CALCULATION OF QUASISYMMETRICAL VEHICLE BODIES BY ASYMMETRICAL MODIFICATION

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1. Introduction

The vehicle structures, to a cursory glance, seem to be symmetrical. However, a closer investigation reveals that the fulfilment of the requirements of their functions (for example, in the case of buses the one-side door-openings) excludes the symmetrical design of the vehicle body in an economical way; in reality, the frame structure is asymmetrical (*Fig.* 1). It is also true that considering the integrity of the structure there are only a few parts which spoil the symmetry. Such structures are called *quasisymmetrical* ones [1].

If the asymmetry is caused by stiffness differences between some structural members situated symmetrically or different designs of some details, then their disturbing effects do not influence significantly the internal load distribution obtained by assuming symmetry. According to the investigations of MICHELBERGER, however, the effects of asymmetrical trimmings of major dimensions cannot be neglected; these may strongly influence the internal load distribution over the structure of the vehicle [2].

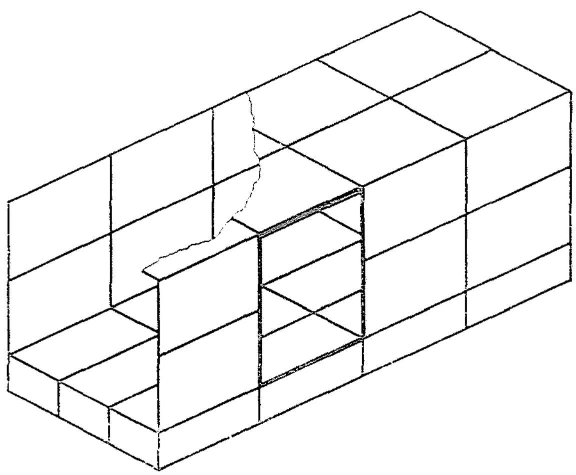

Fig. 1

Nevertheless, the direct investigation of an asymmetrical structure, involving enormous calculation work required by the problem encounters difficulties. Therefore, it may be understood that commonly one has to be satisfied with the assumption of symmetry in investigating a structure. Thus, for example, also BRZOSKA in his otherwise comprehensive research work on integral bus bodies disregarded the disturbing effect of door openings [3].

It was MICHELBERGER who treated of the determination of the internal loads of quasisymmetrical structures by making use of the scalar method [1]. However, no matrix formulation has been found as yet which would better suit computer programming.

2. Internal loads in symmetrical structure

Let us see what simplification might be obtained by assuming structural symmetry in determining the internal loads.

The symmetrical body represented in *Fig.* 2 was obtained by arbitrarily considering one of the symmetrically situated members of different stiffnesses to be of the same stiffness as the other one, and on the places of trimmings, lacking parts were replaced. Calculation is made by the *force method.*

In building up the basic system, symmetry has to be kept in view; redundant connections should be released in the median plane or in symmetrical members *(Fig. 2).* Symmetric self-equilibrating unit loads applied at releases in the median plane generate identical stresses in symmetrical members, and so do pairs of them applied in the same senses at symmetric releases. Similarly, in the symmetrical members, stresses of identical intensity but of opposite senses are generated by the antisymmetrical self-equilibrating unit loads.

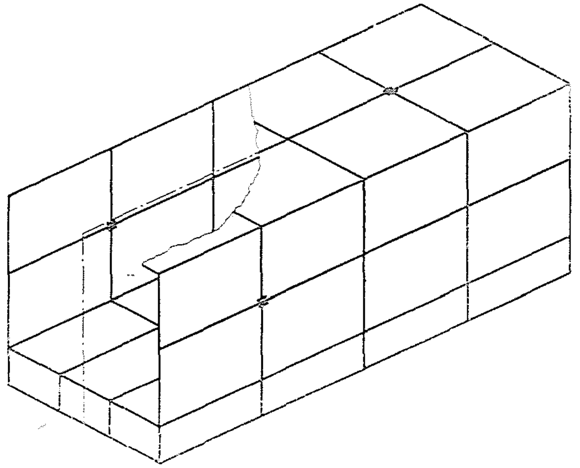

Fig. 2

Furthermore, in establishing the matrix of internal loads, due to the self-equilibrating unit loads by taking into account the elements in the same sequence on either side of the median plane, we obtain the following form

$$\mathbf{B} = \left[\begin{array}{cc} \mathbf{B}_S & \mathbf{B}_A \\ \hline \mathbf{B}_S & -\mathbf{B}_A \end{array}\right]. \tag{1}$$

The general asymmetrical load applied on the symmetrical structure may always be decomposed into a symmetrical and an antisymmetrical part of load. Consequently the internal loads in the basic system are described

— in case of symmetrical external load by the matrix

$$\mathbf{A}' = \left[\begin{array}{c} \mathbf{A}_S \\ \hline \mathbf{A}_S \end{array}\right] \tag{2}$$

— in case of antisymmetrical external load by the matrix

$$\mathbf{A}'' = \left[\begin{array}{c} \mathbf{A}_A \\ \hline -\mathbf{A}_A \end{array}\right]. \tag{3}$$

The flexibility matrix summing up the elasticity characteristics of the elements may, due to the symmetry, be written in the form of the diagonal hypermatrix

$$\mathbf{R} = \langle \overline{\mathbf{R}} \mid \overline{\mathbf{R}} \rangle . \tag{4}$$

Therefore, in the compatibility equation

$$\mathbf{D}\,\mathbf{x} + \mathbf{d} = \mathbf{0} \tag{5}$$

we have

$$\mathbf{D} = \mathbf{B}^*\mathbf{R}\mathbf{B} = \begin{bmatrix} 2\mathbf{B}_S^*\overline{\mathbf{R}}\mathbf{B}_S & \mathbf{0} \\ \mathbf{0} & 2\mathbf{B}_A^*\overline{\mathbf{R}}\mathbf{B}_A \end{bmatrix} = \begin{bmatrix} \mathbf{D}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_A \end{bmatrix} \tag{6}$$

as well as

$$\mathbf{d} = \mathbf{d}' = \mathbf{B}^*\mathbf{R}\mathbf{A}' = \begin{bmatrix} 2\mathbf{B}_S^*\overline{\mathbf{R}}\mathbf{A}_S \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{d}_S \\ \mathbf{0} \end{bmatrix} \tag{7}$$

and

$$\mathbf{d} = \mathbf{d}'' = \mathbf{B}^*\mathbf{R}\mathbf{A}'' = \begin{bmatrix} \mathbf{0} \\ 2\mathbf{B}_A^*\overline{\mathbf{R}}\mathbf{A}_A \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{d}_A \end{bmatrix} \tag{8}$$

according to the symmetrical and antisymmetrical loads, respectively.

Thus, the true internal load distribution of the symmetrical structure is of the form

— in case of a symmetrical load:

$$\mathbf{L}' = \begin{bmatrix} \mathbf{L}_S \\ \hdashline \mathbf{L}_S \end{bmatrix} \tag{9}$$

— in case of an antisymmetrical load:

$$\mathbf{L}'' = \begin{bmatrix} \mathbf{L}_A \\ \hdashline -\mathbf{L}_A \end{bmatrix} \tag{10}$$

and for the case of a general asymmetrical load it may be obtained by superposition of (9) and (10)

$$\mathbf{L} = \mathbf{L}' + \mathbf{L}'' = \begin{bmatrix} \mathbf{L}_S + \mathbf{L}_A \\ \hdashline \mathbf{L}_S - \mathbf{L}_A \end{bmatrix}. \tag{11}$$

Consequently, it can be stated that in case of a symmetrical structure the coefficient matrix ($\mathbf{D}$) of the compatibility equation will be disintegrated and so will be the set of equations into two independent sets of equations according to the cases of the symmetrical and antisymmetrical loads. By this, the calculation work will significantly be reduced due to the approximate halving of the dimensions of the matrices to be inverted. Another advantage of this procedure is that in calculation it is sufficient to use only a part of matrices $\mathbf{B}$, $\mathbf{A}'$, $\mathbf{A}''$ and $\mathbf{R}$ (see the broken lines in their corresponding formulas) relating to the one half of the structure which lessens the preparatory work.

3. Internal loads in the quasisymmetrical structure

The internal loads of the symmetrical structure being known, those induced in the quasisymmetrical structure may be determined by the application of the redundant basic systems. In order to restore the quasisymmetry let us arrange elements which have stiffnesses restoring the original conditions parallel to the elements whose stiffnesses were arbitrarily changed or which were inserted at the trimmings (*Fig. 3*). Accordingly it is seen that generally we have a *connection problem* which may be solved [4, 5]. In the case of statically determined connecting elements the connection problem simplifies to a *modification problem* [5, 6]. In the following this case will be investigated as that being the most significant in the practice.

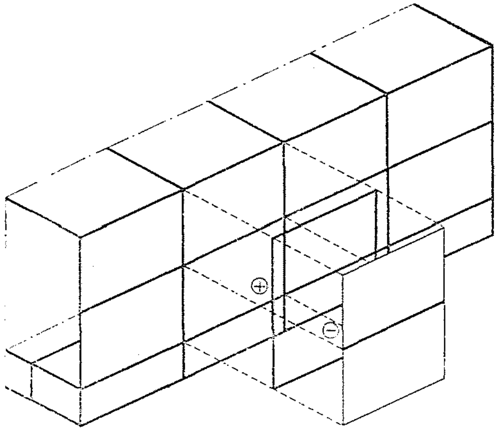

Fig. 3

The internal loads of the modified structure can be determined from the internal loads of the primary structure by the relationship [6]:

$$\mathbf{L}_M = \mathbf{L} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}_v^*(\mathbf{B}_v\mathbf{D}^{-1}\mathbf{B}_v^* + \Delta\mathbf{R}^{-1})^{-1}\mathbf{L}_v \tag{12}$$

Here

$$\Delta\mathbf{R}^{-1} = -\mathbf{R}_v^{-1}(\mathbf{R}_v + \mathbf{R}_M)\mathbf{R}_v^{-1}$$

and

- $\mathbf{L}_M$ — internal loads obtained by modification,
- $\mathbf{L}$ — internal loads of the primary structure,
- $\mathbf{B}$ — matrix of internal loads due to self-equilibrating unit loads of the primary structure,
- $\mathbf{D}^{-1}$ — inverse of coefficient matrix in compatibility equation of the primary structure,
- $\mathbf{B}_v$, $\mathbf{R}_v$, $\mathbf{L}_v$ — submatrices of the primary structure matrices $\mathbf{B}$, $\mathbf{R}$, $\mathbf{L}$ formed from their rows, corresponding to the modified structural members,
- $\mathbf{R}_M$ — flexibility matrix of modifying elements.

In modifying the symmetrical structure investigated in Chapter 2, by virtue of (1) we have

$$\mathbf{B}_v = [\mathbf{B}_{Sv} \quad \mathbf{B}_{Av}] \tag{13}$$

according to (4)

$$\mathbf{R}_v = \overline{\mathbf{R}}_v \tag{14}$$

and

— in case of symmetrical external load, from (9)

$$\mathbf{L}'_v = \mathbf{L}_{Sv} \tag{15}$$

— in case of antisymmetrical external load, from (10)

$$\mathbf{L}_v'' = \mathbf{L}_{Av} \tag{16}$$

— and in case of asymmetrical load, from (11)

$$\mathbf{L}_v = \mathbf{L}_{Sv} + \mathbf{L}_{Av} \tag{17}$$

Accordingly, the internal loads generated in the quasisymmetric structure are obtained by replacement the respective matrices into (12), i.e.,

— in case of symmetrical load

$$\mathbf{L}_M' = \begin{bmatrix} \mathbf{L}_S - (\mathbf{B}_S\mathbf{D}_S^{-1}\mathbf{B}_{Sv}^* + \mathbf{B}_A\mathbf{D}_A^{-1}\mathbf{B}_{Av}^*)\Delta\mathbf{R}_M\mathbf{L}_{Sv} \\ \mathbf{L}_S - (\mathbf{B}_S\mathbf{D}_S^{-1}\mathbf{B}_{Sv}^* - \mathbf{B}_A\mathbf{D}_A^{-1}\mathbf{B}_{Av}^*)\Delta\mathbf{R}_M\mathbf{L}_{Sv} \end{bmatrix} \tag{18}$$

— in case of an antisymmetrical load

$$\mathbf{L}_M'' = \begin{bmatrix} \mathbf{L}_A - (\mathbf{B}_S\mathbf{D}_S^{-1}\mathbf{B}_{Sv}^* + \mathbf{B}_A\mathbf{D}_A^{-1}\mathbf{B}_{Av}^*)\Delta\mathbf{R}_M\mathbf{L}_{Av} \\ -\mathbf{L}_A - (\mathbf{B}_S\mathbf{D}_S^{-1}\mathbf{B}_{Sv}^* - \mathbf{B}_A\mathbf{D}_A^{-1}\mathbf{B}_{Av}^*)\Delta\mathbf{R}_M\mathbf{L}_{Av} \end{bmatrix} \tag{19}$$

— in case of an asymmetric load

$$\mathbf{L}_M = \begin{bmatrix} \mathbf{L}_S + \mathbf{L}_A - (\mathbf{B}_S\mathbf{D}_S^{-1}\mathbf{B}_{Sv}^* + \mathbf{B}_A\mathbf{D}_A^{-1}\mathbf{B}_{Av}^*)\Delta\mathbf{R}_M(\mathbf{L}_{Sv} + \mathbf{L}_{Av}) \\ \mathbf{L}_S - \mathbf{L}_A - (\mathbf{B}_S\mathbf{D}_S^{-1}\mathbf{B}_{Sv}^* - \mathbf{B}_A\mathbf{D}_A^{-1}\mathbf{B}_{Av}^*)\Delta\mathbf{R}_M(\mathbf{L}_{Sv} + \mathbf{L}_{Av}) \end{bmatrix} \tag{20}$$

where

$$\Delta\mathbf{R}_M = (\mathbf{B}_{Sv}\mathbf{D}_S^{-1}\mathbf{B}_{Sv}^* + \mathbf{B}_{Av}\mathbf{D}_A^{-1}\mathbf{B}_{Av}^* + \Delta\mathbf{R}^{-1})^{-1}$$

From the above relationships, the modification is seen to be asymmetrical, i.e., the internal loads in an asymmetrical structure are obtained although in the calculations it is sufficient to take only values for half of the structure into account just as in the case of determination of the internal load distribution in the symmetrical structure. This is favourable if a computer is applied because the storage capacity requirement is significantly reduced and thus a computer of less capacity might be applied.

Still remains a question to be examined, i.e., to what a quasisymmetry the procedure exposed offers an advantage in comparison to the direct solution. No exact answer can be given to this question, however, in case of problems of major extent the assumption might be accepted as a guiding principle that as far as the apparent degree of redundancy caused by the modification, i.e., the dimension of $\Delta\mathbf{R}$ or $\Delta\mathbf{R}_M$ does not surpass the lower of the dimensions of $\mathbf{D}_S$ or $\mathbf{D}_A$ both the preparatory work and the running time may be reduced.

4. Example

The application of the method described is illustrated on the simple structure in *Fig.* 4. The quasisymmetry is caused by the stiffness differences between members 2 and 2'. The basic system of, and the internal loads in one of the associated symmetrical structures are represented in the same figure.

The values of the calculated support moments are listed in *Table I*, where

$$n = J^*/J$$

is the ratio of the stiffnesses which can assume any value within the range -1 to ∞ (E = constant). The value -1 is related to the case of cutout.

Table I

n	a	b	c	d
-1	-2.5	10	0	0
0	-1.9736	7.8947	7.8947	-1.9736
1	-1.8082	7.2327	10.3774	-1.7296
10	-1.5408	6.1633	14.3876	-0.5995
∞	-1.4423	5.7692	15.8654	0

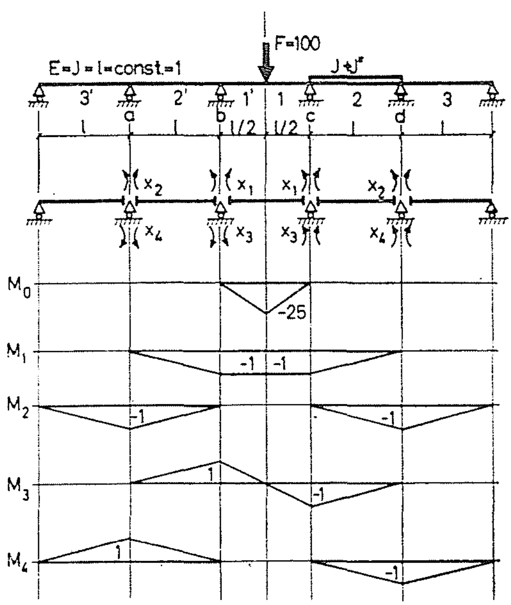

Fig. 4

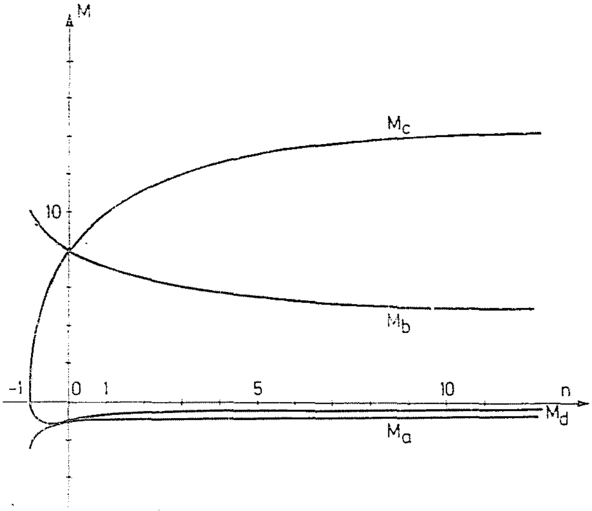

Fig. 5

The effect of the degree of asymmetry exerted upon the values of the support moments are shown in *Fig.* 5.

The result obtained precisely agree with that found by MICHELBERGER [1], which follows from the conceptual identity of both procedures.

Summary

Determination of the internal load distribution in the commonly asymmetrical vehicle bodies encounters difficulties, due to the large extent of the problem. In case where the asymmetry concerns only a few parts of the structure, it might be considered as a quasisymmetrical problem.

By making use of the advantages of deliberately creating symmetry, the internal load distribution in the structure may be determined by applying the force method. The so-called asymmetrical modification of the results leads to the internal load distribution of the quasi-symmetrical structure. In the calculation it is sufficient to take only the half part of the structure into account which means savings in work and time.

The matrix formulation of the problem permits the programming of the algorithm directly to the computer.

References

1. MICHELBERGER, P.: Acta Technica **35—36**, 485, (1961).
2. MICHELBERGER, P.: Stress pattern in bus bodies (Candidate's dissertation). Budapest (1959) (In Hungarian).
3. BRZOSKA, Z.: Archivum budowy maszyn **2**, 321, (1955), **3**, 3, (1956).
4. MICHELBERGER, P.: Per. Pol. Tr. E. **2**, 3, (1974).
5. ARGYRIS, J. H.—KELSEY, S.: Modern Fuselage Analysis and the Elastic Aircraft. Butterworths, London, (1963).
6. NÁNDORI, E.: Per. Pol. Tr. E. **3**, 1, (1975).

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