STABILITY OF CATAMARANS

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There is a variety of methods for determining the statical stability moment of ships and other floating hulls. In Europe the Dargnies- and the Krilov methods are employed among the so-called "numerical" methods based on the same fundamental principle. Both give about the same percentage errors (1 to 3%) in calculating the statical stability moment of a usual shaped single-hull ship. The errors are due to the used simplification in the methods mentioned. For instance, in the Dargnies method such simplification is a neglection in calculating the volume of the wedge-shaped hull part bounded by the side of ship hull and the planes determined by two waterlines at the investigated two neighbouring angles of inclination. The Krilov method applies a simplification in calculating the draught correction. Within some limits, the accuracy of the mentioned methods can be improved by increasing the numerical work by taking more inclinations and more stations into consideration.

The same accuracy is achieved by the so-called planimeter or integrator methods. Also in these cases the errors can be reduced by increasing the number of stations and inclinations.

The generally used above-mentioned methods are developed for the usual shaped ship hull and so it is advisable to apply them only in these cases. The simplifications introduced at the development of these methods are based on the recognition that waterlines differ slightly at two neighbouring inclinations and so do their statical moments and moments of inertia. In case of catamarans these differences are more significant and so the usual stability calculation methods result in higher error of the statical stability moment. Moreover the amount of work of calculation, drawing and planimerisation is much higher than for a single-hull ship.

With catamarans the distance between the middle lines of the two floating hulls B_0 is the multiple of the breadth of one hull B, the B_0/B ratio being about 5 to 8. This fact leads to a simple calculation method for determining the stability moment of catamaran with very small error.

The statical stability moment is known to depend only on the moment of inertia of the waterline area provided the displacement and the position of the centre of gravity of ship weight are constant. The moment of inertia of the waterline area of catamarans can be calculated in two parts. One of them comprises the moments of inertia about the own centre of gravity of the waterline areas of the floating hulls, the other part the waterline areas multiplied by the square of the distance between the axis of one hull and the common axis of the two hulls:

$$J = [J_{01} + J_{02}] + \left[A_1 \left(\frac{B_0}{2}\right)^2 + A_2 \left(\frac{B_0}{2}\right)^2\right].$$

Taking a suitably short part of the ship length Δx, the area of waterline can be considered as a rectangle. So it can be written:

$$\Delta J = \left[\frac{B_1^3}{12}\Delta x + \frac{B_2^3}{12}\Delta x\right] + \left[B_1 \Delta x \left(\frac{B_0}{2}\right)^2 + B_2 \Delta x \left(\frac{B_0}{2}\right)^2\right]$$

where B_1 and B_2 are the breadths of the waterlines of hulls, B_0 is the distance between the axes of two hulls.

Factoring out the values of Δx and B_0^3 from the former equation we got:

$$\Delta J = \Delta x \cdot B_0^3 \left[\frac{1}{12}\left(\frac{B_1}{B_0}\right)^3 + \frac{1}{12}\left(\frac{B_2}{B_0}\right)^3 + \frac{1}{4}\left(\frac{B_1}{B_0} + \frac{B_2}{B_0}\right)\right].$$

This equation suits to find the percentage of the first part (containing the moments of inertia calculated about the own axes) in the total moment of inertia of the waterline area. Here only qualitative results are needed. Thus it can be assumed that the breadth of waterline of two hulls are equal, neglecting their difference owing to the inclination

$$\frac{B_1}{B_0} \cong \frac{B_2}{B_0} = \frac{B}{B_0}.$$

Accordingly, the total moment of inertia:

$$\Delta J = \Delta x \cdot B_0^3 \left[\frac{1}{6}\left(\frac{B}{B_0}\right)^3 + \frac{1}{2}\left(\frac{B}{B_0}\right)\right].$$

Formerly we have mentioned that the distance between the axes of two hulls B_0 is generally about 5 to 8 times the breadth of one hull. The percentage of the first part in the term in brackets

$$h = \frac{\frac{1}{6}\left(\frac{B}{B_0}\right)^3}{\frac{1}{6}\left(\frac{B}{B_0}\right)^3 + \frac{1}{2}\left(\frac{B}{B_0}\right)}$$

has been calculated even for the exceptional value $B_0/B = 3$ and compiled in Table 1.

Table 1

$\frac{B_0}{B}$	h
3	$1/28 = 3.57\%$
4	$1/49 = 2.04\%$
5	$1/76 = 1.32\%$
6	$1/109 = 0.92\%$
7	$1/148 = 0.68\%$
8	$1/193 = 0.52\%$

It is seen in Table 1, that the moments of inertia of water line areas calculated about axis through their own centre of gravity, the first part of the former equation is only a few per cent of the total moment of inertia of the waterline area. So the neglection of the first part involves no significant inaccuracy, in particular when the B_0/B values are high enough. Using this neglection in our stability calculation the calculated value of the moment of stability is less than the real value, thus the neglection is on the side of safety.

For $B_0/B \lesssim 5$, the neglection of the moments of inertia calculated about the own axis of the waterline areas of the hulls causes an error greater than 1 per cent. Trying to improve accuracy, let us see what happens if we do not neglect the moment of inertia calculated about the own axis, but we use their values calculated on a horizontal plane in the height of the mean draughts of the hulls rather than on the real inclined water level.

The side walls of the hulls of the catamaran are approximately vertical near the water level over most of the ship length. The moment of inertia about the own axis of the waterline area $(\Delta x \cdot B)$ in a horizontal plane is:

$$J_k = \frac{\Delta x \cdot B_k^3}{12} .$$

When the side walls are vertical, the breadth of the waterline at an inclination angle φ:

$$B = \frac{B_k}{\cos\varphi} .$$

Hence, the moment of inertia at an inclination angle φ:

$$J = \frac{\Delta x}{12}\left(\frac{B_k}{\cos\varphi}\right)^3 = \frac{1}{\cos^3\varphi} J_k .$$

Thus, substituting the J_k value for J in the stability calculation, the neglection is only $(1 - \cos^3 \varphi).h$ instead of h indicated in Table 1. For this case the expected errors of the results of the hydrostatical stability are shown in Table 2.

Table 2

$\frac{B_0}{B}$	Degrees of inclination			
	10	20	30	40
3	0.16%	0.61%	1.26%	1.98%
4	0.08%	0.34%	0.70%	1.10%
5	0.06%	0.22%	0.49%	0.72%
6	0.04%	0.15%	0.32%	0.50

Thus, for this latter method the expected error is much diminished also for smaller values of B_0/B as compared to the former method.

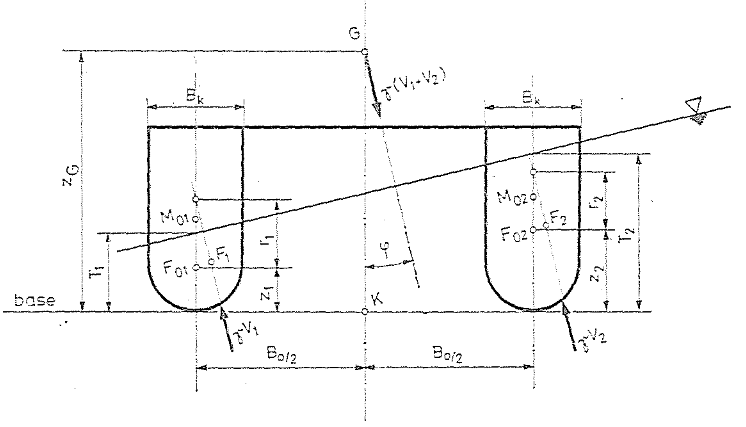

Fig. 1

Fig. 1 represents the forces acting on a catamaran floating in an inclined position (φ is the angle of inclination). Lifting forces $(\gamma \cdot V_1 + \gamma \cdot V_2)$ acting on the two hulls are balanced by the weight acting at the centre of gravity of the catamaran. The resultant of the moments of acting forces written about point K (the statical stability moment) is:

$$M_s = \gamma V_2 \left[\frac{B_0}{2} \cos \varphi + (z_2 + r_2) \sin \varphi \right] -$$

$$- \gamma V_1 \left[\frac{B_0}{2} \cos \varphi - (z_1 + r_1) \sin \varphi \right] - \gamma (V_1 + V_2) z_G \sin \varphi .$$

Knowing the value of total weight and the position of centre of gravity, the third part of the equation is simple and exact to calculate. No approximative method needed but the determination of the first two parts, that is, the moment of the lifting forces. The moment represented by the first two parts is the hydrostatical stability and it is a homogeneous and linear function of the moment of inertia of the waterline area. Calculating their values with the mentioned neglections the results involve errors tabulated in Tables 1 and 2.

In the first case the moments of inertia about the axes through the own centres of gravity of the waterline areas of the floating hulls have been neglected. The horizontal distance between points F_{0i} and F_i is known to be determined by the moments of inertia about the axis through the own centres of gravity of the waterline areas of the hulls.

The lines of action of lifting forces are in fact supposed to pass through points F_{01} and F_{02} (the centres of gravity of the displacements belonging to waterlines parallel to base with draughts T_1 and T_2) contrary to the real case represented in Fig. 1, where the lifting forces pass through points F_1 and F_2. Thus, it is supposed $r_1 = r_2 = 0$. With this supposition, the hydrostatical stability (the first two member of the former equation) can be calculated from the equation

$$M_v \cong M_{va} = \gamma V_2 \left(\frac{B_0}{2} \cos\varphi + z_2 \sin\varphi\right) - \gamma V_1 \left(\frac{B_0}{2} \cos\varphi - z_1 \sin\varphi\right) =$$

$$= \gamma \frac{B_0}{2} (V_2 - V_1) \cos\varphi + \gamma (z_2 V_2 + z_1 V_1) \sin\varphi .$$

The moment calculated with this approximation is smaller than the real one. According to Table 1 the differences are negligible for higher values of the B_0/B ratio.

Supposing the lines of action of the lifting forces to pass through the initial metacenter belonging to waterlines parallel to the base with draughts T_1 and T_2, the values of the moments of inertia of the waterline areas belonging to horizontal floating of hulls are reckoned with, instead of the values belonging to the real inclined floating. Calculating the first two members of the statical stability moment in this way results in errors compiled in Table 2. Accordingly in this second type of approximation we suppose

$$r_1 \cong \frac{J_1}{V_1} \cong \frac{J_{01}}{V_1} \quad \text{and} \quad r_2 \cong \frac{J_2}{V_2} \cong \frac{J_{02}}{V_2}$$

where J_1 and J_2 are the own moments of inertia of waterlines at the inclined position of hulls, J_{01} and J_{02} are the own moments of inertia of waterlines area parallel to base at the mean values of draughts T_1 and T_2.

According to the second type of approximation

$$M_v \cong M_{vb} = \gamma V_2 \left[\frac{B_0}{2} \cos \varphi + (z_2 + r_2) \sin \varphi \right] -$$

$$- \gamma V_1 \left[\frac{B_0}{2} \cos \varphi - (z_1 + r_1) \sin \varphi \right] = \gamma \frac{B_0}{2} (V_2 - V_1) \cos \varphi +$$

$$+ \gamma (z_2 V_2 + z_1 V_1) \sin \varphi + \gamma (J_{02} + J_{01}) \sin \varphi .$$

Thus, the equation for M_{vb} has one term more than the former one (for M_{va}).

These quantities involved in the equation of M_v are simple geometrical properties of the floating hulls of catamaran. Recent practice applies computers for finding these geometrical properties, making their exact values available for the hydrostatic curves of the floating hulls, supplying in turn the statical stability moment of catamaran at a minimum of error.

The statical stability curves (the Reed diagram) of a catamaran can be calculated in the following way:

The sum of assumed couple of V_1 and V_2 belonging together gives the total displacement. The T, z, J_0 values can be read off the hydrostatic curves of the floating hulls as a function of V_1 and V_2. The angle of inclination

$$\varphi = \operatorname{arc\,tan} \frac{T_2 - T_1}{B_0} .$$

These values yield the hydrostatical stability (M_v) and the total statical stability moment (M_s). This method is only valid for an angle of inclination when one of the hulls of the catamaran emerged fully above the water level. The catamaran is generally stable also for greater inclinations, therefore the stability moment is needed for higher values of the angle of inclination.

When one of hulls of catamaran emerged, the moment of statical stability can be written as follows:

$$M_s = \gamma V \left[\frac{B_0}{2} \cos \varphi - (z_G - z_2 - r_2) \sin \varphi \right] .$$

The value of r_2 can be calculated at a satisfactory accuracy by the method for single hull ship. But the value of r_2 of a slender hull is very small related to the difference $(z_G - z_2)$. Accordingly the error is little aggravated by using the following approximation:

$$r_2 \cong \frac{J_{02}}{V} .$$

That is, the initial value of the radius of metacentre is substituted for r_2. The real value of the moment of stability is higher than calculated in this way. Thus the error is on the side of safety.

This method can be used till the deck line touches the water level. For higher inclinations the calculation is unnecessary because the stability analysis becomes meaningless.

Finally in the following, we give detailed results of the stability calculation of a catamaran for presenting the different approximations used.

The total weight of a catamaran is 7.45 Mp, the distance between the center lines of the two hulls $B_0 = 3.80$ m, the breadth of one hull is $B_k = 1.03$ m. Thus, the ratio $B_0/B = 3.69$ is an extremely small value. The center of gravity is in the mid-plane of the catamaran at a height 2.50 m. The calculations are done by using the hydrostatic curves of the hulls made when designing the catamaran.

The values of the displacement and the data read off the hydrostatic curves are compiled in Table 3.

Table 3

	V_1	V_2	T_1	T_2	φ	z_1	z_2	J_{01}	J_{02}
	m^3	m^3	m	m	°	m	m	m^4	m^4
1	3.725	3.725	0.805	0.805	0	—	—	—	—
2	3.450	4.000	0.770	0.838	1.02	0.48	0.53	0.54	0.62
3	3.000	4.450	0.712	0.886	2.62	0.43	0.56	0.48	0.66
4	2.450	5.000	0.641	0.950	4.66	0.38	0.60	0.41	0.73
5	2.000	5.450	0.572	1.000	6.42	0.33	0.64	0.32	0.77
6	1.450	6.000	0.485	1.055	8.53	0.26	0.67	0.20	0.81
7	1.000	6.450	0.390	1.102	10.62	0.20	0.70	0.10	0.85
8	0.450	7.000	0.245	1.160	13.53	0.10	0.73	0.04	0.88
9	0	7.450	0	1.206	17.60	0	0.75	0	0.91

For comparing the two kinds of approximation we calculated the values

$$M_{va} = \gamma \frac{B_0}{2} (V_2 - V_1) \cos \varphi + \gamma (z_2 V_2 + z_1 V_1) \sin \varphi$$

and

$$M_{vb} = M_{va} + \gamma (J_{01} + J_{02}) \sin \varphi$$

and

$$M_G = \gamma z_G (V_1 + V_2) \sin \varphi$$

and with these we determined the statical stability moments of catamaran

$$M_{sa} = M_{va} - M_G$$

and

$$M_{sb} = M_{vb} - M_G$$

values compiled in Table 4.

Table 4

	M_{va} mMp	M_{vb} mMp	M_G mMp	M_{sa} mMp	M_{sb} mMp
1	0	0	0	0	0
2	1.309	1.330	0.336	0.97	0.99
3	2.940	2.992	0.850	2.08	2.13
4	5.149	5.241	1.511	3.64	3.73
5	6.981	7.103	2.089	4.89	5.01
6	9.204	9.354	2.781	6.42	6.57
7	11.045	11.220	3.435	7.61	7.78
8	13.495	13.711	4.385	9.11	9.32
9	15.165	15.440	5.635	9.53	9.80

According to the tabulated values, the differences between the M_{sa} and M_{sb} values are about 2 to 3 per cent. These differences correspond to the differences between errors compiled in Tables 1 and 2.

For 17.60 degrees of inclination, one of the hulls emerges above water level. Table 5 gives the values of

$$M_s = \gamma V \left[\frac{B_0}{2} \cos\varphi - \left(z_G - z_2 - \frac{J_{02}}{V} \right) \sin\varphi \right]$$

as a function of the angle of inclination.

Table 5

φ (degrees)	20	25	30	35	40	45	49.4
M_s (mMp)	9.06	7.72	6.19	4.63	3.14	1.42	0.00

Summary

The numerical and integrator methods used for the stability analysis of vessels of usual form are rather labour consuming for the stability of catamaran boats. Approximations involved in these methods result in calculation errors. These errors are negligible for vessels of usual form but may be considerable for catamarans. Hence this methods of stability analysis is to be avoided for catamaran vessels not only because of the work excess but also for its unreliability. A simple method for the calculation of the statical stability moment as a function of the angle of inclination is here presented. This method requires only data read off the hydrostatic curves determined for the floating hulls of the catamaran.

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