REFLECTIONS ON THE TURBULENT MIXING FLOW OF FLUIDS IN A VORTEX CHAMBER

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Introduction

Devices in which some fluid in turbulent flow is mixed with another fluid are frequently used for mixing, cooling or any other similar purposes. In the flow of the air along the internal side of a cylindrical surface deserves special attention. One side of the cylinder is closed, the opposite side is open. The air mixture inhausted in the centre emerges through the edges of the orifice. Constructions of this type may be suitable to ventilate any room by introducing the ventilating air without any draught even in cases when the temperature of the fresh ventilating air deviates considerably from the temperature of the room to be ventilated. These types of air inducing anemostates — which conveniently may be called vortex chamber induction anemostates — mix room air to the ventilating air, this mixture is then introduced into the room to be ventilated. This way the draught sensation due to the considerable temperature difference between the input and the room air may be completely eliminated.

In order to determine the temperature of the injected air the mixing ratio of the fresh, ventilating (primary) and the room air (secondary) must be known.

The problem involved can be demonstrated by a simple example. Let us introduce fresh, cold air into a room at 26 °C. The minimum temperature of the cold, fresh air to cool temperature of the mixture to +15 °C is wanted. Let the secondary/primary mixing ratio φ of the vortex chamber induction anemostate be unity

$$\frac{Q_s}{Q_p} = \varphi\,. \tag{1}$$

The premissible minimum temperature t_1 of the fresh air is obtained from the temperature equilibrium condition

$$t_1 = (\varphi + 1) \cdot t_{\text{out}} - \varphi \cdot t_{\text{room}}. \tag{2}$$

By substituting

$$t_1 = 2 \cdot 18 - 26 = +4\ ^\circ\mathrm{C}$$

i.e. the minimum temperature of the fresh, ventilating air should be at least $+4\ ^\circ$C.

The example discussed clearly indicates the importance to know the mixing ratio in specifying the temperature of the fresh, ventilating air. In order to discuss the method of calculating of the quantitative relations let

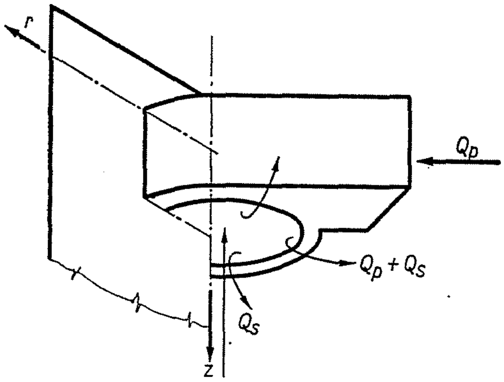

Fig. 1

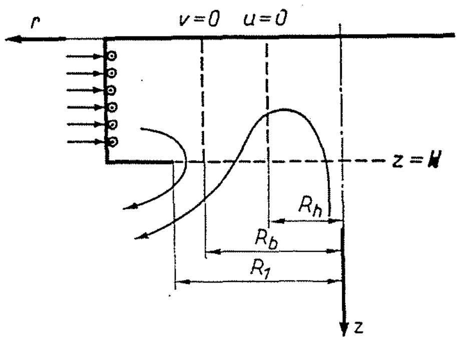

Fig. 2

us investigate the flow produced in the vortex chamber. Fig. 1 depicts the turbulent chamber indicating the input and the output flows. Let us fit the chamber, shown in section in Fig. 2 to a cylindrical co-ordinate system. Assuming the flow in the vortex chamber to be of cylindrical symmetry it is enough to trace one half of the chamber. Accordingly the flow of cylindrical symmetry will be investigated in the cylindrical co-ordinate system r, Θ, z. Fig. 2 showing also the character of the flow clearly demonstrates that the inflow of the secondary air may be accounted for by the injection resulting from the considerable difference in the peripheral velocities between primary and secondary air masses, consequently investigation has to take the frictional flow at least with its peripheral flow component into consideration.

The problem may be solved by the Navier—Stokes equations [1], [6] with the assumptions and simplifications below, interpreting velocity components according to Fig. 3:

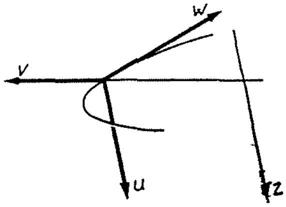

Fig. 3

1. the flow is without friction in the directions z and r;
2. in direction Θ the air flows with friction;
3. the quantity u is of the type $u = f_1\,(r, z)$, and
4. the quantity v is of the type $v = f_2\,(r)$.

With these assumptions the Navier—Stokes equations take the following from [6]:

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = -\frac{1}{\varrho}\frac{\partial p}{\partial z} \tag{3}$$

$$v\frac{\partial v}{\partial r} - \frac{w^2}{r^2} = -\frac{1}{\varrho}\frac{\partial p}{\partial z} \tag{4}$$

$$v\frac{\partial w}{\partial r} + \frac{wv}{r} = (\nu_t + \nu)\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{w}{r^2}\right]. \tag{5}$$

The continuity equation has the following form:

$$\frac{\partial(rv)}{\partial r} + \frac{\partial(ru)}{\partial z} = 0 \tag{6}$$

Calculations procedure of the quantitative relationship

The knowledge of the distribution $z = H$ of velocities u would be enough to calculate the quantitative relation. However, this calculation postulates the knowledge of the pressure distribution, but it is rather difficult to determine.

The simplifications used allow a relatively simple calculation of the peripheral velocities. Consequently the quantitative ratio can be calculated by determining the peripheral velocities taking also the moment of momentum theorem into consideration.

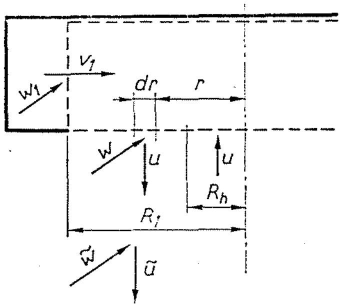

Fig. 4

According to Fig. 4 the following reasoning appears to be justified. Since in the experiment the input is without rotation in the range $0 < r < R_h$, $z = H$:

$$\varrho v_1 H 2 R_1 \pi w_1 R_1 = \varrho \tilde{u} \, (R_1^2 - R_h^2) \, \pi \, (\widetilde{wR}) \tag{7}$$

where $(\widetilde{wR})$ and $\tilde{u}$ denote average values within the range $R_h < r < R_1$. In Eq. (7) on the endplate of the vortex chamber the friction considered to be negligible.

$$v_1 H 2 R_1 \pi = Q_p \tag{8}$$

and

$$\tilde{u}(R_1^2 - R_h^2)\pi = Q_p + Q_s \tag{9}$$

it follows that

$$(\widetilde{wR}) = w_1 R_1 \frac{1}{1 + \varphi} \tag{10}$$

and one finally has

$$\varphi = \frac{w_1 R_1}{(\widetilde{wR})} - 1 \tag{11}$$

According to Fig. 4, the emergent momentum is

$$\varrho u \, 2 \, r^2 \, \pi w \, dr$$

and the average emergent momentum for $z = H$ in the range $R_h < r < R_1$:

$$(\widetilde{wR}) \, (Q_p + Q_s) \, \varrho$$

Consequently

$$(\widetilde{wR}) = \frac{2\pi}{(1+\varphi)Q_p} \int_{R_h}^{R_1} wur^2\, dr \tag{12}$$

From the above reasoning it follows that if for $z = H$ the values of w can be calculated, and an approximation of u is known in the range $R_h < r < R_1$ $(\widetilde{wR})$ and φ may be obtained from Eqs (11), and (12), respectively.

Calculation of the peripheral velocity

Let us investigate the possibility of calculating the w values and determining the relationships between the individual velocity components. The results of experiments on spatial velocity disribution according to *G. Gaulier* [4], [5] using a DISA CTA anemometer enable to approximate the values of the velocity components in equation form: $u = (ar^2 + b)\, z/H$ in the range $z = H$, $R_h < r < R_1$.

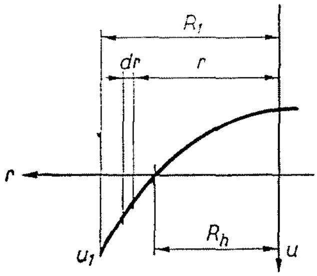

Fig. 5

The constants a and b are found from the boundary conditions (see Fig. 5):

$$\begin{aligned} r &= R_h \qquad u = 0 \\ r &= R_1 \qquad u = u_1 \end{aligned} \tag{13}$$

With these values

$$u = \frac{u_1}{R_1^2 - R_h^2}(r^2 - R_h^2)\frac{z}{H}\,. \tag{14}$$

The unknown value u_1 may be eliminated by expressing u in terms of the primary and secondary (Q_p and Q_s) air masses in $z = H$.

$$Q_p + Q_s = \int_{R_h}^{R_1} 2\, r\pi u(r)\, dr\,, \tag{15}$$

Substituting:

$$Q_p + Q_s = Q_p(1+\varphi) = \int_{R_h}^{R_1} 2\pi \frac{u_1}{R_1^2 - R_h^2} (r^3 - R_h^2 r)\, dr \tag{16}$$

By interpreting the quantitative relationship $Q_s/Q_p = \varphi$ and, solving the integral applying (14), Eq. (16) results in:

$$u = \frac{2Q_p(1+\varphi)}{\pi(R_1^2 - R_h^2)^2} \cdot \frac{z}{H} \cdot (r^2 - R_h^2)\,. \tag{17}$$

Since u and v must satisfy the continuity equation the distribution of v can be determined from the assumed u value. For this purpose one has to substitute Eg. (17) into (6) and solving the differential equation which for v is homogeneous and linear, we obtain:

$$v = \frac{C}{r} - \frac{Q_p(1+\varphi)(r^3 - 2\,R_h^2\, r)}{2\pi H(R_1^2 - R_h^2)^2}\,. \tag{18}$$

In the above equation C represents the constant of integration, R_h is unknown, but can be determined from the following boundary conditions:

$$\begin{aligned} r &= R_1 \quad & v &= -\frac{Q_p}{2\pi R_1 H} \\ r &= 0 \quad & v &= 0\,. \end{aligned} \tag{19}$$

It follows from the first condition that $C = 0$. The second condition yields:

$$R_h = R_1 \sqrt{\varphi\left(\sqrt{1+\frac{1}{\varphi}} - 1\right)} \tag{20}$$

from which R_h can be determined.

The relationships obtained permit to determine the significant radius R_b in Fig. 2. Obviously R_b is the radius where v vanishes. From this condition one has

$$R_b = R_h\sqrt{2}\,. \tag{21}$$

According to this approach, there exist points where components of u or v the axial or radial velocity change sign. Roughly, $r = R_b$ may be said to represent the central line of mixing, whereas $r = R_h$ in $z = H$ separates the fluid elements emerging from or entering the chamber.

With the approximation obtained for u and v Eq. (5) can be solved, bearing in mind that the flow is absolutly turbulent, consequently the viscosity cannot be considered as constant. If the value of the turbulent viscosity is accounted from experiments, and [2]:

$$\nu_t \gg \nu \,. \tag{22}$$

Eq. (5) takes the form:

$$v\frac{dw}{dr} + \frac{wv}{r} = \nu_t\left\{\frac{d^2w}{dr^2} + \frac{1}{r}\,\frac{dw}{dr} - \frac{w}{r^2}\right\}. \tag{23}$$

According to DISA stream measurements the peripheral velocities increase monotonously, the sign of the derivative of the velocity is always positive. If the value of the velocity derivative with respect to r is not zero on rearranging, in the range $R_h < r < R_1$ the following differential equation results:

$$\frac{d^2w}{dr^2} = \frac{vdw}{\nu_t} - \frac{dw}{rdr} + \frac{w}{r^2} + \frac{v\,w}{\nu_t\,r}\,. \tag{24}$$

The boundary conditions are

$$\begin{aligned} r &= R_h & w &= 0 \\ r &= R_1 & w &= w_1 \end{aligned} \tag{25}$$

Consequently the solution reduces to a boundary condition problem.

In the differential equation denotes ν_t the turbulent viscosity, its value can be determined by experimentally with a DISA anemometer. The experimental evaluation of the turbulent viscosity may be conveniently carried out by means of Eq. (24). Nevertheless some viewpoints must be thought of when evaluating the point where v vanishes. At this point the ν_t value can be defined only if at the same point also the value

$$\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr} - \frac{w}{r^2}$$

vanishes, and the indetermined term has a finite value.

According to investigations with the Bernoulli-L'Hospital rule, the turbülent viscosity can be defined also for the point where v vanishes.

The boundary condition problem represented by Eqs (24) and (25) was solved by means of the relaxation method [3].

As a calculation example see Fig. 6.

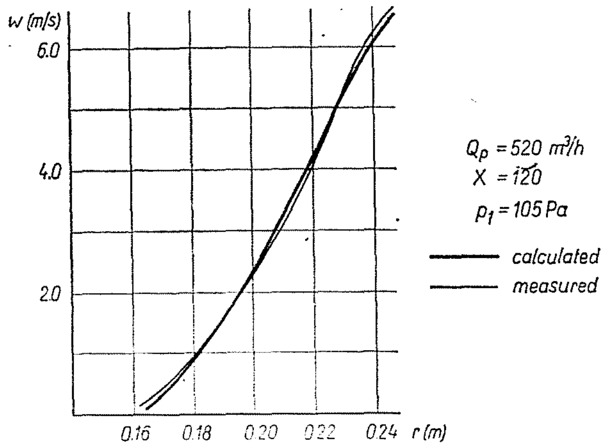

Fig. 6

An expression to calculate the quantitative relationship

Let us attack now the quantitative relationship mentioned under 2. With the aid of the approximative equation (17) Eq. (12) can be reduced to a simple form:

$$(\widetilde{wR}) = \frac{4}{(R_1^2 - R_h^2)^2} \int_{R_h}^{R_1} (r^4 - R_h^2 r^2)\, w\, dr \tag{26}$$

Putting (26) into (11) and taking the results under 3. into consideration results in:

$$\varphi = \frac{R_1\, w_1 (R_1^2 - R_h^2)^2}{4 \int_{R_h}^{R_1} (r^4 - R_h^2 r^2) w dr} \tag{27}$$

suitable to determine the quantitative relationship. The computations were carried out with a program written in BASIC on the WANG computer of the department of Fluid Mechanics Technical University of Budapest.

Summary

Devices in which some fluid is mixed with another fluid in a vortex chamber are frequently used for mixing, cooling or any other purposes. The author shows how important it is to determine the mixing ratio of the primary and the secondary fluid. The author recommande a method to computate the mixing ratio, by solving the Navier-Stokes equation. The turbulent viscosity was taken from experiments. The above has been illustrated with an example too.

Symbols

w	peripheral velocity
u	axial velocity
v	radial velocity
r, θ, z	cylindrical co-ordinates
R_1	half value of the lower orifice of the chamber
R_h	co-ordinate value at $u = 0$
R_b	co-ordinate value at $v = 0$
Q_p	input (primary) fluid flow
Q_s	secondary fluid flow
ν	Newtonian viscosity
ν_t	turbulent viscosity

References

1. KOVAL, MIKHAILOV: Teploenergetika Nr 2. 25–27, 1972.
2. LAUNDER, SPALDING: Lectures in Mathematical Models of Turbulence, Academic Press, London, 1972.
3. COLLATZ, L.: The Numerical Treatment of Differential Equations, Springer, (Wien, Berlin, New York), 1966.
4. GAULIER, G.: DISA Information No 21, 16–20, 1977.
5. JØRGENSON, L.: DISA Information No 11, 12–18, 1971.
6. WHITE, O.: Viscous Fluid Flow, Mc.Graw-Hill New York, 1975.

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