Abstract

The variable-geometry suspension system is in the focus of the paper. The advantages of the variable-geometry system are the simple structure, low energy consumption and low cost. During maneuvers the variable-geometry system modifies the camber angle of the front wheels in order to improve road stability. The system affects both the chassis roll angle and the half-track change. Moreover, the tracking error of the reference yaw rate can also be reduced. In the paper the challenges and possibilities of the variable geometry suspension system are analyzed.

Keywords

variable geometry suspension · driver assistance systems · automotive control

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1 Introduction

In recent decades several new researches and development tendencies have evolved [13]. The automotive industry put emphasis on urban mobility and transport, alternative fuels, electrification of the vehicle safety applications in co-operative systems, suitable materials, environment-friendly and efficient manufacturing. In some of these systems the driver is supported by assistance systems to meet the performance specifications. Several important journal and conference papers have been presented in this topic, see e.g. [17], [16].

A new possibility in automotive safety control is variable geometry suspension systems. The suspension determines such critical components as the height of the roll center and the half track change. The advantages of the variable geometry suspension are the simple structure, low energy consumption and low cost compared to other mechanical solutions such as an active front wheel steering, see [3], [9]. Since various safety and economy properties of the vehicle are determined by the suspension geometry it has significant influence on the control design. The control input of variable geometry systems is camber angles of the front and rear wheels, with which the driver is supported to perform the various vehicle maneuvers, such as a sharp cornering, overtaking or double lane changing. The control system must guarantee various crucial vehicle performances such as trajectory tracking, roll stability and geometry limits.

Several papers for various kinematic models of suspension systems have been published. A review of the variable geometry systems was presented by [19]. The control system varied the leverage ratio between the spring/damper unit and the road wheel assembly. A nonlinear model of the McPherson strut suspension system was published by [4]. By using this model the kinematic parameters such as camber, caster and king-pin angles were examined. The kinematic design of a double-wishbone suspension system was examined by [21]. Seeking to meet the performance requirements often leads conflict situations and requires a compromise considering the kinematic and dynamic properties, see [21]. The vehicle handling characteristics based on a variable roll center suspension was proposed by [12]. A rear-suspension active toe control for the enhancement of driv-
ing stability was proposed by [6]. The main focus on these methods is on the construction solution and the control design has received little attention. However, besides performances, the control design must handle important tasks such as disturbance attenuation and robustness against uncertainties.

In our project, which focuses on the integrated vehicle control systems, the variable geometry suspension system has several possibilities, see [8]. It has been shown that suspension control design is in interaction with the construction of the system [15]. Therefore it is possible to formulate a common control and construction design task, which leads to an optimal variable geometry suspension systems [14]. In this paper some further aspects of the design of variable geometry suspension system are presented.

This paper is organized as follows. In Section 2 the motivation of the variable geometry suspension system is presented. In Section 3 the possibilities of the variable geometry suspension system, i.e., the effects of suspension construction, are analyzed in more detail. In Section 4 the effects of the suspension system on the performance specifications are analyzed. Finally, in Section 5 the concluding remarks are summarized.

2 Motivation example

In this section a motivation example of the efficiency of a variable geometry suspension is proposed. Fig. 1 shows a double-wishbone suspension with three different arrangements of suspension arms. The positions of arms have an important role in vehicle dynamics. Different aims in suspension design result in different suspension constructions. Fig. 1(a) illustrates a construction of a variable-geometry suspension which is able to minimize the roll angle of the chassis. The second construction (Fig. 1(b)) minimizes the half-track change, while it is possible to reach minimal actuation using a third suspension (Fig. 1(c)). The effects of different suspension types in variable geometry suspension control can be seen in Fig. 2. In this motivation example the aim is to track a predefined yaw-rate signal, which induces a change in the wheel camber [14]. According to simulations it is confirmed that the control system of Suspension 1 is able to reduce the roll of the chassis compared to the other suspension types (Fig. 2(a)). The half-track change of the suspensions is illustrated in Fig. 2(b). It is shown that the variable-geometry suspension controller can achieve reduced half-track change if Suspension 2 is used. Fig. 2(c),(d) show control input \( a_i \) and the wheel camber of the system. Minimal control input is actuated in Suspension 3.

Although all of these suspension types result in different roll angles, half-track changes and control inputs, they can track the predefined yaw-rate signal accurately, see Fig. 2(e). After these statements the following questions have arisen: How is it recommended to design the construction of the suspension to improve the performances of the controlled system? What is the relationship between construction and control design? What are the main factors in variable geometry suspension system design?

3 Construction aspects

In this section the effects of suspension construction are analyzed.

3.1 Suspension types

Variable geometry suspension systems can be built in several constructions. In this paper two of them are presented: the double-wishbone and the McPherson suspension. Both suspensions are widely used in passenger cars, and they can be reconstructed to active systems easily. Besides, the positions of arms of these suspension types depend on the movement of the tire and the chassis, therefore it is possible to formulate their kinematic relationships without complex high-fidelity Finite Element Methods.

The kinematic model of the variable-geometry suspension based on the double wishbone suspension system is presented in Fig. 3. In this type of suspension the actuator is positioned in point A, therefore point A is able to move in the lateral direction. This actuation affects the modification of wheel camber angle. In the kinematic modeling of the double-wishbone suspension the masses, inertias and the elasticity of the construction elements are ignored, and the arms of the suspension are modeled as bar elements. The suspension is analyzed in a coordinate system which is fixed to the chassis. Consequently the rolling of the chassis and the road irregularities have the same effect in terms of the moving of the wheel compared to the chassis. The deduction of the formalization of the wheel camber angle depends on the geometric position of the suspension points, road irregular-
This paper is organized as follows. In Section 2 the motivation of the variable geometry suspension system is presented. In Section 3 the possibilities of the variable geometry suspension system are analyzed. Finally, in Section 5 the conclusion and remarks are summarized.

2 Motivation example

In this section a motivation example of the efficiency of suspension construction is presented. In Figure 2(a) it is shown that the control system of Suspension 1 is able to reduce half-track change if Suspension 1 is used. Figure 2(b) shows that Suspension 2 is able to reduce half-track change if Suspension 2 is used. Figure 2(c) shows that Suspension 3 is able to reduce half-track change if Suspension 3 is used.

3 Construction aspects

In this section the effects of suspension construction are analyzed. The vertical forces of the suspension movements are similar to those of road irregularities. The transformation of the double-wishbone suspension parameters to the chassis are similar to those of road irregularities. The efficiency and the input of the mechanism. The vertical forces of the suspension are considered as an indirect way in the modeling of the suspension movements. The effects of the movement of the chassis are similar to those of road irregularities. The transformation of the double-wishbone suspension parameters to the parameters of a quarter-car model is presented by [17].

Another possible suspension construction is illustrated in Figure 2(e). After these statements the following questions have arisen: How is it recommended to design the construction of the suspension to improve the performance of the controlled system? What is the relation between construction and control design? What are the main factors in variable geometry suspension system design?

Fig. 2. Results of different suspension systems

Fig. 4. In this case the control input is the lateral movement of point C. However it is a possible construction for variable geometry system, it also has a disadvantage. In a double wishbone suspension there are two lateral arms, which transfer lateral loads, while McPherson system has only one. Subsequently the actuator in point C in the McPherson system must actuate an increased force compared to the double wishbone one.
3.1 Suspension types

Variable geometry suspension systems can be built in several constructions. In this paper two of them are presented. The efficiency of the front wheel camber angle on lateral vehicle dynamics is more significant than the rear wheel camber angle. This factor explains why in the paper the variable-geometry suspension of the front wheel camber angle is used. It should also be noted that $C_{1,y}$ depends on velocity.

3.2 Front-rear wheels

An important point in the analysis of the variable-geometry suspension is the efficiency of the position of controlled suspension. Nonzero wheel camber affects the lateral force in the tire-ground contact, which is formulated by the next expression at small side-slips [13]:

$$F_y = C_y \gamma$$

where $\gamma$ is the wheel camber angle, $C_y$ is a coefficient which represents the stiffness of the camber. The lateral dynamics of vehicle is extended with [11], thus linearized tire model is the following:

$$F_{y,i} = C_i \alpha_i + C_{i,y} \gamma,$$  

where $i \in [1,2]$ represents the front and rear suspensions, $C_i$ is cornering stiffness, $\alpha_i$ is the side-slip angle and $C_{i,y}$ relates to front and rear $C_y$. Consequently, it is possible to actuate both the front and the rear suspensions. Therefore it is necessary to analyze if there is any difference between the actuation in the front and rear suspensions.

The efficiency of wheel camber actuation is analyzed using a high-fidelity vehicle simulation software in order that the complex Pacejka Magic-formula tire model is compared to the formulated control oriented linear tire model. The tire parameter of linear model $C_{1,y}$ is approximated by using a least square method from simulated signals when changing the front and rear camber angles. Front and rear wheel parameters $C_y$ are estimated at different velocities, see Fig. 5. Note that the value of parameter $C_{1,y}$ is higher than that of $C_{2,y}$. It demonstrates that the efficiency of the front wheel camber angle on lateral vehicle dynamics is more significant than the rear wheel camber angle. This factor explains why in the paper the variable-geometry suspension of the front wheel camber angle is used. It should also be noted that $C_{1,y}$ depends on velocity.

3.3 Double effect: camber & toe angles

There is another aspect of suspension control on the rear wheels. [11] proposes a system architecture in which the suspension geometry is modified to realize active toe angle on real wheels. In this concept the goal is to improve driver performances using a driver assistance system: the driver steers the front wheels and a controller assists the driver during the rear wheel toe angle modification.

![Double wishbone suspension system](image1)

**Fig. 3. Double wishbone suspension system**

![McPherson suspension system](image2)

**Fig. 4. McPherson suspension system**

![Estimation of parameters C_y](image3)

**Fig. 5. Estimation of parameters $C_y$**

The geometry of the suspension determines the rotation of the wheel at camber modification. In the case of double wishbone suspension at camber modification the wheel rotates around an axis, which is determined by the steering track-rod end and the connection point of the lower arm, see Fig. 5. It means that the position of the track-rod end has an important role in the rotation of the wheel. Angle $\varepsilon$ represents the angle of the axis, around which the wheel rotates at actuation. The consequence of angle $\varepsilon$ is the relationship between the camber angle and the toe angle. During actuation there is camber angle modification and an additional toe angle. It means that a suitable suspension geometry can improve the lateral force on the tire not only by the camber angle, but also by the toe angle.

The angle $\varepsilon$ is determined by the position of the track-rod end and connection point of the lower arm. The lower arm position can be determined by other suspension construction performances [11], therefore it is necessary to influence the height and length of the track-rod. The length of track-rod plays a role in...
steering design [5], therefore the height of track-rod is chosen to influence wheel rotation. An appropriate choice of this height can improve the lateral force in the tire-road contact with the common camber and toe angle.

3.4 Actuator forces

The control input of the system is the lateral movement of a suspension point in the given construction. In a real implementation this movement is realized using a hydraulic actuator [11], [10],[2] or an electric motor [3]. In both systems it is necessary to determine force resistances, which influence the necessary power of the actuator.

The in-built hydraulic actuator must compensate for different resistances at the generation of the wheel camber angle. In order to modify the camber angle of the rotated wheels it is necessary to generate energy against the gyroscopic effect. The torque of the rotation of the wheel around its longitudinal axis is formulated by the following assumption:

\[ M_{gy} = 2J_w v/r_w \gamma \] (3)

where \( J_w \) is the inertia of the wheel on the rotation axis, \( v \) is the velocity of the vehicle and \( r_w \) is the wheel radius.

During camber actuation the position of the wheel is modified. Since in most cases the tire can not be pushed into the road (except sand), the vertical movement of the tire-road contact point induces the movement of the chassis. It means that the hydraulic cylinder must increase the potential energy of the system and compensate for the energy dissipation of the damper. The formulation of this resistance depends on vehicle roll dynamics, see [15].

The lateral movement of actuator cylinder can result in a lateral movement of tire-road contact area in the plane of the road, see Fig. 6. In Section 3.3 the importance of \( \varepsilon \) is established in the aspect of lateral forces. The rotation of the wheel also induces the movement of the tire-road contact area, which results in increased tire wear. The position of the rotation axis influences the position of the wheel, and during it the movement of the tire-road contact. An increased lateral movement of the contact area requires increased frictional energy \( E_{fric} \), which must be generated by an actuator force. \( E_{fric} \) depends on the position of the wheel rotation axis:

\[ E_{fric} = f(\varepsilon) \] (4)

In this section several factors of variable geometry suspension actuator forces have been proposed. When the wheel camber angle is modified, the electro-hydraulic cylinder must actuate energy to equalize the mentioned resistances. It is also deduced that resistances depend on the construction of the suspension.

4 Performances and design aspects

4.1 Performance specifications

In this section the performance specifications concerning both the construction of the variable-geometry suspension and the design of the control are formulated. In normal cruising maneuvers the steering control assists the driver in following the trajectory, while the variable-geometry suspension control also focuses on other performances. It minimizes the chassis roll angle by modifying the roll center of the vehicle. Moreover, the half-track change can also be minimized by using the variable-geometry suspension system. Consequently, the performance requirements are related to the yaw-rate tracking, the roll angle and the half track change. Besides, control input must also be reduced.

**Trajectory tracking**

In the trajectory tracking control the vehicle must follow the reference yaw rate. The goal is to minimize the difference between the reference yaw rate and the measured yaw rate of the vehicle:

\[ z_1 = |\dot{\psi}_{ref} - \dot{\psi}| \rightarrow min \] (5)

**Minimization of chassis roll angle**

It has also been shown that the roll center depends on controller actuation and road disturbances. The height of the roll center has an important role in the vertical dynamics of the vehicle, it determines roll motion. A possible way to minimize the chassis roll angle is the minimization of the height of the roll center \( h_M \). In this case the difference between the roll center and the center of gravity must be minimized:

\[ z_2 = |h_{CG} - h_{M,ref}| \rightarrow min \] (6)

In the aspect of \( z_2 \) performance it can be established that the height of roll center in steady state is determined by the suspension construction. Besides, the vertical movement of the roll center is determined by \( t_z \) and \( a_z \), where \( a_z \) is control input. It means that the minimization of the roll center is determined by the construction and control of the suspension simultaneously.
Half-track change minimization

An additional important economy parameter is the half-track change $\Delta B = f(t_i, a_i)$. The lateral movement of the contact point is relevant from the aspect of tire wear \cite{7}, when the suspension moves up and down while the vehicle moves forward. By using an appropriate variable geometry control these unnecessary movements can be eliminated:

$$z_3 = |\Delta B| \rightarrow \min \quad (7)$$

Control input minimization

During the control tasks it is necessary to prevent large control input, which is the lateral movement of a suspension arm $a_i$ depending on the suspension construction. It has construction limits, therefore the fourth performance focuses on the minimization of the input displacement:

$$z_4 = |a_i| \rightarrow \min \quad (8)$$

4.2 Suspension construction and control design

In the case of a variable geometry suspension the previous performances must be guaranteed. The performance vector of the system is:

$$Z = [z_1 \ z_2 \ z_3 \ z_4]^T$$

It is necessary to design an appropriate construction and a controller for suspension system which is able to guarantee $z$ performances simultaneously. However, the minimization of each performance requires different control inputs. Therefore it is necessary to find a design technique, which is able compute a construction and a controller, by which it is possible to achieve a balance between performances. It is realized by the weighting of performances, see \cite{14}. Then a cost function $J$, which depends on weights $W_i$, suspension control $K$ and some construction variables $h_i$ is formulated. The optimization task of the variable geometry suspension system design is the following:

$$\min_{K, h_i} J(W_i, K, h_i) \quad (10)$$

5 Conclusion

In the paper the challenges and possibilities of the variable geometry suspension system have been analyzed in detail. The interaction between suspension construction and control design has been presented and several aspects for the construction effects have been analyzed. The performance specifications have also been presented and the optimization task has been formalized.

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