

# GROUND AND NEAR-GROUND MANEUVERS OF AIRCRAFTS WITH MINIMAL EXHAUST EMISSION

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## I. Introduction, scope\*

In the struggle against air pollution the importance of the emission produced by the aircrafts keeps growing. In the vicinity of airports of considerable traffic, the emission tends to approach that produced by industry.

Essentially, the investigations of emission problems of aircrafts and those of imission problems of airports attained the state of surveying and registration [1—8]. The moderate results obtained in the reduction of the emission of exhaust gases show that there is still a lot to do in this field. More significant results are expected mainly from modifying the construction of the combustors.

But there is another possibility to reduce the emission of the aircraft, seldom examined so far. By analyzing the ground and near-ground movements of the aircraft as well as the emission of the engine operating modes of these movements, an aircraft movement can be defined which provides the minimal total emission during different maneuvers (e.g.: take-off run, climbout, taxiing between assigned points of the airport, etc.). Provided the maneuvers, optimal from this point of view, are realizable in other respects too, the air pollution in airports can be reduced without any modification of the construction of the engine.

Considering the above, the aim of this study is to determine the maneuvers for the aircraft optimum for the air pollution. Further on, aircraft will be understood as conventional fixed wings airplane (chiefly turbojet engine-powered high performance commercial aircraft) and thus, the conditions of other aircraft — e.g.: helicopter, VTOL- and STOL-aircraft, propeller aircraft powered with low performance piston engine, etc. — will be neglected.

The air pollution problem will be investigated from several aspects. The following are considered as air pollutants: total mass of exhaust gases, mass of special exhaust constituents (CO, CH, NO<sub>x</sub>, particulates etc.), as well as total

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mass of exhaust constituents. On the basis of the investigations made according to the above view-points, different kinds of optimal aircraft maneuvers can be determined. Thereby the results of these investigations may permit to control the ratio of air pollutant components in the exhaust gases — even if restricted — by modifying the maneuvers of the aircraft. Accordingly, in case of airports with most unfavourable back-ground (industrial) pollution in respect of certain-components (e.g. CO), it is prospectively imaginable to elaborate aircraft maneuvers involving engine emissions of a minimum content of the given component (in our case CO).

## 2. Emission of the aircraft turbojet engine as a function of the engine operating conditions

### 2.1 Rise of the emission, its relation to the engine operating condition

The emission of the aircraft turbojet engine is associated with the imperfect-combustion process in the combustor. The exhaust gases contain the following emission-components:  $\text{NO}_x$  (oxides of nitrogen), CH (hydrocarbons), CO (carbon monoxid), solid particulates (smoke) and  $\text{SO}_x$  (sulphur oxides) in negligible quantities.

For a given combustor construction, emission components depend on the following characteristics:

- air temperature and pressure at the combustor inlet,
- air to fuel ratio (dilution), i.e. the highest temperature at the turbine entry,
- released heat in the combustor, i.e. the specific or absolute thermal load of the combustor,
- velocities of the air and of the gases in the combustor, i.e. the air flow entering the combustor,
- temperature, pressure and humidity of the ambient air (of little significance).

All the characteristics but the humidity depend definitely on the operation mode of both the power plant and the airframe, therefore a correlation exists between the emission of the turbojet engine and its operation.

The  $\text{NO}_x$  emission relates to the characteristic features of the combustion of the hydrocarbons. In addition to the main and highest temperatures of the combustor which have a direct connection with the dilution in the combustor, i.e. with the operation of the engine, the rise of  $\text{NO}_x$  depends on the following: situation of the high-temperature zones within the combustor, temperature of flame, rate of chemical reactions,  $\text{N}_2$  content of fuel and humidity of intake air.

The reaction of oxygen and nitrogen takes place in the so-called “primary construction zone” of the combustor (the zone immediately before the spray

nozzles). In this zone the temperature of the exhaust gases is the highest and the rate of  $\text{NO}_x$  formation is determined by this highest temperature, while the other parameters influence it just slightly. The highest temperature in the combustor depends definitely on the operation mode of the engine since it can not be controlled but by modifying the highest temperature in the combustor, i.e. the dilution of the combustion. Both the specific and absolute values of  $\text{NO}_x$  emission will be the highest at the maximum loading of the engine, at idle (at low combustor temperature) there is practically no  $\text{NO}_x$  emission at all.

The *CO emission* originates from imperfect combustion and  $\text{CO}_2$  dissociation. The formation of mixture of unequal quality, low temperature or sudden cooling of the combustor walls facilitate the CO emission as well. These phenomena occur mainly at idle operation mode; this statement refers especially to the formation of mixture.

The *CH emission* is due to the imperfect formation of mixture, non-uniform distribution of the fuel within the combustor and the sudden cooling of the combustor walls. Most of these phenomena take place in idle or similar operation modes of the engine as well and thus it can be proved that both the CO and CH emissions increase when the loading of the engine decreases to achieve their maximum specific, and in most cases also the maximum absolute values in the idle operation mode.

Essentially, the burnt gases contain tiny *carbon particles* with some organic content. Smoke will develop in the combustor zones with rich fuel-air mixtures. Here the reactions between the hydrocarbons and the oxygen stop or slow down causing hydrocarbons to vaporize, in consequence of the too rich a mixture and the lack of oxygen. Upon reaction stop, the vapours cool, dehydrogenation and polymerization take place while microscopical particles will be produced which visibilize exhaust gases. Smoking achieves the maximum absolute value in full loading, the maximum specific smoke, however, arises only at about 50 p.c. loading.

As our further investigations refer to two-flows turbojet engines, the emission characteristics of engines of these types will be reported from the literature [2, 3].

In the Figures the emission characteristics of the following two-flows turbojet engines are shown:

*JT9D*  $F_{P_0} = 21\ 320$  kp (maximum thrust)

*JT3D*  $F_{P_0} = 8\ 172$  kp

*JT8D*  $F_{P_0} = 6\ 350$  kp

*Spey 511*  $F_{P_0} = 5\ 171$  kp

Fig.1 shows the specific  $\overline{\text{CO}}$  emission  $\frac{\text{pond emission}}{\text{kp/thrust} \cdot \text{h}/\text{hour}}$  as a function of the engine power setting (loading):  $\varphi = \frac{F_P}{F_{P_0}}$  ( $F_{P_0}$  is the thrust at full throttle).

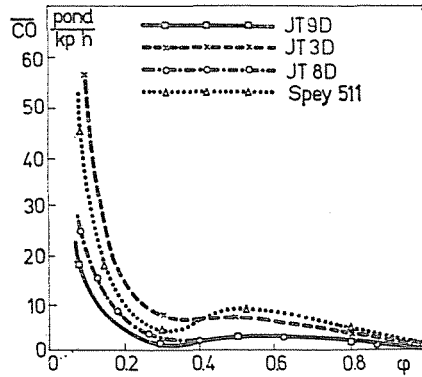


Fig. 1

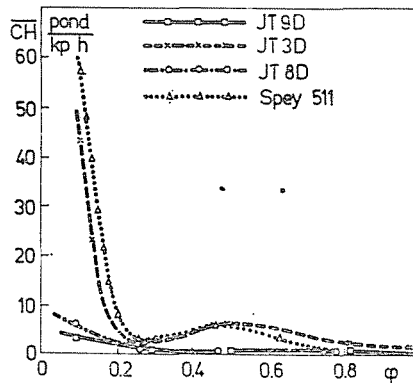


Fig. 2

In Fig. 2 the  $\overline{CH}$  values are plotted likewise against the engine power setting. Both emission components increase definitely by decreasing the engine power setting, by increasing the engine size (thrust) the emission, however, generally decreases. This latter is in connection with the more favourable combustion process due to the greater dimensions of the combustor.

In Fig. 3 the characteristics of the  $\overline{NO_x}$  emission are shown. In conformity with the above considerations, the  $\overline{NO_x}$  emission increases with the increase of the engine power setting; this latter, namely, can only be realized by increasing the highest temperature of the combustor. The dimensions of the engine hardly influence the  $\overline{NO_x}$  value, as it is the material of the turbine blades which the highest temperature depends on, rather than the size of the engine.

Fig. 4 shows the specific particulate content (smoke) of the exhaust gases. The maximum specific particulate emission arises at 40 to 60 p.c. loading. Namely in this range the insufficient formation of mixture enables the local

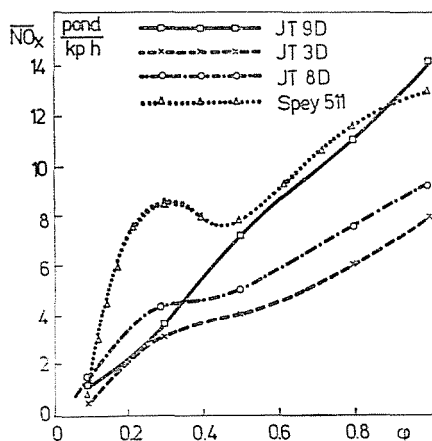


Fig. 3

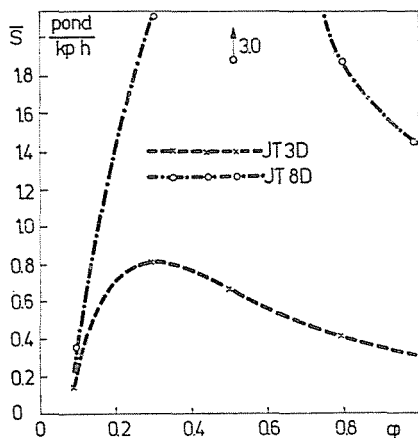


Fig. 4

development of rich mixture areas in the combustor and the great mass of secondary air still available will rather lead to the breaking off and slowing down of the combustion reaction of the rich mixture zones.

## 2.2 Similarity of the emission characteristics of the engines, approximate definition of the emission characteristics

Between the emission of the exhaust gases of the turbojet engines and the working cycle realized in the engines, i.e. the engine loading, there is a rather definite correlation. The dependence of the combustion process (isobaric heat release) affecting the emission on the thermal cycle enables to set up the above

relations and provides the similarity between emission characteristics of engines of different types. The cycle of the engine, i.e. its operation (loading) can only be varied by modifying the temperature of the working mass behind the combustor (at the turbine entry), i.e. by changing the dilution of combustion. As a consequence, not only the full combustion but the emission characteristics will be modified as well.

The similarity of combustion processes taking place in the combustor within the turbojet engines of different types are proved by their fully identical principles, as follows:

*a)* The air entering the combustor in each turbojet combustion process of constant pressure being at a lower temperature than that of fuel ignition, the essential combustion process comes only about at a definite dilution ( $m \approx 1$ ).

*b)* The continuity of combustion in each combustor is accomplished by creating vortex flow within the combustion zone, provided hereby a continuous and mutual mixing of the burning and yet unburned mixture.

*c)* In each up-to-date combustor, the atomization will be produced by a vortex spray nozzle at 60 to 100 atm. pressure. The cone angle of the spray, its quality, the co-ordination of the spray nozzle and the combustor are more or less the same in each combustor.

These considerations prove that there is a close similarity between the emission characteristics of certain engines (mainly in case of the same maximum thrust).

Further on the following theorem will be made use of: the specific emission characteristics of the engines of identical types (one- and two-flow turbojet engines, turboprop engines etc.) and of nearly the same maximum thrust (performance) will be taken to be the same as a function of loading.

### 3. Possibility of emission reduction in ground and near-ground flight operations

#### 3.1 *Basic relations. Data of the aircraft TU-134 and emission characteristics of its power plant*

In the following, the gases exhausted from the turbojet engines in different operation phases (take-off, landing, taxiing) will always be weighed as a first step. Hereupon the conditions to minimize this weight will be investigated. Finally, on the basis of the emission characteristics of the engine already available, the weight of certain pollutant components in the affected operation phase will be calculated.

The weight (in kp) of the exhaust per second is supposed to be proportional to the engine thrust  $F_p$ :

$$\dot{E} = CF_p .$$

The thrust  $F_p$  is expressed as

$$F_p = \varphi F_{P_0} ,$$

where  $F_{P_0}$  is the maximum (take-off) thrust of the engine, taken as constant (independent of the flight velocity  $V$ ), the dimensionless coefficient  $\varphi$  is a constant value, too, *characteristic of the operation phase* corresponding essentially to the throttle setting. The maximum value of  $\varphi$  is, by definition, 1.0 (full throttle).

In the above expression of  $\dot{E}$  neither the proportional coefficient  $C$  is constant but it depends on the operation mode of the gas-turbine engine, i.e. on  $\varphi$  as well:

$$C = C_0 \bar{C}(\varphi) .$$

Here  $C_0$  is the corresponding value of the operation mode,  $\varphi = 1.0$  (full loading),  $\bar{C}(1) = 1.0$ .  $[C] = [C_0] = 1/s$  and thus,  $\bar{C}(\varphi)$  is dimensionless.

By the above expressions we get:

$$\dot{E} = C_0 \bar{C}(\varphi) \varphi F_{P_0} . \quad (1)$$

From the evaluation of the relevant data of several existing turbojet engines it follows that for  $\bar{C}(\varphi)$  the following simple relationship can be applied with good approximation:

$$\bar{C}(\varphi) = \frac{1}{\varphi^\nu} . \quad (2)$$

The exponent  $\nu$  is a constant value, characteristic of the type of engine. Thus,

$$\dot{E} = C_0 \varphi^{1-\nu} F_{P_0} = B \varphi^{1-\nu}, \quad (B = C_0 F_{P_0})$$

and the weight of the total exhaust emitted during the operation phase:

$$E = \dot{E} t = B \varphi^{1-\nu} t . \quad (3)$$

$t$  is the time of the considered operation phase in seconds.

The results of the following investigations will be applied to the commercial aircraft type *TU-134* for getting a clear numerical picture of the emission conditions. Basic data of the aircraft involved in the investigation:

Maximum take-off weight:  $G_{t_0} = 44\,000$  kp

Maximum landing weight:  $G_l = 40\,000$  kp

Area of the wing:  $A = 115$  m<sup>2</sup>

Take-off thrust:  $F_{P_0} = 2 \cdot 6600 = 13\,200$  kp

(rather than  $2 \cdot 6800 = 13\,600$  kp as  $F_{P_0}$  begins to decrease somewhat with the speed.)

Idle thrust:  $F_{Pid} = 2 \cdot 450 = 900$  kp.

The polar curves of the aircraft for the various flight operations are found in "Elements of the Aerodynamics of the Aircraft TU-134" by T. I. Ligum.

The characteristic values of one engine of the aircraft TU-134 are given in Table 1.

Table 1

$\varphi$	1	0,3	0,6	0,4	0,3	0,1
$\dot{E}$ [kp/s]	63	57.3	50.5	41.8	35.7	17.5
$F_p$ [kp]	6600	5280	3960	2640	1980	660
$C$ [1/s]	0.00955	0.0108	0.01275	0.01582	0.01805	0.0265
$\bar{C}(\varphi)$ [—]	1	1.132	1.336	1.659	1.89	2.78

According to the data in the Table,  $C_0 = 0.00955$  1/s and the function  $\bar{C}(\varphi)$  is

$$\bar{C}(\varphi) = \frac{1}{\varphi^{\nu}} = \frac{1}{\varphi^{0.55}}$$

This approximate relationship provides sufficient accuracy in the range  $\varphi = 1-0.1$ .

The power plant of the aircraft *TU-134* is a two-flow turbojet engine as well, its thrust is just the same as that of the engine *JT8D*. The thrust of the engine *JT3D* is greater, the thrust of engine *Spey 511*, however, is less. Making use of the theorem of similarity in Item 2, the emission characteristics of the engine of the aircraft *TU-134* are listed in Table 2. This Table contains the specific emission values relating to the thrust ( $\overline{CO}$ ;  $\overline{CH}$ ;  $\overline{NO_x}$ ;  $\overline{S}$ ), the hourly emission of an engine ( $\underline{CO}$ ;  $\underline{CH}$ ;  $\underline{NO_x}$ ;  $\underline{S}$ ) and the specific values of the different emission components referred to the weight of the total exhaust ( $\overline{\overline{CO}}$ ;  $\overline{\overline{CH}}$ ;  $\overline{\overline{NO_x}}$ ;  $\overline{\overline{S}}$ ). In the bottomline the sum ( $\overline{\overline{\Sigma}}$ ) of the specific emission is given.

Table 2

$\varphi$	1	0,8	0,6	0,4	0,1
$\overline{CO}$ [pond/kph]	1.3	2.5	4.0	6	30
$\overline{CH}$ [pond/kph]	0.5	1.2	6	4	15
$\overline{NO_x}$ [pond/kph]	12.5	10	8	5	1
$\overline{S}$ [pond/kph]	1.0	1.3	1.6	1.5	0.4
$\underline{CO}$ [kp/h]	8.59	13.2	15.85	15.85	19.8
$\underline{CH}$ [kp/h]	3.3	6.34	23.8	10.6	9.9
$\underline{NO_x}$ [kp/h]	82.5	52.8	31.7	13.2	0.66
$\underline{S}$ [kp/h]	6.6	6.86	6.34	3.96	0.264
$\overline{\overline{CO}} \cdot 10^3$ [—]	0.0378	0.064	0.0871	0.10	0.315
$\overline{\overline{CH}} \cdot 10^3$ [—]	0.0145	0.0307	0.1309	0.0705	0.158
$\overline{\overline{NO_x}} \cdot 10^3$ [—]	0.363	0.256	0.1743	0.0876	0.0105
$\overline{\overline{S}} \cdot 10^3$ [—]	0.0291	0.033	0.0348	0.0263	0.0042
$\overline{\overline{\Sigma}} \cdot 10^3$ [—]	0.4444	0.3837	0.4271	0.2844	0.4877

In conformity with the above considerations, among the specific emission components referred to the total exhaust, the value of  $\overline{\overline{CO}}$  and  $\overline{\overline{CH}}$  increases,  $\overline{\overline{NO_x}}$  decreases, while  $\overline{\overline{S}}$  remains rather constant when  $\varphi$  is decreasing.



### 3.2 Possibility of exhaust reduction in take-off

#### 3.21 Take-off run with minimum exhaust

Formula (3) is valid for this operation phase, too

$$E_{t_0} = B\varphi_{t_0}^{1-\nu} t_{t_0}. \quad (3 \text{ to})$$

The time  $t_{t_0}$  of take-off run on the ground (for  $\varrho_0 = 0.125 \text{ kp m}^{-4}\text{s}^2$ ) is given by

$$t_{t_0} = 2 \frac{G_{t_0}}{g} \int_0^{q_{t_0}} \frac{dq}{F\sqrt{q}}.$$

$q_{t_0}$  is the dynamic pressure of the *lift-off speed* as given in the Flight Manual (hereinafter: Fl.Ma.) of the aircraft,  $F$  the resultant accelerating force in take-off run. The accelerating force linear in  $q$  can be written as [9]:

$$F = F_0 - \frac{F_0 - F_{t_0}}{q_{t_0}} q = \frac{F_0 q_{t_0} - (F_0 - F_{t_0}) q}{q_{t_0}},$$

where

$$\begin{aligned} F_0 &= \varphi_{t_0} F_{P_0} - \mu G_{t_0} \\ F_{t_0} &= \varphi_{t_0} F_{P_0} - X(q_{t_0}) = \varphi_{t_0} F_{P_0} - c_{xt_0} q_{t_0} A \\ q_{t_0} &= \frac{G_{t_0}}{c_{yt_0} A}. \end{aligned}$$

$c_{xt_0}$  and  $c_{yt_0}$  are the coefficients of aerodynamic forces in the polar of the operation phase (extended landing gear, the angle of deflection of the flap:  $\delta_F = \delta_{F_{t_0}}$ ) at the angle of attack  $\alpha_{t_0}$  of take-off run. The layout of the undercarriage gives  $\alpha_{t_0}$  as the aircraft effects the take-off run in "three point" position.

Substituting the former expressions in the above formula of  $t_{t_0}$  and integrating we obtain

$$t_{t_0} = \frac{4,606 G_{t_0} q_{t_0}}{g \sqrt{D_2 \varphi_{t_0} + D_3}} \log \frac{\sqrt{D_2 \varphi_{t_0} + D_3} + D_4}{\sqrt{D_2 \varphi_{t_0} + D_3} - D_4}, \quad (4)$$

where  $D_2 = D_1 F_{P_0} q_{t_0}$ ;  $D_3 = -D_1 q_{t_0} \mu G_{t_0}$ ;  $D_4 = D_1 \sqrt{q_{t_0}}$   
and

$$D_1 = c_{xt_0} q_{t_0} A - \mu G_{t_0}.$$

$D_1$  is practically always positive, as  $\alpha_{t_0}$  is a small angle ( $1^\circ$  to  $2^\circ$ ).

The weight of the exhaust during the take-off run can be calculated by formula (3 to), in consideration of (4).

What is now the throttle setting  $\varphi_{t_0}$  required for the take-off run in order to get a minimum weight of exhaust. Of course, the acceptability of  $\varphi_{t_0}$  got hereby must be checked from other operation points of view.

The minimum of  $E_{t_0}$  has to be evaluated by differentiating the expression (3 to) with respect to  $\varphi_{t_0}$ . Zeroing the derivative,  $\varphi_{t_0}$  giving the minimum can only be determined graphically from the obtained transcendent equation.

Instead of this usual procedure, it is more expedient in this case to plot the function  $E_{t_0} = f(\varphi_{t_0})$  according to (3 to), i.e. of (4) and to read off  $\varphi_{t_0}$  giving the minimum and  $E_{t_0\min}$ .

Physically the maximum of  $\varphi_{t_0}$  is 1.0 corresponding to the take-off run at full throttle. The minimum of  $\varphi_{t_0}$  (i.e.  $\varphi$ ) corresponds, from the turbojet engine point of view, to the value at idle thrust

$$\varphi_{\min 1} = \frac{F_{id}}{F_{P_0}}$$

In take-off run, when the aircraft is just going to run:

$$\varphi_{\min 2} = \frac{\mu G_{t_0}}{F_{P_0}}$$

For attaining, however, in take-off run at least the lift-off speed  $V_{l_0}$  corresponding to  $q_{l_0}$  — viz. just to carry out the lift-off maneuver — in extreme case the thrust of engines has to equal the sum of drag and rolling resistance at  $q_{l_0}$ :

$$F_{P_{t_0\min}} = \varphi_{t_0\min} F_{P_0} = c_{x_{t_0}} q_{l_0} A + G_{t_0} \frac{q_{l_0} - q_{l_0}}{q_{l_0}},$$

hereby, using the former symbols:

$$\varphi_{t_0\min} = \frac{D_1 q_{l_0}}{F_{P_0} q_{l_0}} + \frac{\mu G_{t_0}}{F_{P_0}} = \frac{D_4^2 - D_3}{D_2}. \quad (5)$$

The same result will be got even mathematically from relationship (4). The lowest limit of the domain of definition of the function  $E_{t_0} = f(\varphi_{t_0})$  is exactly  $\varphi_{t_0\min}$  as the logarithm is not defined for negative numbers. (The mathematical upper limit of the domain of definition is  $+\infty$  and the physical upper limit is 1.0.)

If  $\varphi_{t_0}$  tends to  $\varphi_{t_0\min}$  from above, in accordance with (4) and (3 to), both  $t_{l_0}$  and  $E_{t_0}$  tend to the infinity. Consequently, the thrust  $F_{P_{t_0\min}}$  can just attain

the lift-off speed (with  $\varphi_{t_{0\min}}$ ) though in an infinite long time and, of course, producing infinite exhaust meanwhile.

When  $\varphi_{t_0} \rightarrow +\infty$ , then  $t_{t_0} \rightarrow 0$  and  $E_{t_0} \rightarrow 0$ . Between  $q_{t_{0\min}}$  and  $q_{t_0} = +\infty$ , however, a local minimum can be got, this is of practical importance only for a  $q_{t_0}$  value lower than 1.0.

### 3.22 Application to the aircraft TU-134

This aircraft effects the take-off run at an angle of attack  $\alpha_{t_0} = 1^\circ$  and at an angle of deflection  $\delta_{F_{t_0}} = 20^\circ$  of the flap. Then, according to Fig. 18 in Ligum's book

$$c_{x_{t_0}} = 0.067 \text{ and } c_{y_{t_0}} = 0.46.$$

According to Fl. Ma.:

$$V_{t_0} = 260 \text{ km/h} = 72.3 \text{ m/s, thus } q_{t_0} = 326 \text{ kp/m}^2.$$

Fig. 5 shows the variation of  $t_{t_0}$  and  $E_{t_0}$  as a function of  $\varphi_{t_0}$  using the above numerical data and the already given value  $\nu = 0.55$  on the basis of the relationships (4) and (3 to).

As it appears from the Figure, the curve  $E_{t_0} = f(\varphi_{t_0})$  will have its minimum at the value  $\varphi_{t_0} = 1.0$  viz., the take-off run effected at take-off thrust (corresponding to Fl. Ma.) produces the minimum of exhaust. This is, however, not necessarily valid for other types of aircraft.

It is remarkable that the first, rough approximation with the assumption of  $C = \text{const} = C_0$  (i.e.  $\nu = 0$ ) for the aircraft TU-134 would give a minimum of  $E_{t_0}$  at  $\varphi_{t_0} = 0.8$ . This minimum, however, is only by some 2.4 p.c. less than the value corresponding to  $\varphi_{t_0} = 1.0$ .

At the same time, the time of take-off run (unaffected by  $\nu$ ) would increase by 6.3 s according to Fig. 5; thus, the take-off run would be lengthened by about 420 m ( $\sim 40$  p.c.). Under these conditions, it is not worth while to strive after the minimum of  $E_{t_0}$ .

Hereupon, let us examine the quantities (kp) of emission components discharged by the aircraft in take-off run at different  $\varphi_{t_0}$  values, using data in the last five lines of Table 2. Being

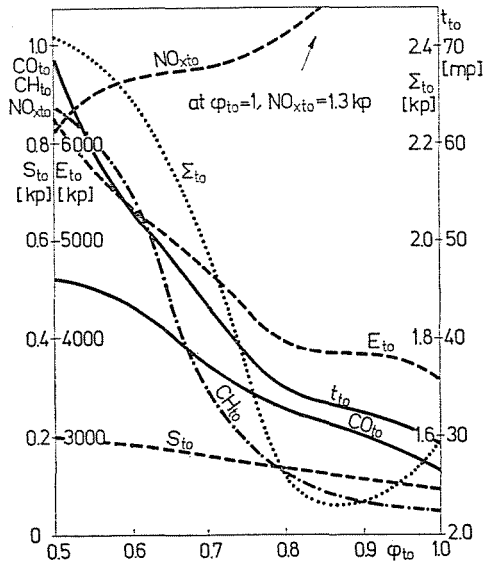


Fig. 5

aware of the specific emission components for the exhaust ( $\overline{\text{CO}}$ , etc.) and of the total exhaust ( $E_{t_0}$ ) belonging to the given  $\varphi_{t_0}$  values, the emission components discharged in take-off run ( $\text{CO}_{t_0}$ ,  $\text{CH}_{t_0}$ ,  $\text{NO}_{x t_0}$ ,  $\text{S}_{t_0}$ ) can be defined. The results are also shown in Fig. 5. The definition of  $E_{t_0}$  already involved the aircraft *TU-134* to be powered with two engines. In conformity with our considerations to now, the aircraft *TU-134* emits the minimum  $\text{CO}_{t_0}$ ,  $\text{CH}_{t_0}$  and  $\text{S}_{t_0}$  emissions in a take-off run according to operation mode  $\varphi_{t_0} = 1$ . With decreasing  $\varphi_{t_0}$ , however, the  $\text{NO}_x$  emission decreases continuously as well, consequently a long take-off run is favourable. It follows also from this variation of the  $\text{NO}_x$  emission that the minimum value of the total emission  $\Sigma_{t_0}$  will be at  $\varphi_{t_0} \cong 0.86$ . Hence, evidently, the optimum take-off runs are remarkably different from the point of view of various emission components.

### 3.23 Climbout with minimum exhaust

The climbout following the take-off run will be considered to 1000 m altitude only, as according to the relevant technical literature the exhaust of the turbojet engines above that altitude can already be neglected from the point of view of the air pollution in airports.

The aircraft carries out this section of climbout at a safety speed (indicated air speed)  $V_c = 1.2 V_{\min}$ .  $V_{\min}$  is to calculate with  $c_{y\max}$  corresponding to the retracted landing gear and the closed flap position ("smooth" aircraft) as the retraction of the landing gear begins already at 5–10 m altitude and both retraction of the landing gear and closing of the flap will be complete at about 100 m.

In climbout the thrust of the engine is the *nominal thrust*, 92 to 94 p.c. of the take-off thrust ( $\varphi_{\text{nom}} = 0.92$  to 0.94).

Throttling down the engine — keeping of course the safety speed  $V_c = \dot{E}_c$  will decrease. The aircraft, however, will climb on a flatter path, consequently the climbing time will increase.

The formula (3) is valid for that operation phase, too:

$$E_c = B\varphi_c^{1-\nu} t_c. \quad (3c)$$

The climbing time with rate of climb  $w_{c\varphi}$  is

$$t_c = \frac{10^3}{w_{c\varphi}} = \frac{10^3 G_{t_0}}{(\varphi_c F_{P_0} - X_c) V_c} \quad (6)$$

where  $X_c$  is the drag of the "smooth" aircraft at a speed  $V_c$ .

The exhaust will be

$$E_c = \frac{10^3 B G_{t_0} \varphi_c^{1-\nu}}{(\varphi_c F_{P_0} - X_c) V_c}. \quad (7)$$

In case of  $\nu < 1.0$  (practical value), the function (7) would have its extreme value at negative  $\varphi_c$  if the function is defined for negative numbers at all. This, however, is irrelevant in practice.

According to (7),  $E_c$  varies in the positive  $\varphi_c$  range hyperbolically. The vertical asymptote of the hyperbola will be at

$$\varphi_{c_{\min}} = \frac{X_c}{F_{P_0}}.$$

Here both  $t_c$  and  $E_c$  are infinite as the aircraft flies level (on a path of  $0^\circ$  slope).

The minimum of  $E_c$  is given by  $\varphi_{\text{nom}}$  owing to the hyperbolic decrease of  $E_c$ .

This infinite mass of exhaust belonging to  $\varphi_{c_{\min}}$  arises on an infinite long flight path. Present paper, however, deals with the air pollution in the airport's vicinity. Characterizing the vicinity of the airport by the distance  $L_{vi}$  from the lift-off point, we can easily realize that the hyperbolic variation of  $t_c$  and  $E_c$  is valid only for the section between  $\varphi_{\text{nom}}$  and  $\varphi_{vi}$ , where, according to (6) and taking into account that  $t_c = \frac{L_{vi}}{V_c}$

$$\varphi_{vi} = \frac{10^3 G_{t_0} + X_c L_{vi}}{F_{P_0} L_{vi}}.$$

For  $\varphi_c < \varphi_{vi}$ ,  $t_c$  can be taken as constant, consequently  $E_c$ , according to (3c), decreases as a  $(1 - \nu)$ -degree parabola.

Now, we have to compare  $E_c$  arising on the allowable flattest climbing path (e.g. one engine inoperative) with  $E_c$  belonging to  $\varphi_{\text{nom}}$ ; first of all as to the weight, than regarding the pollutant components, as the engine's operating conditions differ greatly in the two cases.

### 3.24 Application to the aircraft TU-134

For the "smooth" aircraft  $c_{y_{\max}} = 1.45$  (see Fig.13 of the quoted Ligum book).  
Hereby

$$V_c = 1,2V_{\min} = 1,2 \sqrt{\frac{2G_{t_0}}{\rho_0 A c_{y_{\max}}}} = 78 \text{m/s} \quad \text{and} \quad q_c = 380 \text{kg/m}^2.$$

The lift coefficient in climbing flight is

$$c_{y_c} = \frac{G_{t_0}}{q_c A} = 1,01$$

involving  $c_{x_c} = 0,075$  in "smooth" polar (Fig.14 by Ligum).

Thus,

$$X_c = c_{x_c} q_c A = 3280 \text{kp},$$

the time of climb is  $t_c = 61,7 \text{ s}$  see (6) and according to (3c) using the value  $\varphi_{\text{nom}} = 0,94$

$$E_{c_{\text{nom}}} = 7540 \text{kp}.$$

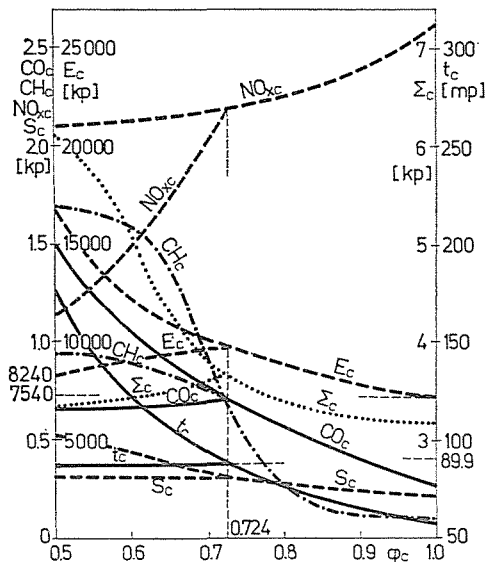


Fig. 6

Confining ourselves to the vicinity of the airport let us take  $L_{vi} = 7000$  m. Then  $t_c = \frac{L_{vi}}{V_c} = \frac{7000}{78} = 89.9$  s and — applying the above formula —  $\varphi_{vi} = 0.724$ .

The weight of the exhaust for  $\varphi_c < \varphi_{vi}$  will be

$$E_{cvi} = B t_c \varphi_c^{1-\nu} = 11280 \varphi_c^{0.45}.$$

The branching out of  $t_c$  and  $E_c$  (and the emission components as well) are plotted by the same lines for the case  $\varphi_c < \varphi_{vi}$  in Fig. 6.

Taking the allowable flattest climbing path identical with that of one engine inoperative, the value  $\varphi_c = 0.5$  enters in calculating  $E_{cvi}$ . So we get

$$E_{cvi} = 11280 \cdot 0.5^{0.45} = 8240 \text{ kp.}$$

The values of  $t_c$ ,  $E_c$  and of the partial emissions in function of  $\varphi_c$  are shown in Fig. 6. The weights of  $E_{cnom}$  and  $E_{cvi}$  do not differ essentially. In case of airports with too much of  $\text{NO}_x$  component in the background pollution, the take-off run and the climbout have to be fulfilled at the possible lowest  $\varphi$  value (dependent on the circumstances of the airport) in order to reduce the  $\text{NO}_x$  content. In consequence of the considerations so far, the minimum value of  $\varphi$  will be determined mainly by the area of the airport.

### 3.3 Possibility of exhaust decrease in landing

#### 3.31 Approach with minimum exhaust

In landing the aircraft descends to 500 m at idle running of the engines. Thereafter, at the “First Marker” located at a certain distance (8 to 15 km) from the threshold of the runway it turns to the smooth *ILS*-path. The aircraft is able to glide on that flat path only if the engines produce adequate thrust.

Consequently, the exhaust per unit length of the *ILS*-path in gliding is essentially higher than that in descent.

In approach with thrust, the angle of path ( $\Theta$ ) and the thrust ( $F_P = \varphi_a F_{Po}$ ) are related as:

$$\sin \Theta = \frac{X_a - F_P}{G_l} = \frac{X_a - \varphi_a F_{Po}}{G_l}, \quad (8)$$

where  $X_a$  is the drag at the constant (indicated) approach speed  $V_a$  as given in Fl. Ma.

$$\text{From } Y_a \cong G_l \quad c_{ya} = \frac{G_l}{q_a A}$$

taking the value of  $c_{xa}$  from the polar of the aircraft belonging to extended landing gear and flap angle of deflection in approach according to Fl. Ma. Hence:

$$X_a = c_{xa} q_a A.$$

The time of approach is

$$t_a = \frac{500}{V_a \sin \Theta} = \frac{500 G_l}{(X_a - \varphi_a F_{Po}) V_a} \quad (9)$$

and the weight of exhaust, from (3):

$$E_a = B \varphi_a^{1-\nu} t_a = \frac{500 B G_l \varphi_a^{1-\nu}}{(X_a - \varphi_a F_{Po}) V_a}. \quad (10)$$

Essentially, this formula is structurally identical with (7), only that the corresponding members in the denominator are exchanged.

It is valid for the expression (10) as well that in the practical case of  $\nu < 1.0$  the extreme value of  $E_a$  is at a negative  $\varphi_a$  (if the function is defined for negative numbers at all), which, however, is irrelevant.

$$E_a \rightarrow \infty \text{ for } \varphi_{a_{\max}} \rightarrow \frac{X_a}{F_{Po}}.$$

The positive branch of the curve  $E_a$  passing through the origin of the coordinate system tends hyperbolically to a vertical asymptote at  $\varphi_{a_{\max}}$ . Thus, the minimum of  $E_a$  is at the idle thrust  $\varphi_{\min 1} = \varphi_{id}$

$$E_{a_{\min}} = B \varphi_{id}^{1-\nu} t_a \quad (11)$$

(Now in (9)  $\varphi_a = \varphi_{id}$ ).

### 3.32 Application to aircraft TU-134

According to Fl. Ma.,  $V_a = 250$  km/h = 69.5 m/s, thus,  $q_a = 300$  kp/m<sup>2</sup>. The angle of deflection of the flap is 38°, that of the fuselage flap 40°, the landing gear is extended. The polar of aircraft TU-134 for this operation mode is shown in Fig. 38 by Ligum.

$$c_{ya} = \frac{C_L}{q_a A} = 1.16 \text{ hence } c_{xa} = 0.2 \text{ from the polar and so}$$

$$X_a = c_{xa} q_a A = 6900 \text{ kp.}$$

The angle of approach path according to (8):  $\Theta = 8^\circ 38'$ .

According to (9) the time of approach is  $t_a = 96$  s (here calculating with 1000 m instead of 500 m as also the descent will be at  $F_{Pid}$ ).

Length of the approach path:  $1000/\sin 8^\circ 38' = 6700$  m considered already to be the vicinity of the airport.

The minimum exhaust is calculated from  $\varphi_{id} = F_{Pid}/F_{P0} = 0.0682$ , hence according to (10):

$E_{a_{min}} = 3610$  kp (assuming  $\bar{C}(\varphi) = \frac{1}{\varphi^{0.55}}$  to be valid for  $\varphi_{id}$  as well). The approach path gradient  $8^\circ 38'$  being too steep, it is unlike to fit the approach of the aircraft. In recent years the application of  $6^\circ$  ILS-paths has been considered — mainly for noise abatement — the airplane pilots are, however, still averse to accept it.

Let us calculate the approach data for the nowadays applied  $2^\circ 40'$  ILS-path. In this case, the altitude to be considered for the approach path 7000 m long (rather similar to the former case) is  $7000 \cdot \operatorname{tg} 2^\circ 40' = 326$  m.

The required thrust is, according to (8),  $F_P = 5040$  kp and hence:

$$\varphi_a = \frac{5040}{13200} = 0.382.$$

From (9)  $t_a = 100.8$  s (taking 326 m instead of 500 m). From (10)  $E_a = 8240$  kp.

It has to be pointed out that approach on a  $2^\circ 40'$  ILS-path produces an exhaust weighing 2.28 times that in a  $8^\circ 38'$  path giving the minimum.

From the foregoing it is evident that by decreasing the gradient of approach path, the quantity of exhaust increases.  $E_{a_{min}}$  belongs, however, to the idle operation of engines where the values of  $\bar{CO}$  and  $\bar{CH}$  are especially high, while practically neither  $\bar{NO}_x$  nor  $\bar{S}$  result. As  $\bar{\Sigma}$  is approximately constant, the minimum total emission  $\Sigma$  will be got in approaching in idle operation. Table 3 compiles the final conclusions of the above considerations.

Table 3

$\Theta$	$\varphi_a$	$t_a$ (s)	$E_a$ (kp)	CO(kp)	CH(kp)	NO <sub>x</sub> (kp)	S(kp)	$\Sigma$ (kp)
$8^\circ 38'$	0.0682	96.0	3610	1.262	0.636	0.018	0.007	1.973
$2^\circ 40'$	0.382	100.8	8240	0.826	0.578	0.717	0.216	2.335

### 3.33 Landing run with minimum exhaust

The conditions in landing run are similar to those in take-off run; the braking force is a linear function of  $q$ , just as the accelerating force in take-off run.

Many airliners are equipped with a brake parachute as well; this will only be released on a wet slippery runway.



The braking force linear in  $q$  can be written as:

$$F = \frac{F_o q_l + (F_l - F_o) q}{q_l}$$

where

a) for a dry runway

$$\left. \begin{aligned} F_o &= \mu_d G_l - F_{Pid}, \\ F_l &= X_l + \mu_d(G_l - Y_l) - F_{Pid} = c_{xl} q_l A + \mu_d(G_l - c_{yl} q_l A) - F_{Pid} \end{aligned} \right\} \quad (12)$$

b) for a wet runway

$$\left. \begin{aligned} F_o &= \mu_w G_l - F_{Pid}, \\ F_l &= X_l + \mu_w(G_l - Y_l) + X_{Bp} - F_{Pid} = \\ &= c_{xl} q_l A + \mu_w(G_l - c_{yl} q_l A) + c_{xBp} q_l A_{Bp} - F_{Pid} \end{aligned} \right\} \quad (13)$$

(In both cases  $F_l$  is generally greater than  $F_o$ .)

Here,  $q_l$  is the dynamic pressure of the *touch-down speed* as given in Fl. Ma.,  $\mu_d$  and  $\mu_w$  are coefficients of braking resistance on dry and wet runway resp., (owing to the braking of the main wheels, even the less valued  $\mu_w$  is much greater than  $\mu$  in the take-off run)  $c_{Bp}$  and  $A_{Bp}$  are drag coefficient and reference area of the released brake parachute, resp.,  $c_{xl}$  and  $c_{yl}$  are the respective coefficients of aerodynamic forces belonging to the angle of attack  $\alpha_l (= \alpha_{i0})$  in the polar for extended landing gear and for the angle of deflection prescribed for the flaps and for the spoiler in landing run.

During landing run, the exhaust depends on  $F_{Pid}$  of a constant value. Consequently it will be of a minimum weight in the shortest landing run, i.e. if the pilot takes the best advantage of the possibility of braking.

The time of landing run can be written (with  $q_0$ ) as

$$t_l = 2 \frac{G_l}{g} \int_0^{q_l} \frac{dq}{F \sqrt{q}} = 2 \frac{G_l}{g} q_l \int_0^{q_l} \frac{dq}{[F_o q_l + (F_l - F_o) q] \sqrt{q}},$$

after integrating:

$$t_l = \frac{4G_l \sqrt{q_l}}{g \sqrt{F_o(F_l - F_o)}} \operatorname{arc} \operatorname{tg} \sqrt{\frac{F_l - F_o}{F_o}}. \quad (14)$$

The exhaust during the landing run is:

$$E_l = B \varphi_{id}^{1-r} t_l. \quad (15)$$

### 3.34 Application to aircraft TU-134

According to Fl. Ma. the touch-down speed is  $V_l = 234 \text{ km/h} = 65 \text{ m/s}$ ; thus  $q_l = 264 \text{ kp/m}^2$ .

For the coefficients of braking resistance assume  $\mu_d = 0.25$ ,  $\mu_w = 0.13$ ,  $\alpha_l (= \alpha_{l0}) = 1^\circ$  (three point position), the angles of deflection of the wing flaps:  $38^\circ$ , of the fuselage flap:  $40^\circ$  and of the spoiler:  $52^\circ$ . Now, according to Fig. 38 by Ligum:

$$c_{xl} = 0.22, \quad c_{yl} = 0.37.$$

$A_{Bp} = 40 \text{ m}^2$  and according to the literature (e.g. Journal of Aircraft Vol. 6. No. 3. p. 269) can be taken:  $c_{xBp} = 0.6$ .

a) *Dry runway* [according to (12), (14), (3l)]

$$\begin{aligned} F_o &= 9100 \text{ kp}, \quad F_l = 12\,970 \text{ kp} \\ t_{ld} &= 25.9 \text{ s} \\ E_{ld} &= 975 \text{ kp} \end{aligned}$$

b) *Wet runway* [according to (13), (14), (3l)]

$$\begin{aligned} F_o &= 4300 \text{ kp} \\ F_l &= 15\,880 \text{ kp} \\ t_{lw} &= 38.5 \text{ s} \\ E_{lw} &= 1147 \text{ kp} \end{aligned}$$

Considerably less exhaust is seen to be produced in landing run than in the former operation phases.

Determining the emission components some 99 p.c. of the emission is seen to be CO and CH. Consequently, in case of a background pollution containing a considerable quantity of CO and CH, the run of the engines has to be minimized even from this aspect.

## 3.4 Possibility of exhaust reduction in ground

### 3.41 Taxiing with minimum exhaust

The aircraft taxis from the foreground of the airport to the take-off point of the runway and at landing after the landing run to the foreground. (Between the foreground and the standing place the aircraft is mostly trailed.)

The taxiing is carried out at relatively low speed  $V_t$  (which may be taken for constant), acted upon the thrust  $\varphi_t F_{Po}$ . The very short section of acceleration to  $V_t$  can be neglected. In taxiing the flap is in closed position.

For the practical values of  $V_t$  the aerodynamic forces can be disregarded so much the more as in the polar belonging to that operation mode (extended landing gear, closed flap)  $c_{yt}$  is just zero and  $c_{xt}$  is of a very small value.

So, the thrust has to equalize only the rolling resistance

$$\varphi_t F_{Po} = \mu G_{t0} \quad (\text{after landing: } \mu G_t)$$

whence

$$\varphi_t = \frac{\mu G_{t0}}{F_{Po}} \quad (\text{the former } \varphi_{\text{min}2}).$$

For a taxiing distance (e.g. to the take-off point)  $L_t$  the time of taxiing will be

$$t_t = \frac{L_t}{V_t} \quad \text{and} \quad E_t = B\varphi_t^{1-r} t_t. \quad (15)$$

### 3.42 Application to the aircraft TU-134

In Fl. Ma. no factual numerical value is given for the taxiing speed  $V_t$ , it is merely prescribed for the pilot to choose it in accordance with the conditions (width and quality of taxiway, obstructions, weather conditions etc.).

The average taxiing speed of the aircraft TU-134 can be taken as  $V_t = 40 \text{ km/h} = 11.1 \text{ m/s}$ . Let the average distance to the take-off point be  $L_t = 2000 \text{ m}$ . With these data

$$\begin{aligned} \varphi_t &= \varphi_{\min 2} = 0.0808 \text{ and according to (15)} \\ t_t &= 180 \text{ s hence } E_t = 7300 \text{ kg} \\ &\text{(2.5 kp CO; 1.3 kp CH).} \end{aligned}$$

Assuming, as a first approximation, the same quantity of exhaust in taxiing after landing, 14 600 kg exhaust will arise in a total flight cycle.

It is interesting to conclude that in the closer vicinity of the airport the taxiing produces the most exhaust, involving — as practically it takes place at idle running — the maximum quantity of CO and CH emission components. Increasing the taxiing speed, the mass and emission of the exhaust may be decreased, the reality of such considerations, however, is restricted by traffic safety view-points.

The emission arisen in taxiing could be reduced most effectively by trailing the aircrafts. This method, however, inevitably reducing the traffic capacity of the airport is inadmissible.

### 3.43 Emission in starting and load testing the engines

For these operations there are very detailed and varied prescriptions in Fl. Ma. referring to the wide-ranging variation of the throttle setting in very short periods. Hence, there is little to speak of the problem from the aspect of emission conditions.

## Summary

The relationship between the ground and nearground maneuvers of the aircraft and the emission of the engines is investigated first in general, then relating to the aircraft TU-134. A definite relationship has been proved to exist between the different maneuvers and the exhausted emission and each maneuver proved to be possible with the operation mode of minimum emission.

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