.882436E 01	0.617149E 01	0.382322E 01
0.128697E 02	0.106457E 02	0.614000E 01	0.254994E 01
0.158511E 02	0.121662E 02	0.526420E 01	0.528195E 00
0.191135E 02	0.132466E 02	0.337034E 01	0.199820E 01
0.225495E 02	0.136666E 02	0.601602E 00	—.429255E 01
0.262510E 02	0.131192E 02	—.286222E 01	—.539506E 01
0.305931E 02	0.110751E 02	—.664443E 01	—40.9732E 01
0.365164E 02	0.602217E 01	—.959945E 01	0.137351E 01
0.460288E 02	—.620724E 01	—.640572E 01	0.102329E 02
0.523679E 02	—.164507E 02	0.293336E 01	0.207737E 01
0.551283E 02	—.214637E 02	0.956794E 01	—.980970E 01

5. Conclusion

A useful FORTRAN Computer program is presented giving only the required modes of vibration of multi-story framed structures. The required lowest modes: shape vectors, and the corresponding frequencies, have been obtained. In this numerical procedure the solution of the polynomial of high degree was avoided, so the computer time is highly saved.

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```
     REAL AI(20,20),DI(10,20),M(20),CI(10,11),XI(10),DII(20)
    ..XX(20),CC(20),B(10,20)
   1 FORMAT(24H SMALLEST EIGENVALUE IS ,E14.8/)
   2 FORMAT(42H THE ASSOCIATED EIGENVECTOR COMPONENTS ARE/)
   3 FORMAT(12)
   4 FORMAT(E10.4)
   6 FORMAT(4(5X,E12.6))
     READ 3,NI
     READ 4,EPSII
     NO=NI/4
     DO 8 I=1,NI
     DO 8 J=1,NO
     READ 6,(AI(I,4* (J—1)+K),K=1,4) AI(I,4*(J—1)+K),K=1.4)
   8 PRINT 6,(AI(I,4*(J—1)+K),K=1,4)
     DO 9 I=1,NO
   9 READ 6,(M(4*(I—1)+K),K=1,4)
     DO 10 I=1,NI
     DO 10 J=1,NI
  10 AI(I,J)=AI(I,J)*M(J)
     K=NI
     KO=0
  16 CALL EIGEN (AI,XX,CC,DII,K,EPSII,EIGI)
     PRINT 1,EIGI
     K1=KO+1
     DO 19 J=1,K
  19 DI(KO+1,J)=DII(J)
     IF(KO)37,37,18
  18 KI=NI—KO
     DO 34 I=1,KO
     CI(I,K1)=0.
     DO 34 J=1,KI
  34 CI(I,K1)=CI(I,K1)—DI(I,J)*M(J)*DI(K1,J)
     NK=NI—KO+1
     DO 35 I=1,KO
     DO 35 J=NK,NI
     JO=J—NI+KO
  35 CI(I,JO)=DI(I,J)*M(J)
     CALL GAUSS(KO,CI,XI)
     DO 36 J=NK,NI
     JO=J—NI+KO
  36 DI(K1,J)=XI(JO)
     IF(K—NI+3)12,12,13
  12 GO TO 17
  13 CONTINUE
     DO 14 I=1,K1
     DO 14 J=1,NI
  14 B(I,J)=DI(I,J)*M(J)
     DO 5 L=1,KO
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```
      LI=L+1
      DO 5 I=LI,K1
      NU=NI—L
      DO 5 J=1,NU
    5 B(I,J)=B(I,J)—B(L,J)*B(I,NU+1)/B(L,NU+1)
      GO TO 28
   37 CONTINUE
      DO 33 J=1,NI
   33 B(1,J)=DI(1,J)*M(J)
   28 CONTINUE
      KO=KO+1
      K=K—1
      DO 15 I=1,K
      DO 15 J=1,K
   15 AI(I,J)=AI(I,J)—AI(I,K+1)*B(K1,J)/B(K1,K+1)
      GO TO 16


   17 CONTINUE
      PRINT 2
      NV=KO+1
      DO 11 J=1,NI
      PUNCH 6,(DI(I,J),I=1,NV)
   11 PRINT 6,(DI(I,J),I=1,NV)
      STOP
      END


      SUBROUTINE GAUS(NSS,AS,XS)
      DIMENSION AS(10,11),XS(10)
      MS=NS+1
      L=NS—1
      DO 52 KS=1,L
      JJ=KS
      BIG=ABS(AS(KS,KS))
      KP1=KS+1
      DO 47 I=KP1,NS
      AB=ABS(AS(I,KS))
      IF(BIG—AB)46,47,47
   46 BIG=AB
      JJ=I
   47 CONTINUE
      IF(JJ—KS)48,50,48
   48 DO 49 J=KS, MS
      TEMP=AS(JJ,J)
      AS(JJ,J)=AS(KS,J)
   49 AS(KS,J)=TEMP
   50 DO 51 I=KP1,NS
      QUOT=AS(I,KS)/AS(KS,KS)
      DO 51 J=KP1,MS
   51 AS(1,J)=AS(1,J)—QUOT*AS(KS,J)
      DO 25 I=KP1,NS
   52 AS(I,KS=0.
      XS(NS)=AS(NS,MS)/AS(NS,NS)
      DO 54 NN=1,L
      SUM=0.
      I=NS—NN
      IP1=I+1
      DO 53 J=IP1,NS
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   53 SUM=SUM+AS(I,J)*XS(J)
   54 XS(I)=(AS(I,MS)—SUM)/AS(I,I)
      RETURN
      END




      SUBROUTINE EIGEN(A,X,C,D,N,EPSI,EIG)
      DIMENSION A(20,20),X(20),O(20),D(20)
      DO 20 I=1,N
   20 X(I)=1.
C      CALCULATE COMPONENTS OF THE VECTOR(1./LAMBDA*X)
   21 DO 22 I=1,N
      O(I)=0.
C      DO 22 J=1,N
   22 C(I)=C(I)+A(I,J)*X(J)
      NORMALIZE THE VECTOR(1./LAMBDA*X)
      DO 23 I=1,N
   23 D(I)=C(I)/C(1)
C      CHECK TO SEE IF REQUIRED ACCURACY HAS BEEN ATTAINED
      DO 24 I=1,N
      DIFF=X(I)—D(I)
      IF(ABS(DIFF)—EPSI)24,25,25
   24 CONTINUE
      GO TO 27
   25 DO 26 I=1,N
   26 X(I)=D(I)
      GO TO 21
   27 EIG=1,C(1)
      RETURN
      END
```

Summary

The method presented in this paper can be summarized in the following steps:

1. Arrange the flexibility matrix for the structure and feed it to the computer with the mass vector of the structure.

2. The program applies the iteration method given by the subroutine EIGEN obtaining the first mode vector and the fundamental frequency.

3. The orthogonality equation is applied using the elements of the first mode.

4. Solving the above equation with the motion equations, a new system of motion equations is got.

5. Applying the iteration method on the new equations, the second mode vector and its frequency are obtained.

6. Substituting the elements of the vector obtained in step 5 on the orthogonality equation of step 3, the eliminated element of the second mode vector is got.

7. Repeating the steps from 3 to 6 for the following higher mode, this mode can be got. In the step №. 6 a system of linear equations is formed. Solving these equations by use of GAUSS subroutine, the unknown elements of the vector are got. The higher modes are obtained by the same procedure.

8. The output of the program is the square value of each frequency in radian/sec. and the corresponding mode vectors.

Notations

a_{ij} — Elements of the flexibility matrix
m_i = Masses of the structure at concentrated levels (floor levels)
y_i = Horizontal deflections of the levels of masses in the plane of vibration

where $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n$

w = Frequency of vibration
n = Number of stories of the structure

Input Data of the Computer Program

AI = Flexibility matrix
M = Vector of masses

Output Data

EIGI = Eigen value
DI = Matrix gives the modes of vibration

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Abd El Fattah El AKABAWI, H-1521 Budapest