MATHEMATICAL VIBRATION ANALYSIS OF BUSES OPERATING IN TOWNS

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Introduction

Vibration comfort, wheel to ground contact of motor cars can be much improved, dynamic stresses in both car body and undercarriage significantly reduced by aptly choosing suspension parameters (damping and spring characteristics) by means of vibration analysis.

Practically [1], mathematical and instrumental vibration analysis of cars in motorways are usually made under steady driving conditions (constant load, homogeneous road quality and constant speed).

Nevertheless, buses operate under unsteady driving conditions. According to observations [2], under usual conditions, buses run between two stations partly at a stochastic acceleration or deceleration, and partly at a quasi stationary speed.

Ignoring random fluctuations of speed and acceleration, this mode of operation is well described by the so-called trapezoidal time speed diagrams (Figs 1a and 1b).

Since buses represent an important rolling stock, it is advisable to investigate the righteousness to reckon with an unsteady operation mode in mathematical vibration analysis.

On the other hand, since in certain cases, representing non-linear vibration systems may be treated a statistically equivalent linear ones, intermediating an adequate mathematical method [3], and application of spectral relationships for time-invariant linear systems in steady state provides for a fast analysis of standard deviation and frequency, compared to analysis in the

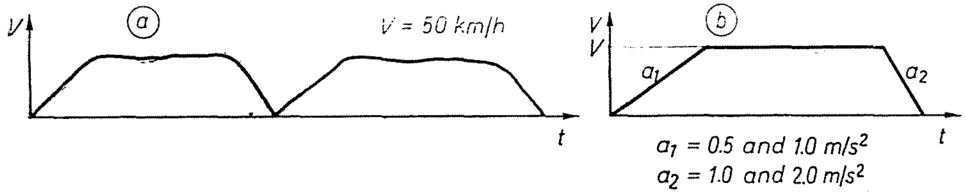

Fig. 1

time domain, thus, it seems advisable to find a transformation method starting from vibration parameters calculated for steady speeds and permitting to approximate the effect of urban, unsteady mode of operation.

1. Simulation of vibration phenomena

The above considerations underlaid investigations into the necessity of reckoning with the unsteady mode of operation. Thereupon a method of transformation from steady to unsteady mode of operation has been developed. Starting assumptions:

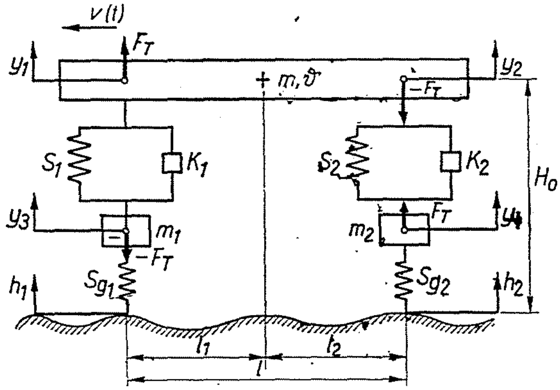

Fig. 2

1. The analysis was made on the linear plane model of four degrees of freedom of a 11 m. bus made by IKARUS (Fig. 2) performing vertical vibrations according to a differential equation system written in matrix form:

$$M \cdot \ddot{y} + K \cdot \dot{y} + S \cdot y = F_T^* + S_q \cdot h \tag{1}$$

Eqs (1) differ from the usual ones exclusively by inertia vector F_T^*

$$F_T^* = \begin{bmatrix} -\dfrac{l_1 \cdot l}{\vartheta^2} \cdot F_T \\ \dfrac{l_2 \cdot l}{\vartheta^2} \cdot F_T \\ F_T \\ -F_T \end{bmatrix} \tag{2}$$

where:

M mass matrix
K damping matrix

S spring stiffness matrix
S_q tyre spring stiffness matrix
F_T force of inertia due to acceleration

$$F_T = \frac{1}{l}\left[H_0 + \frac{1}{l}(l_1 \cdot y_2 + l_2 \cdot y_1)\right] \cdot (m + m_1 + m_2)\ \frac{dv}{dt}. \tag{3}$$

H_0 static height of gravity centre
m_1; m_2; m masses with and without spring.

Other symbols are seen in Fig. 2.

Numerical values for the considered bus type were:

$m = 5.932\ \mathrm{kps^2\,cm^{-1}}$; $m_1 = 0.866\ \mathrm{kps^2\,cm^{-1}}$; $m_2 = 1.58\ \mathrm{kps^2\,cm^{-1}}$

$\vartheta^2 = 70\,000\ \mathrm{cm^2}$; $l_1 = 260$ cm; $l_2 = 290$ cm; $h = 90$ cm

$S_1 = 315.3$ kp/cm; $S_2 = 531.8$ kp/cm; $S_1 = 2800$ kp/cm

$S_{g2} = 5600$ kp/cm; $k_1 = 16$ kps/cm; $k_2 = 32$ kps/cm

2. Assuming urban roads to be paved with asphalt, road profile spectrum exciting the linear model was produced by means of formula (4):

$$S_h(\omega) = D_h^2 \cdot 2\pi\left[\frac{0.054 \cdot V}{\omega^2 + 0.04\,V^2} + \frac{0.0024 \cdot V(\omega^2 + 0.36 \cdot V^2)}{(\omega^2 - 0.36\,V^2)^2 + 0.0036\,V^2}\right] \tag{4}$$

where: $D_h = 1$ cm standard deviation of the road roughness
V velocity, m/sec
ω circular frequency, rad/sec

3. Vibration analysis in the time domain was made by digital simulation [5]. The road roughness function needed for excitation based on the road spectral density function was produced by means of Eq. 5

$$h(t) = \sqrt{\frac{S_h(o)}{2T}} + \sum_{k=1}^{\infty} \sqrt{\frac{2}{T} S_h(k\omega_0)} \cdot \sin(k\omega_0 t + \psi_k)] \tag{5}$$

where: $2T$ excitation period time, sec
$S_h(\mathrm{o})$ spectral value for $\omega_0 = 0$
$\omega_0 = \pi/T$ fundamental mode rad/sec
$\psi_k(-\pi, \pi)$ random variable of uniform distribution.

4. Operation of the bus was represented by simplified [2] trapezoid diagram (Fig. 1b), with accelerations of 0.5 and 1 m/sec², decelerations of 1 and 2 m/sec², and constant speed $V = 50$ km/h.

2. The necessity to reckon with unsteady mode of operation

To find out whether the unsteady mode of operation needs to be reckoned with or not, digital simulation has been applied to "drive" the model first in steady, then in unsteady mode of operation, taking both acceleration and deceleration into consideration. Standard deviations of the vertical amplitudes D_{yi}, speeds $D_{\dot{y}i}$ and accelerations $D_{\ddot{y}i}$ of gravity centres of the vibrating system

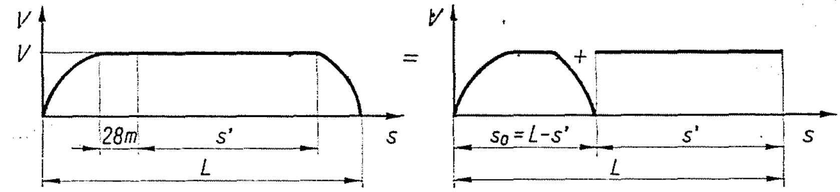

Fig. 3

(y_i, $i = 1, 2, 3, 4$) have been determined and compared. To reduce the important running time of the simulation procedure a method consisting of a single simulation, recording the data and calculating standard deviations for different distances L between stops has been developed. This method consist essentially in dividing the distance L between two stops in two parts: a steady and an unsteady part (Fig. 3). The so-called nodding vibration of decaying character, due to the force of inertia of acceleration, has been taken into consideration by inserting a 28 m section of constant speed in the unsteady part. A single simulation of the unsteady and the steady part each was applied to separately determine functions $y = f(t)$, output signals of the systems, stored separately in the storage unit of the computer. Thereafter expected values $\bar{y}^{I}$ and $\bar{y}^{II}$ of the function $y = f(t)$ consisting of n_1 and n_2 discrete values, respectively, have been determined and stored for each stop distance.

Expected values for the entire process resulted from averaging:

$$y = \frac{1}{n} [n_1 \cdot \bar{y}^{I} + n_2 \cdot \bar{y}^{II}] \tag{6}$$

where: n_1 number of discrete values in the unsteady section;

n_2 number of discrete values in the steady section;

$n = n_1 + n_2$.

Since in this formula, variation of the distance between stops entrains only the variation of the number n_2 of discrete values in the steady section, averaging $\bar{y}$ for an arbitrary length L is much simplified.

Knowledge of the expected value permits to determine the standard deviation of output signals of the system:

$$D_y = \sqrt{\frac{1}{n}\left[\sum_{j=1}^{n_1}(y'_j - \bar{y})^2 + \sum_{j=1}^{n_2}(y''_j - \bar{y})^2\right]} \tag{7}$$

where: y'_j, y''_j discrete function values of unsteady and steady lengths, respectively.

Analysis results have been plotted in Figs 4 to 7. Difference percentages between standard deviations of unsteady and steady modes of operation have been compiled in Table 1. Their comparison yields the following conclusions:

1. In unsteady mode of operation–expect diversions–standard deviations of vertical vibrations of gravity centres are less than at constant speed $V = 50$ km/h.

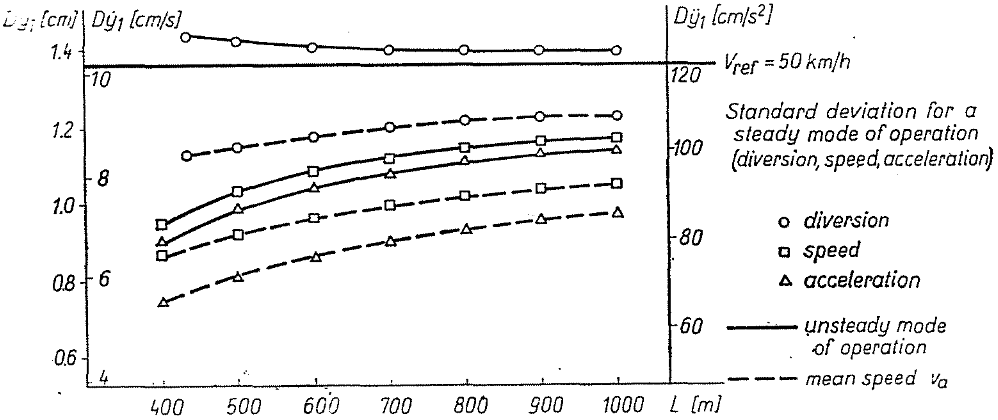

Fig. 4

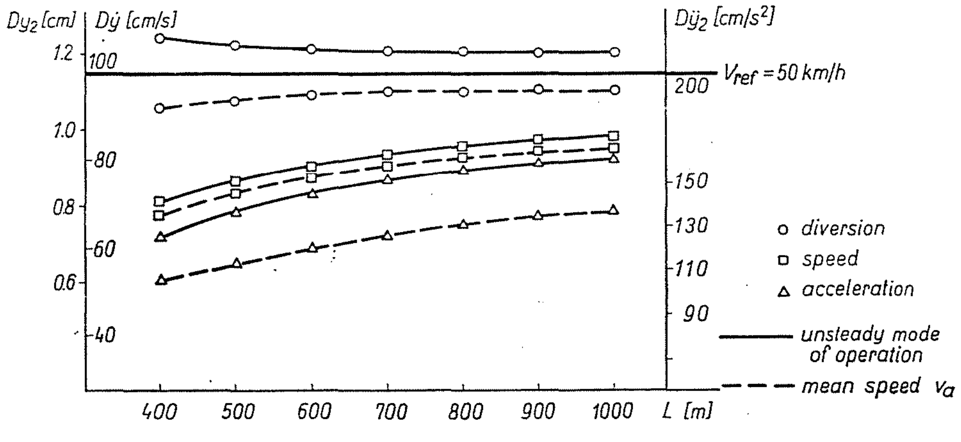

Fig. 5

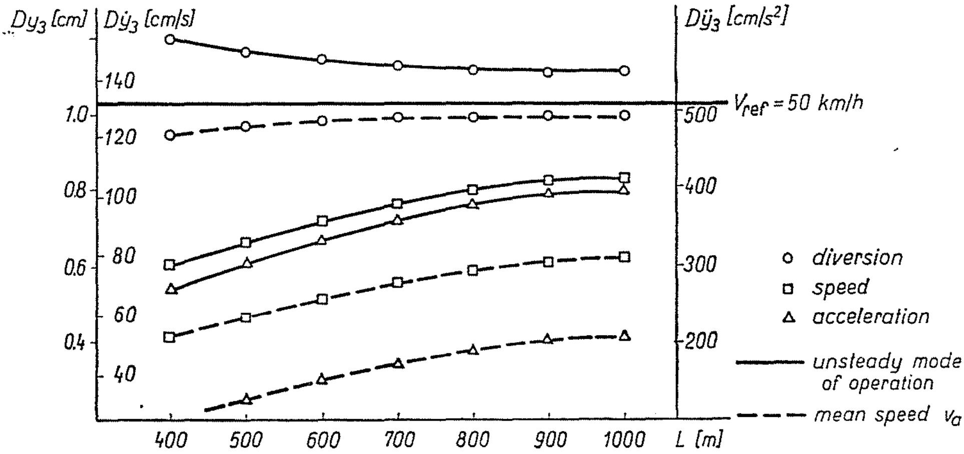

Fig. 6

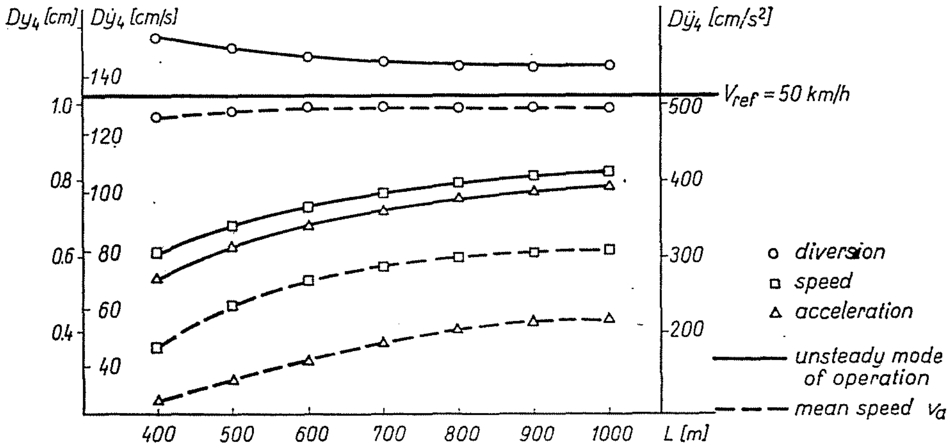

Fig. 7

Table 1

Percentage difference between standard deviations of unsteady and steady modes of operation:
$H_{yi} = 100\ (D_{yi} - \bar{D}_{yi})/D_{yi}$

i	$L = 400$ m			$L = 1000$ m		
	H_{yi}	$H_{\dot{y}i}$	$H_{\ddot{y}i}$	H_{yi}	$H_{\dot{y}i}$	$H_{\ddot{y}i}$
1	21.67	11.11	18.75	12.14	9.09	13.86
2	14.52	5.63	18.55	8.33	2.37	15.85
3	20.17	31.17	62.26	10.81	24.53	46.25
4	41.03	27.50	59.26	10.81	23.15	42.50
Average	24.35	18.85	39.71	10.52	14.78	29.62

2. With increasing distance L between two stops, the difference between standard deviations of the two operation modes monotonously decreases.

3. Difference percentages between standard deviations of the two operation modes are significant even taking the practically possible longest distance $L = 1000$ m into consideration, averaging at 10%, 15% and 30% for vibration amplitudes, vibration velocities and vibration accelerations, respectively.

Standard deviation values hint to the expediency to reckon with the real, unsteady mode of operation in certain cases, e.g. in analysing the dynamic stresses in the car body.

4. The digital simulation procedure being lengthy and costly it seems advisable to develop a transformation method rapidly determining standard deviations by means of spectral relationships.

3. Transformation method from unsteady to steady mode of operation

The rather time consuming simulation method required to determine bus vibration characteristics ($D_{yi}, D_{\dot{y}i}, D_{\ddot{y}i}$) in unsteady mode of operation urged to develop a transformation method permitting to treat the unsteady mode of operation as a steady one, hence, based on a linear vibration system, to apply the much simpler spectral method. To this aim, the trapezoid diagram in Fig. 1b, describing the unsteady mode of operation of period T has been applied. By varying period T, the quoted simulation method has been applied to determine the standard deviations of the vibration characteristics (D_y and $\bar{D}_y$) in the unsteady mode of operation described by the trapezoid diagram, as well as in the steady one of mean speed V.

Ration δy results from dividing standard deviations D_y for the unsteady mode of operation by $\bar{D}_y$ for the steady mode at speed V:

$$\delta_y = \frac{D_y}{\bar{D}_y} \tag{8}$$

Numerical analysis of the simulation tests yielded a fairly approximating linear relationship:

$$\delta_y = 1 + C_y \cdot K \tag{9}$$

where: C_y output signal constant, independent of the distance between

$$K = \frac{t_1 + t_2}{T}$$

dynamic characteristic (depending on the distance between stops), to be determined according to the trapezoid diagram in Fig. 1b.

In possession of the above, it is rather simple to convert from the steady mode of operation at speed V to the unsteady mode described by the trapezoid diagram, namely:

$$D_y^* = \delta_y \cdot \bar{D}_y = (1 + C_y \cdot K) \cdot \bar{D}_y \tag{10}$$

Eq. (9) being a linear relationship between δ_y and K, and besides, condition

$$\lim_{K \to 0} \delta_y = 1$$

is met, (9) may be written from a single determination of δ_y for K. This means that for a given acceleration "a", it is sufficient to perform the time analysis for a trapezoid diagram of drive and to determine the δ_y values, since standard deviation values for different stop length are easy to obtain from (10).

Correctness of this method was checked on an actual model. Utilizing technical ratings of the selected 11 m bus, vibration characteristic constants been determined as described above (Table 2).

Table 2

Vibration characteristic constants

i	C_{yi}	$C_{\dot{y}i}$	$C_{\ddot{y}i}$
1	0.311/0.447	0.177/0.204	0.315/0.374
2	0.208/0.227	0.064/0.088	0.340/0.383
3	0.267/0.307	0.673/0.738	2.00/2.01
4	0.249/0.300	0.608/0.662	1.833/1.834

Remark: Denominator values refer to acceleration $a = 1$ m/sec^2. Numerator values to acceleration $a = 0.5$ m/sec^2

Digital simulation and the suggested transformation method have been applied to determine standard deviations of vibration characteristics of the plane model of four degrees of freedom.

Table 3

Vibration characteristics of front suspension

L [m]	D_{yi}^{x} [cm]	D_{y1} [cm]	Rel. error [%]	D_{y3}^{x} [cm]	D_{y3} [cm]	Rel. error [%]
500	1.412	1.418	0.45	1.159	1.160	0.05
600	1.414	1.411	—0.22	1.147	1.142	—0.45
900	1.406	1.399	—0.51	1.116	1.120	0.34

L [m]	$D^x_{\dot{y}_1}$ [cm/s]	$D_{\dot{y}_1}$ [cm/s]	Rel. error [%]	$D^x_{\dot{y}_3}$ [cm/s]	$D_{\dot{y}_3}$ [cm/s]	Rel. error [%]
500	7.747	7.710	−0.47	9.045	8.700	−3.97
600	8.027	8.010	−0.22	9.427	9.300	−1.36
900	8.493	8.61	1.36	10.287	10.420	1.27

L [m]	$D^x_{\ddot{y}_1}$ [cm/s²]	$D_{\ddot{y}_1}$ [cm/s²]	Rel. error [%]	$D^x_{\ddot{y}_3}$ [cm/s²]	$D_{\ddot{y}_3}$ [cm/s²]	Rel. error [%]
500	87.273	86.500	−0.89	316.036	312.000	−1.29
600	91.066	91.200	0.15	340.557	341.000	0.13
900	97.841	100.00	2.16	402.984	395.000	−2.02

As an illustration, front suspension data have been compiled in Table 3. Comparison of standard deviations D_y and D_y^x obtained by digital simulation and by transformation, respectively, shows the relative error percentage to be below 5%, irrespective of the stop distance, indicating a good approximation by the transformation method.

Let us remark here that the transformation method can be extended to arbitrary run curves. Then:

$$K = \frac{\int_{-\infty}^{\infty} v \cdot f(v) dv}{V} \tag{11}$$

where: $f(v)$ speed density function

V maximum speed.

In this case, however, linearity of the function $\delta_y = \varphi(K)$ has to be decided over by further research.

Summary

1. In view of real distances between bus stops, standard deviations of output signals significantly differ between real unsteady and arbitrary steady operation modes. Therefore it is advisable to reckon with unsteady modes of operation for buses.

2. The digital time-function method, rather running time-consuming for unsteady modes of operation, is fairly substituted by the much simpler spectral method, in possession of the described transformation method.

3. The described procedure suits an exacter determination of the dynamic stresses and vibration comfort of town buses.

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