METHOD FOR DETERMINING THE LOSSES OF MOTOR VEHICLES DURING ACCELERATION

By

J. HEGEDŰS

Department of Motor Vehicles, Technical University Budapest
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Presented by Prof. Dr. Z. LÉVAI

Mathematical equations of the method

From the speed—time diagram plotted for accelerating from stationary position to maximum speed, the acceleration can be determined in the whole range and — complemented by further data — so can be the pure accelerating force.

The momentary acceleration of the motor vehicle can be determined from the following differencial equation:

$$\frac{dv}{dt} = \frac{F - E}{m + \Sigma I \left(\frac{i}{r}\right)^2} \; [\mathrm{m/sec^2}] \tag{1}$$

where

F [kp]	the tractive effort available on the circumference of the driving wheels
E [kp]	the running resistance of the motor vehicle
$m \left[\frac{\mathrm{kp\ sec^2}}{m}\right]$	the mass of the motor vehicle
$\Sigma I \left(\frac{i}{r}\right)^2 \left[\frac{\mathrm{kp\ sec^2}}{m}\right]$	the moment of inertia of all rotating parts reduced to the circumference of the driving wheel.

The moment of inertia of the rotating parts is taken in consideration with a mass factor

$$m' = m + \Sigma I \left(\frac{i}{r}\right)^2 = m \cdot k_m \left[\frac{\mathrm{kp\ sec^2}}{\mathrm{m}}\right]. \tag{2}$$

The value of the mass factor k_m is shown by Fig. 1 as a function of the quotient of the motor *rpm* by the relevant vehicle speed.

n [1/min]	*rpm* of the motor built in,
v [km/h]	the speed pertaining to the motor *rpm* at the min. gear transmission.

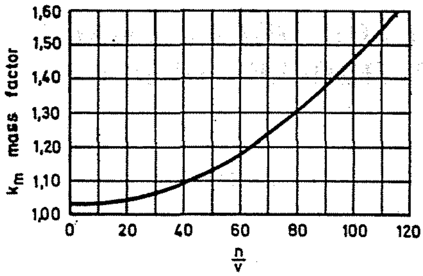

Fig. 1

The tractive effort available on the driving wheels is:

$$F = F_m - F_\eta - F_d, \quad [\text{kp}] \tag{3}$$

where

F_m [kp] the theoretical tractive effort on the circumference of the driving wheels computed from the characteristics of the motor, measured on the test stand:

F_η [kp] the total loss on the different part — groups and equipment,—reduced to the circumference of the driving wheels,

F_d [kp] dynamic loss, difference of static and dynamic characteristics reduced to the circumference of the driving wheel.

The theoretical tractive effort F_m can be determined from the motor's external characteristics:

$$F_m = \frac{P_m \cdot i_{\text{total}}}{n_m \cdot r_g} 716{,}2 \qquad [\text{kp}] \tag{4}$$

where

P_m [HP] motor performance measured on test stand

n_m [1/min] motor *rpm*

i_{total} [—] all gear transmissions ratio between the motor and the driving wheel

r_g [m] dynamical radius of the driving wheel.

The movement on level road of the motor vehicle is opposed by the following running resistances:

$$E = F_w + F_f, \qquad [\text{kp}] \tag{5}$$

where

F_w [kp] air resistance

F_f [kp] rolling resistance.

The air resistance of the motor vehicle:

$$F_w = \frac{\gamma}{2g} \cdot C_w \cdot A \cdot v^2, \qquad [\mathrm{kp}] \tag{6}$$

where

γ [kp/m^3]	specific density of the air
g [m/sec^2]	gravitational acceleration
C_w [—]	form factor of the car-body
A [m^2]	frontal surface of the motor vehicle
v [m/sec]	vehicle speed.

The rolling resistance of the motor vehicle:

$$F_f = G_e \cdot f_e + G_h \cdot f_h, \qquad [\mathrm{kp}] \tag{7}$$

where

G_e and G_h [kp]	loads on the front and rear axle, resp.
f_e and f_h [—]	rolling resistance factor of the front and rear wheels, resp.

The rolling resistance factor of each axle is computed according to the method elaborated by Dr. KOJI SEKI [5] with the relationship:

$$f = \left[\frac{0.98}{p_v\, 0.75} \quad \frac{0.14 \left(\frac{v}{100}\right)^{4.4}}{p_v\, 1.55} \right] G_v^{\,0.44} \cdot 0.28\, G_m^{\,-0.48} \cdot b \tag{8}$$

where

$$p_v = \frac{p}{p_m}; \tag{9}$$

$$G_v = \frac{G}{G_m}; \tag{10}$$

p [kp/cm^2]	tyre pressure
p [kp/cm^2]	suggested tyre pressure
G [kp]	load on the tyre
G_m [kp]	suggested load on the tyre
b [—]	wheel-disc width factor

$$b = -\,0.05 \cdot e + 1.25 \quad [-] \tag{11}$$

where

e [inch]	width of the wheel-disc.

Determination of losses F_η and F_d

Starting from the differential equation

$$\frac{dv}{dt} = \frac{F - E}{m \cdot k_m} \tag{12}$$

the motor vehicle's equation of motion on level road is the following:

$$F_m - F_d - F_\eta - F_w - F_f - \frac{dv}{dt} \cdot m \cdot k_m = 0. \tag{13}$$

To determine the dynamic and the mechanic losses F_d and F_η, resp, the values of the acceleration and of the accelerating force are needed.

For the sake of illustrating the method of determining the acceleration and the accelerating force from the tested motor vehicle's speed—time diagram (Fig. 2):

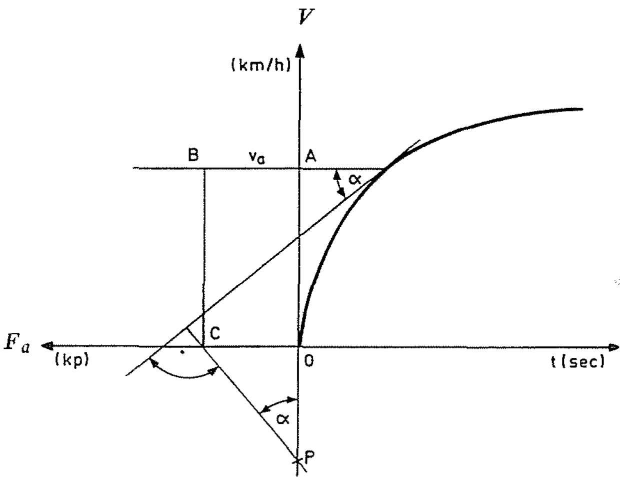

Fig. 2

The acceleration pertaining to speed v_a is equal to the tangent of the angle α.

A perpendicular dawn to the tangent pertaining to speed v_a cuts point C on the horizontal axis.

According to triangle POC:

$$\text{tg}\,\alpha = \frac{\overline{OC}}{\overline{OP}} = \frac{F - E}{m \cdot k_m} = \frac{F_a}{m \cdot k_m}$$

$\overline{OC} = \overline{AB}$ and

$$\overline{OP} = \frac{x}{y} \cdot z \cdot m \cdot k_m ,$$

where

$y\left[\frac{\text{mm}}{\text{m/sec}}\right]$ the scale of the speed axis

$x\left[\frac{\text{mm}}{\text{sec}}\right]$ the scale of the time axis

$z\left[\frac{\text{mm}}{\text{kp}}\right]$ the scale of the accelerating force axis

F_a[kp] the available accelerating force.

Since the distance $\overline{OP}$ is proportional to $m \cdot k_m$, to the mass of the motor vehicle increased by the inertia moment of the rotating masses, the distance $\overline{OC}$ will be identical to the accelerating force pertaining to the investigated speed.

Knowing the forces F_m, F_f, F_w and F_a, the diagram of the tractive effort, running resistance and accelerating force of the tested motor vehicle can be plotted (Fig. 3.). At the speed $v_{\max}$ the acceleration is zero, thus

$$F_m(v_{\max}) - F_f(v_{\max}) - F_w(v_{\max}) - F_\eta(v_{\max}) = 0\,, \tag{14}$$

that is

$$F_\eta(v_{\max}) = F_m(v_{\max}) - F_f(v_{\max}) - F_w(v_{\max})\,. \tag{15}$$

The lost force F_η contains not only losses in the drive train, but all other losses related to the operation of the motor vehicle, to be met by the motor, such as losses of force due to the output demand for the eventual servo-system, air conditioning, wisher, ventilator, etc.

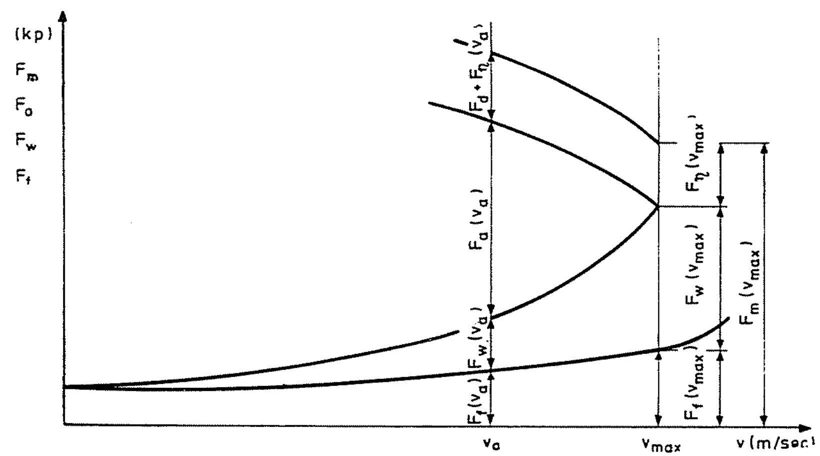

Fig. 3

At a speed v_a where the vehicle can be accelerated, the equation of motion will be different:

$$F_m(v_a) - F_f(v_a) - F_w(v_a) - F_a(v_a) - F_d(v_a) - F_\eta(v_a) = 0 \tag{16}$$

that is

$$F_d(v_a) + F_\eta(v_a) = F_m(v_a) - F_f(v_a) - F_a(v_a) - F_w(v_a)\,. \tag{17}$$

At all speeds where the motor vehicle can be accelerated, an additional loss occurs impairing performance.

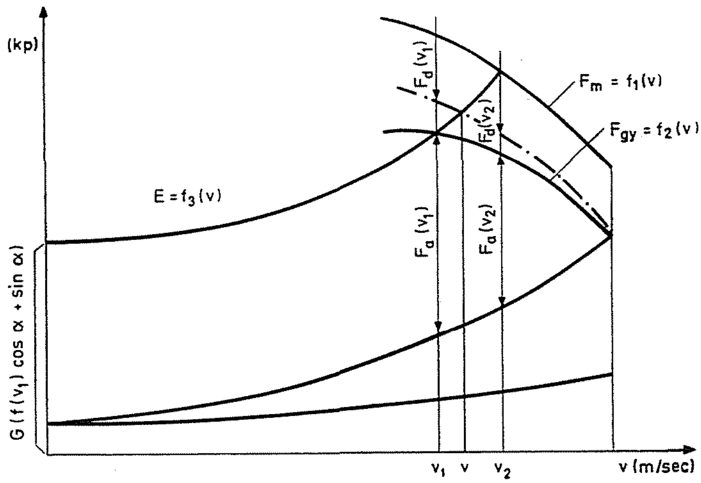

Fig. 4

This loss is due to mixture formation and thermodynamical conditions different during acceleration from those in stable circumstances in a measurement on the test stand.

If the motor vehicle is driven from the level road to a gradient of theoretically any inclination, then obviously a gradient may be found where the final velocity of the vehicle equals the v_a value. Since here the acceleration stops the dynamical loss F_d will necessarily stop, too.

Hence in case of an acceleration according to Fig. 3, at a speed v_a it can be written:

$$F_m(v_a) - F_d(v_a) - F_\eta(v_a) - F_f(v_a) - F_w(v_a) - F_a(v_a) = 0 \tag{16}$$

and for a constant speed:

$$F_m(v_a) - F_\eta(v_a) - F_w(v_a) - G[f(v_a)\cdot\cos\alpha + \sin\alpha] = 0\,. \tag{18}$$

Equalizing the equations we get:

$$F_d(v_a) = G\cdot\sin\alpha - G\cdot f(v_a)[1-\cos\alpha] - F_a(v_a)\,. \tag{19}$$

To determine the F_d value in the speed range $v_{\max} > v_a > 0$ the interrelated speed and α values are needed.

According to Fig 4, if the tested motor vehicle driven at a speed v_1 to a gradient of the angle α where

$$G \cdot [f(v_1) \cdot \cos \alpha + \sin \alpha] = F_a\ (v_1) \tag{20}$$

the accelerating capacity of the motor vehicle is off. Since however, the dynamic loss F_d was an actual one, it has previously to end. This is manifest by the operating point on the new running resistance curve shifting towards greater speeds.

After the dynamical loss ended, the conditions will be stabilized and the vehicle will run at the constant speed v.

Supposing that the loss F_d is proportional to the acceleration, or to the generating accelerating force F_a, it seems to be likely that the deviation of the speed v compared to the speeds v_1 and v_2 — determined by the intersection point of the $E = f_3\ (v)$ curve with the $F_{gv} = f_2(v)$ and $F_m = f_1(v)$ curves — will be inversely proportional to the accelerations pertaining to the latter two speeds.

Hence, it can be written:

$$\frac{v - v_1}{v_2 - v} = \frac{a(v_2)}{a(v_1)} = \frac{F_a(v_2)}{F_a(v_1)} = \frac{F_d(v_2)}{F_d(v_1)} \tag{21}$$

and the speed v where the vehicle will run evenly:

$$v = \frac{v_2 \cdot a_2 + v_1 a_1}{a_1 + a_2}, \qquad [\mathrm{m/sec}] \tag{22}$$

where

a_1 and a_2 are the accelerations (m/sec^2) pertaining to the speeds v_1 and v_2.

Starting from Eqs 20 and 22, the boundary speed curve of the motor vehicle under test is:

$$v_{\text{bound}} = f_1(\alpha), \tag{23}$$

yielding in the speed range $v_{\max} > v > v_{\min}$ the curve

$$F_d = f_2(v) \tag{24}$$

from Eq. 19, and

$$F_\eta = f_3(v) \tag{25}$$

from the relationship

$$F_\eta(v) = F_m(v) - F_f(v) - F_w(v) - F_a(v) - F_d(v)\,. \tag{26}$$

Analysis of the mechanical and dynamical losses of the motor built into the motor vehicle requires a complex and sophisticated computation.

To be acquainted with, and to generalize these losses, many measurement data were needed of a great number of current up-to-date passenger motor cars.

The high number of data justified the use of a digitalal computer ODRA 1204.

Results

Computer analysis granted the losses F_d and F_η of the tested motor vehicles as well as their specific values as a function of speed and acceleration.

Since the final speeds of vehicles of different categories are widely different, the use of the specific values of the losses according to the speed was considered as impractical.

In the seconds after starting, the value of the maximum acceleration of each motor vehicle, irrespective of category, is determined by the rate of the adhesion between wheel and road; their difference is negligible. Therefore it seemed to be more expedient to analyse, and to generalize the specific values of the losses as a function of acceleration.

Since the conclusions drawn from the investigations are best used at the design stage where the acceleration of the vehicle is still unknown, the presentation of the specific values F_d and F_η as a function of the actual acceleration would entrain inaccuracy.

Initially the theoretical tractive effort computed from the motor's moment and the running resistances are only known. These values and the vehicle rating permitted to determine the theoretically possible acceleration as a function of speed.

The theoretical acceleration is

$$a_e = \frac{F_m - E}{m \cdot k_m} \cdot [\mathrm{m/sec^2}] \tag{27}$$

Accordingly, the scatter of theoretical acceleration

$$\frac{F_d}{F_a + F_d} = f_4(a_e) \tag{28}$$

and

$$\frac{F_\eta}{F_m} = f_5(a_e) \tag{29}$$

has been generalized by an analytic program and the results illustrated in Fig. 5 as dynamical efficiencies as a function of the theoretical acceleration

$$\eta_m = 1 - \frac{F_\eta}{F_m} \tag{30}$$

mechanical and

$$\eta_d = 1 - \frac{F_d}{F_a + F_d} \tag{31}$$

dynamical efficiencies.

Application of the results

Application of the method of preliminary planning will be illustrated on a Peugeot 504 passenger car with a carburettor Otto engine built in.

According to the investigated vehicle specification, the theoretical acceleration may be determined as a function of speed

$$a_e = \frac{F_m - E}{m \cdot k_m} \qquad [\text{m/sec}^2] \tag{27}$$

As a matter of course this acceleration cannot be achieved under normal circumstances its values are, however, needed in following computations.

At the zero acceleration pertaining to each speed in case of stationary operation, the actual tractive force value is given by the relationship:

$$F_v = F_m \cdot \eta_m \quad [\text{kp}] \tag{32}$$

where the force F_m is multiplied by η_m pertaining to the theoretical acceleration value generated by the theoretical tractive force F_m (Fig 5.)

In case of acceleration, the dynamical loss consumes the tractive force excess $F_v - E$.

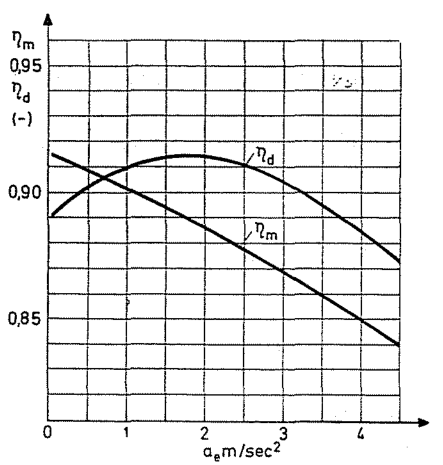

Fig. 5. Mechanical and dynamical efficiencies in accelerating operation of Otto engines built into vehicles

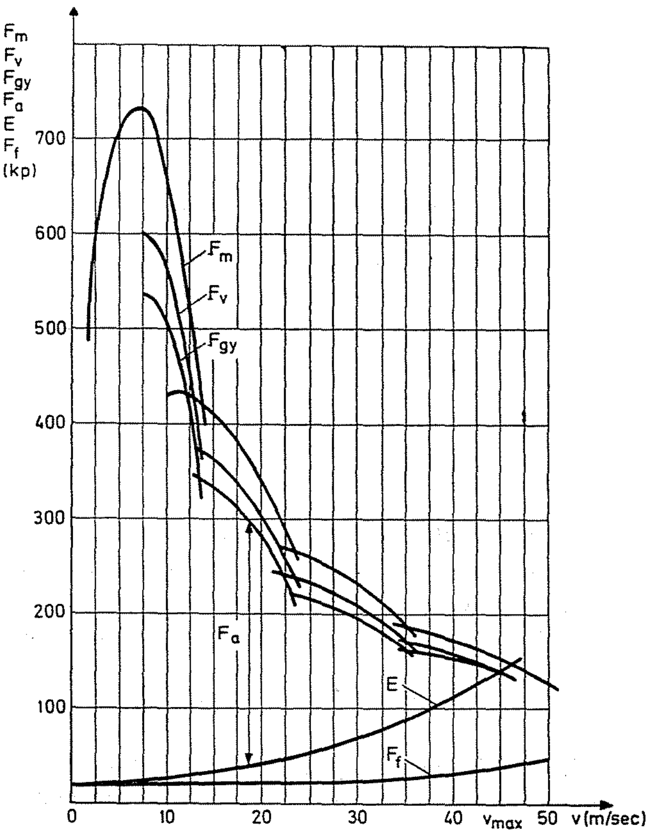

Fig. 6. Tractive force, accelerating force and running resistance diagrams of a passenger car type Peugeot 504

The actual pure accelerating force is, similarly to the aforesaid:

$$F_a = (F_v - E) \cdot \eta_d \cdot \quad [\text{kp}] \tag{33}$$

The value η_d pertaining to the vehicle speed in question will be found in this case also on the base of the theoretical acceleration (Fig. 5).

The relevant values are shown in Fig. 6.

The intersection point of curves E and F_v indicates the final speed v possible for the vehicle on level road. If the running resistance changes, the new intersection point will give the changed final speed.

The intersection points of the curves F_{gy} pertaining to the different gears present the ideal spots for gear shifting.

The distance between the curves E and F_{gy} shows the neat accelerating force F_a.

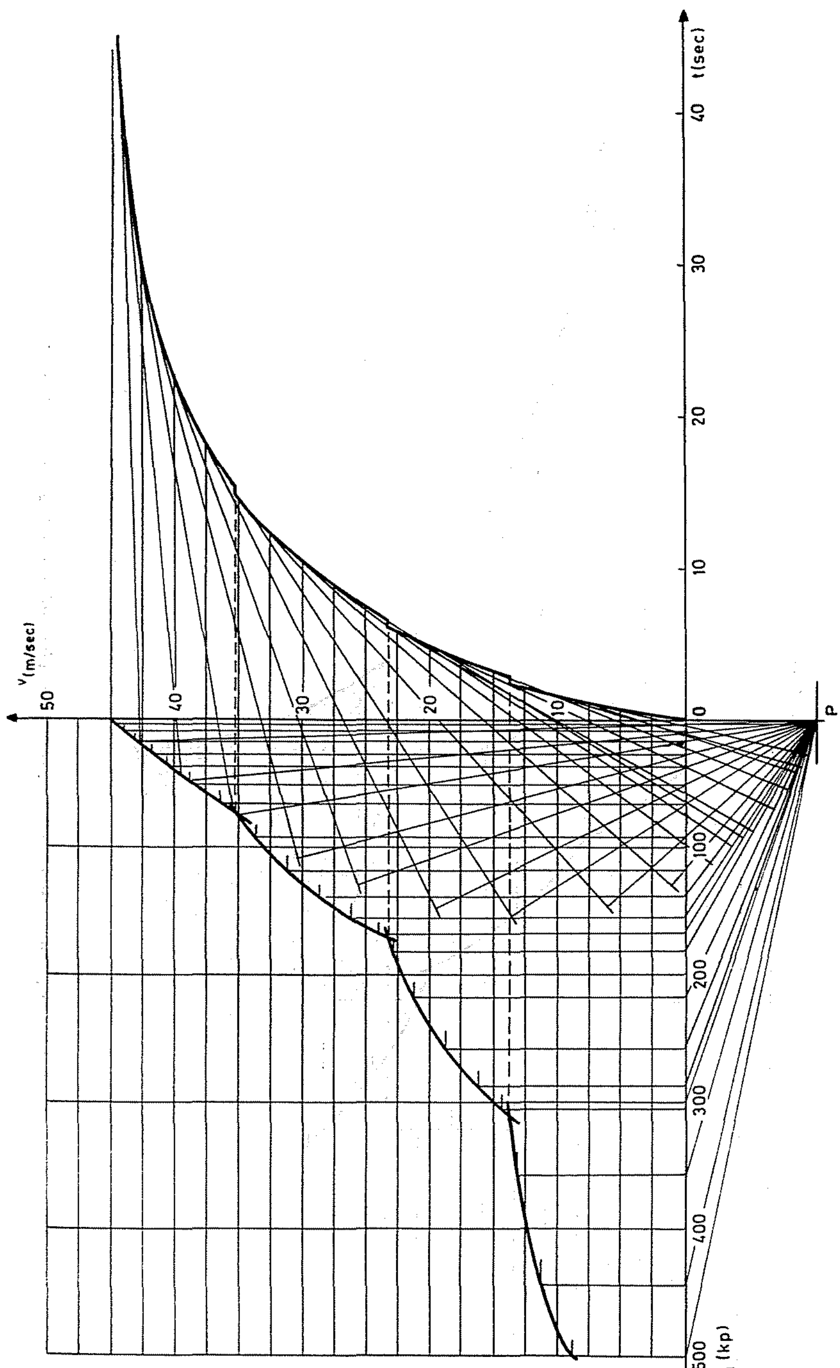

Fig. 7. **Speed—time diagram of a passenger car type Peugeot 504**

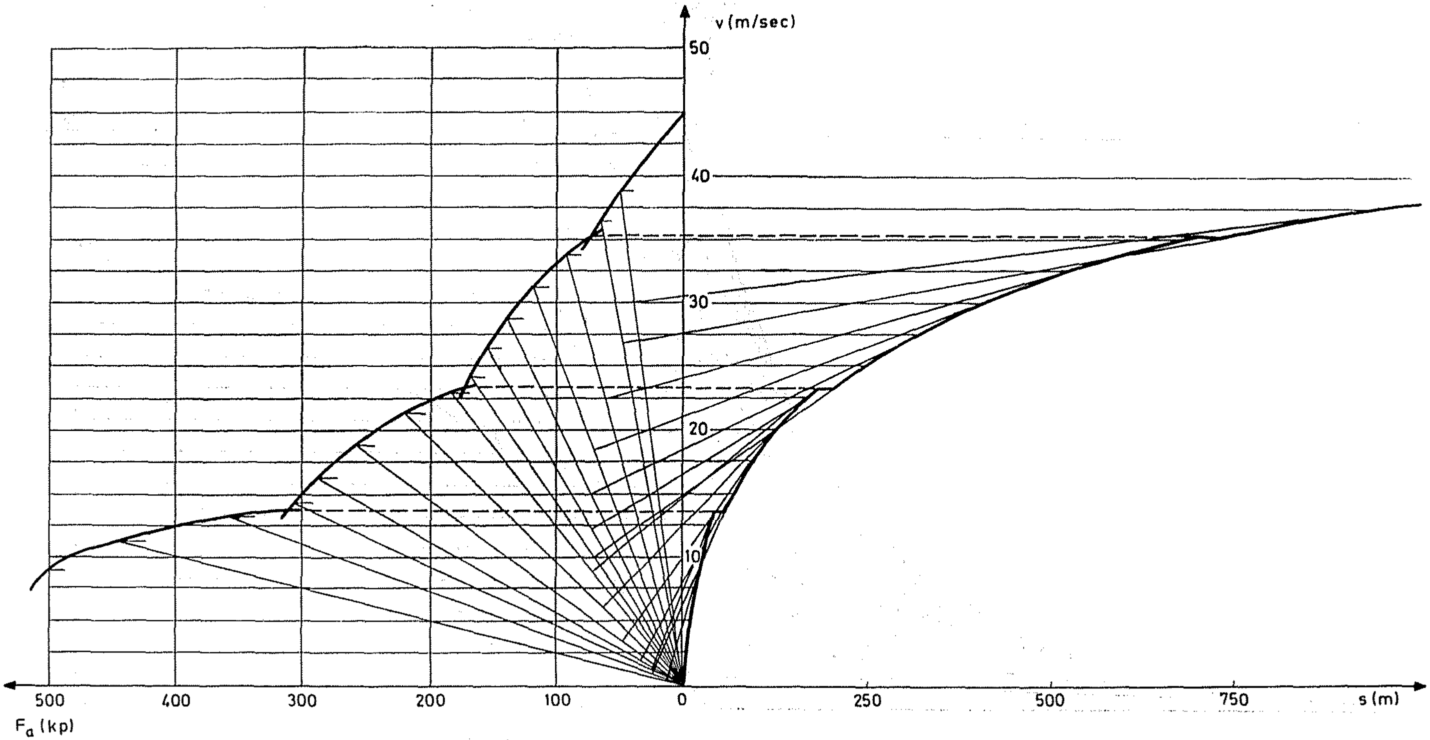

Fig. 8. **Speed — distance diagram of a passenger car type Peugeot 504**

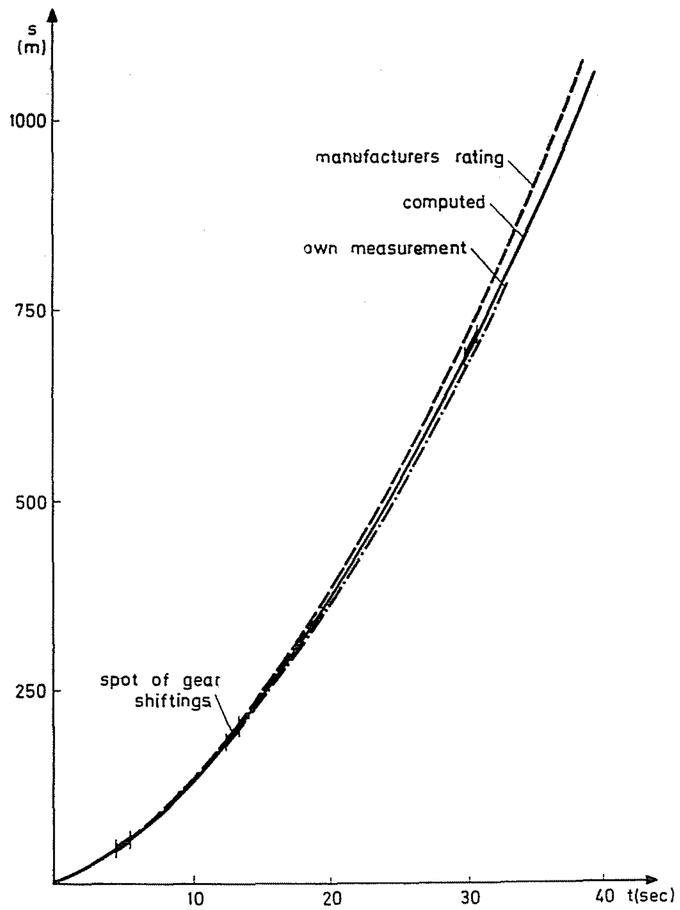

Fig. 9. Distance—time diagram of a passenger car type Peugeot 504

The actual situation in the period immediately after the start is somewhat hard to follow. The method gives a trustworthy information only after the practically slipfree connection of the clutch.

Figs 7 and 8 illustrate the neat accelerating force values of the tested vehicle as a function of speed, the speed—time and the speed—distance diagrams have been plotted by means of the already described graphical differentiation.

The time and distance value pairs pertaining to identical speed values and contained in the speed—time and speed—distance diagrams have been applied to plot the distance—time diagram of the motor vehicle (Fig. 9) where also the values taken and measured on road, using a passenger car of identical type made available by the manufacturers, have been indicated.

Summary

The described method makes easier and more accurate the work of predicting the running qualities of motor vehicles in the design stage or to select the one most appropriate for given running qualities without preliminary operation.

The running qualities of a motor vehicle are determined jointly by the motor, drive, wheels, body, etc. as subsystems of a system.

The said units are rather various in construction but varying their assembly offers further possibilities for the designing engineers.

Several kinds of motors are built into the same body or the same motor is built into different bodies. The running qualities of the described varieties are of course different and so will be the losses.

The pure accelerating force needed for the linear acceleration of a given motor vehicle running on a speedway is reflected by the vehicle's speed—time diagram.

Since the external characteristics of the motor built into the vehicle and measured on test stand are known and the running resistances can be calculated, the difference of the accelerating force determined from the speed—time diagram and of the loss-free tractive force theoretically available on the driving wheels, reduced by the value of the running resistance, will yield the sum of the losses occurring during acceleration.

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József Hegedűs, 1450, Budapest, P.O.B. 93. Hungary