DETERMINATION OF THE YARN FORCE ARISING ALONG THE BALLOON FORMED IN A SHUTTLE

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The phenomenon of balloon formation has been the subject of extended investigations fro the last 100 years but no exact description of it is available so far. Various experimental formulae have been established for the forces arising along the balloon, however, no calculation of its value taking into account all the forces involved has been carried out as yet. Also, investigations on and experimental approaches of the phenomenon of the balloon were primarily concerned with the spinning balloon, while work is hardly found in the literature discussing balloon formation in a shuttle. The reason for this may lie in the fact that balloon formation in a shuttle is influenced by a number of factors and thus it is a more complex phenomenon than that of the spinning balloon.

Since neither the form of the shuttle balloon is known, nor are there any formulae available in the literature establishing it or the yarn tension, the determination of the yarn forces arising in weawing, be it with the help of an approximating formula, would be of considerable interest in respect of both the number of end breakages and the improvement of fabric quality.

In a previous paper an equation has been established for it in the form of a Legendre type elliptical integral by solving the system of differential equations of the shuttle balloon. Let us now use the same equation to determine the yarn force arising during winding-off the pirn, which appears to have a considerable influence on the number of weft breakages.

To begin with, let us establish the function giving the yarn tension force arising at an arbitrary point of the plane curve of the balloon formed in a shuttle.

Observations on the shuttle balloon have shown that the condition of equilibrium for any arch element of the yarn is determined by the system of differential equations

$$\frac{S(x) \cdot y'}{\sqrt{1 + y'^2}} = c$$

$$-\frac{\sqrt{1 + y'^2}}{y'} = \frac{\sigma}{c} \cdot \left(\frac{\omega^2}{2} \cdot x^2 + g \cdot x\right) + c_1 .$$

Taking into account the initial conditions, the value of the integration constant c_1 in the second equation can be determined. At the point A(O, H) of the balloon being $x = 0$

$$\frac{\sqrt{1 + y_0'^2}}{y_0'} = c_1 .$$

Since at the point A(O, H) y_0' lies in the direction of the yarn force S_0 and considering that $yo' = \operatorname{tg} \alpha_0$, the value of the left-side quotient can be determined.

$$c_1 = -\frac{\sqrt{1 + y_0'^2}}{y_0'} = -\frac{\sqrt{1 + \operatorname{tg}^2 \alpha_0}}{\operatorname{tg} \alpha_0} = -\frac{\dfrac{1}{\cos \alpha_0}}{\dfrac{\sin \alpha_0}{\cos \alpha_0}} = -\frac{1}{\sin \alpha_0} =$$

$$= -\frac{1}{\sin(90^\circ + \gamma_0)} = -\frac{1}{\cos \gamma_0} = -\frac{1}{\dfrac{V_0}{S_0}} = -\frac{S_0}{V_0} .$$

On the other hand, according to the first equation with respect to the point A(O, H):

$$\frac{S_0 \cdot y_0'}{\sqrt{1 + y_0'^2}} = S_0 \cdot \frac{\dfrac{\sin \alpha_0}{\cos \alpha_0}}{\dfrac{1}{\cos \alpha_0}} = S_0 \cdot \sin \alpha_0 =$$

$$= S_0 \cdot \sin (90^\circ + \gamma_0) = S_0 \cdot \cos \gamma_0 = S_0 \cdot \frac{V_0}{S_0} = V_0 = c .$$

Multiplying the differential equations defining the condition of equilibrium for the yarn

$$- S(x) = \sigma \cdot \left(\frac{\omega^2}{2} \cdot x^2 + gx\right) + c \cdot c_1$$

and taking into account the values of the integration constants c and c_1

$$- S(x) = \sigma \cdot \left(\frac{\omega^2}{2} \cdot x^2 + g \cdot x\right) - S_0$$

the yarn force at any point of the plane curve of the shuttle balloon is

$$S(x) = S_0 - \sigma \cdot \left(\frac{\omega^2}{2} \cdot x^2 + gx\right) .$$

Thus, for a planar case, yarn tension force in addition to being dependent on the momentary balloon radius, depends on yarn density, gravitational acceleration and on the angular velocity of the rotation. From function $S(x)$ defining

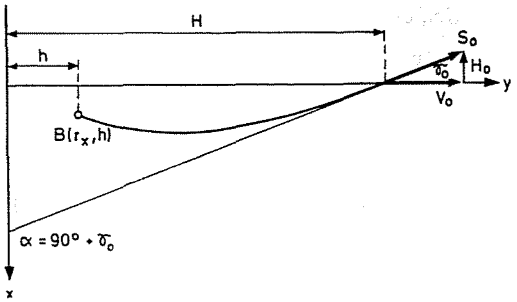

Fig. 1

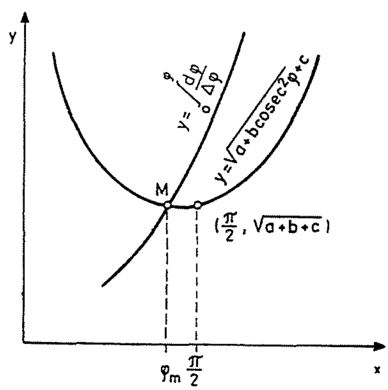

Fig. 2

yarn tension force it is seen — since the second term in the right side is always positive — i.e.

$$\sigma \cdot \left(\frac{\omega^2}{2} \cdot x^2 + gx\right) > 0\,,$$

that the highest yarn tension force will always be found at that point of the balloon where the second term of the right-side expression is zero. However, this can only occur for $x = 0$. Going backwards from the balloon apex A(0, H) along the axis y up to the point where the radius of the balloon is at its maximum, yarn tension force is found to be decreasing, which is evident, because the yarn lengths to be kept in equilibrium are also decreasing.

The maximum value of the yarn tension force determined in this way is, however, a relative, i.e. a local extreme value, valid only in the range of the balloon height $H—h$. The maximum value of the yarn tension force may, of course, fall outside this interval, just as in the present case.

The extreme value of the yarn tension force $S(x)$ may occur at any point x for which the first differential quotient of the function with respect to its variable x is zero, thus

$$\frac{dS(x)}{dx} = -\sigma \cdot (\omega^2 \cdot x + g) = 0$$

whence

$$x = -\frac{g}{\omega^2}$$

while the second differential quotient of the function $S(x)$ with respect to its variable x is given as

$$\frac{d^2S(x)}{dx^2} = -\sigma \cdot \omega^2 < 0$$

hence yarn tension force is found to be at its maximum at

$$x = -\frac{g}{\omega^2}.$$

However, that point lies outside the range of the balloon height $H-h$, and the maximum yarn tension force at this point x is

$$S(x)_{\max} = S_0 - \sigma \cdot \left(\frac{\omega^2}{2} \cdot \frac{g^2}{\omega^4} - \frac{g^2}{\omega^2}\right) = S_0 + \frac{\sigma \cdot g^2}{2 \cdot \omega^2}.$$

Remaining within the height of the balloon curve, maximum yarn tension will occur when $x = 0$ and the maximum yarn tension force at this point is

$$S(x)_{\max} = S_0,$$

to be measured in the apex of the balloon.

Let us consider furthermore to what extent the initial yarn force S_0 measured at the guide eye depends on the balloon height $H-h$, on the angular velocity, on the momentary radius measured at the winding-off point, on the yarn density and gravitational acceleration.

The equation of the plane curve of the shuttle balloon during winding-off the yarn is of the form:

$$y - H = \pm \frac{2V_0}{\sigma \cdot \omega^2 \cdot B} \cdot \left\{\int_0^{\arcsin \frac{x+C}{A}} \frac{d\varphi}{\Delta\varphi} - \int_0^{\arcsin \frac{C}{A}} \frac{d\varphi}{\Delta\varphi}\right\}$$

where

$$A = \sqrt{\left(\frac{g}{\omega^2}\right)^2 + 2 \cdot \frac{S_0 - V_0}{\sigma \cdot \omega^2}}$$

$$B = \sqrt{\left(\frac{g}{\omega^2}\right)^2 + 2 \cdot \frac{S_0 + V_0}{\sigma \cdot \omega^2}}$$

$$C = \frac{g}{\omega^2}$$

$$k = \frac{A}{B}.$$

The root before the brackets in the right side requires a negative sign, since $H > y$ and hence $y-H$ is always negative. The right side is, thus, also to be of negative sign. First of all, let us consider the correctness of the negative sign of the equation obtained for the balloon curve. Reducing the right-side elliptical integral to

$$y - H = -\frac{2V_0}{\sigma \cdot \omega^2 \cdot B} \cdot \int\limits_{\arcsin \frac{C}{A}}^{\arcsin \frac{x+C}{A}} \frac{d\varphi}{\sqrt{1 - k^2 \cdot \sin^2 \varphi}}$$

whence the equation of the balloon curve takes the form:

$$y = -\frac{2V_0}{\sigma \cdot \omega^2 \cdot B} \cdot \int\limits_{\arcsin \frac{C}{A}}^{\arcsin \frac{x+C}{A}} \frac{d\varphi}{\sqrt{1 - k^2 \cdot \sin^2 \varphi}} + H .$$

Be $\arcsin \dfrac{x+C}{A} = z$, then the derivative of the equation of the balloon curve with respect to x:

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = -\frac{2V_0}{\sigma \cdot \omega^2 \cdot B} \cdot \frac{1}{\sqrt{1 - k^2 \left[\sin \arcsin \dfrac{x+C}{A}\right]^2}} \cdot \frac{1}{\sqrt{1 - \left(\dfrac{x+C}{A}\right)^2}} \cdot \frac{1}{A}$$

The value of the derivative at the guide being at the point A(O, H)

$$y'_0 = \left(\frac{dy}{dx}\right)_{x=0} = -\frac{2V_0}{\sigma \cdot \omega^2 \cdot B} \cdot \frac{1}{\sqrt{1 - \dfrac{A^2}{B^2} \cdot \dfrac{C^2}{A^2}}} \cdot \frac{1}{\sqrt{1 - \dfrac{C^2}{A^2}}} \cdot \frac{1}{A} =$$

$$= -\frac{2V_0}{\sigma \cdot \omega^2 \cdot A \cdot B} \cdot \frac{1}{\sqrt{\left(1 - \dfrac{C^2}{A^2}\right)\left(1 - \dfrac{C^2}{B^2}\right)}} =$$

$$= -\frac{2V_0}{\sigma \cdot \omega^2 \cdot A \cdot B} \cdot \frac{A \cdot B}{\sqrt{(B^2 - C^2)(A^2 - C^2)}} .$$

Substituting the values of A, B and C gives

$$y'_0 = \left(\frac{dy}{dx}\right)_{x=0} = -\frac{2V_0}{\sigma \cdot \omega^2 A \cdot B} \cdot \frac{A \cdot B}{\sqrt{2 \cdot \dfrac{S_0 + V_0}{\sigma\omega^2} \cdot 2 \cdot \dfrac{S_0 - V_0}{\sigma\omega^2}}} =$$

$$= -\frac{V_0}{\sqrt{S_0^2 - V_0^2}} = -\frac{V_0}{H_0} .$$

where H_0 denotes the component of the tensioning force S_0 along the axis in the balloon apex.

For the purpose of checking the results let us determine the value of the derivative for the apex A (O, H) of the balloon on the basis of direct geometric considerations

$$y_0' = \left(\frac{dy}{dx}\right)_{x=0} = \operatorname{tg} \alpha_0 = \operatorname{tg}(90^\circ + \gamma_0) = -\operatorname{ctg} \gamma_0 = -\frac{V_0}{\sqrt{S_0^2 - V_0^2}} = -\frac{V_0}{H_0},$$

which is in agreement with the result obtained and verifies the correctness of the equation established for the balloon curve formed in a shuttle.

Thus, the equation of the plane curve of the balloon formed during winding-off the pirn in a shuttle may be written as:

$$y - H = -\frac{2V_0}{\sqrt{(\sigma \cdot g)^2 + 2 \cdot \sigma \cdot \omega^2 (S_0 + V_0)}} \cdot \left\{ \int_0^{\arcsin \frac{r_x + C}{A}} \frac{d\varphi}{\Delta\varphi} - \int_0^{\arcsin \frac{C}{A}} \frac{d\varphi}{\Delta\varphi} \right\}.$$

This equation applies for all the curves passing through the apex $A(O, H)$ of the balloon. From these curves the one also passing the momentary winding-off point $B(r_x, h)$ is to be chosen. For this curve

$$h - H = -\frac{2V_0}{\sqrt{(\sigma \cdot g)^2 + 2 \cdot \sigma \cdot \omega^2 \cdot (S_0 + V_0)}} \cdot \left\{ \int_0^{\arcsin \frac{r_x + C}{A}} \frac{d\varphi}{\Delta\varphi} - \int_0^{\arcsin \frac{C}{A}} \frac{d\varphi}{\Delta\varphi} \right\}$$

The last two equations together give the equation of the curves passing through the points $A(O, H)$ and $B(r_x h)$, by which the functional relationship between the yarn force S_0 measured at the guide eye in the apex, the balloon length H, the angular velocity ω, and the yarn τ density is also given in an implicit form.

Denoting the common coefficient of the integrals by E, i.e.

$$\frac{2V_0}{\sqrt{(\sigma \cdot g)^2 + 2 \cdot \sigma \cdot \omega^2 \cdot (S_0 + V_0)}} = E$$

furthermore writing for the limits of the elliptical integrals

$$\arcsin \frac{C}{A} = K$$

$$\arcsin \frac{r_x + C}{A} = \varphi,$$

then

$$\sin \varphi = \frac{r_x + C}{A},$$

whereby

$$\frac{H-h}{E}=\int_0^{\varphi}\frac{d\varphi}{\Delta\varphi}-\int_0^{K}\frac{d\varphi}{\Delta\varphi}.$$

The second integral in the right side is constant, i.e.

$$\int_0^{K}\frac{d\varphi}{\Delta\varphi}=G$$

$$\frac{H-h}{E}+G=\int_0^{\varphi}\frac{d\varphi}{\Delta\varphi}=F(k,\varphi).$$

The equation thus obtained has to be solved for φ. The Legendre type elliptical integral of the first kind in the right side of the equation can be represented directly with the use of Table F.

Assuming V_0, the component of the yarn tension along the axis in the apex, in the left side function, to be measured, thus a given value, and by eliminating the initial yarn force, then, since

$$A=\frac{r_x+C}{\sin\varphi}=\frac{\left(r_x+\frac{g}{\omega^2}\right)}{\sin\varphi}$$

and substituting A in the equation by its value, furthermore, raising both sides to the second power, we get

$$\left(\frac{g}{\omega^2}\right)^2+2\cdot\frac{S_0-V_0}{\sigma\cdot\omega^2}=\frac{\left(r_x+\frac{g}{\omega^2}\right)^2}{\sin^2\varphi}$$

whence the yarn force arising at the guide eye

$$S_0=\frac{\sigma\cdot\omega^2}{2}\cdot\left\{\frac{\left(r_x+\frac{g}{\omega^2}\right)^2}{\sin^2\varphi}-\left(\frac{g}{\omega^2}\right)^2\right\}+V_0.$$

Thus the equation

$$y=\frac{H-h}{E}+G$$

is of the following form

$$y=\frac{\sqrt{(\sigma\cdot g)^2+2\cdot\sigma\cdot\omega^2\cdot\left\{2V_0+\frac{\sigma\cdot\omega^2}{2}\cdot\frac{\left(r_x+\frac{g}{\omega^2}\right)^2}{\sin^2\varphi}-\left(\frac{g}{\omega^2}\right)^2\right\}}}{2V_0}\cdot(H-h)+G. \quad (1)$$

Carrying out the operations prescribed and reducing on the left side, leads to the equation

$$y = \sqrt{a + b \cdot \operatorname{cosec}^2\varphi} + C$$

Representing both the function

$$y = F(k, \varphi)$$

and the function obtained last in a rectangular coordinate system of the same axis (φ, y), the curves shown in the figure are obtained. The abscissa φ_m of the point of intersection M of the curves gives the approximate value of the root of the equation. In order to determine the root with the required accuracy, use can be made of an approximate method, as the chord method, or Newton's tangent method.

Having the value of φ_m approximated with the accuracy required, V_0 being measured, the initial yarn force S_0 arising at the guide eye can be calculated on the basis of (1).

Finally, replacing the expression of the initial yarn force S_0 given by (1) in the formula of the tension force $S(x)$, the yarn force at any point of the balloon curve formed in a shuttle is obtained in the form

$$S(x) = \frac{\sigma \cdot \omega^2}{2} \cdot \left\{ \frac{\left(r_x + \frac{g}{\omega^2}\right)^2}{\sin^2\varphi} - \left(\frac{g}{\omega^2}\right)^2 \right\} + V_0 - \sigma \cdot \left(\frac{\omega^2}{2} \cdot x^2 + gx\right).$$

The space curve of the shuttle balloon will be the subject of further investigations.

Summary

An approximate formula is given for calculating the yarn force arising during winding-off the pirn in the shuttle of a loom.

For the present, the balloon formed in a shuttle is considered as a plane curve and yarn force is determined for that case.

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