Periodica Polytechnica
Transportation Engineering

Distributed parameter modeling of single-mast stacker crane structures

42(1), pp. 1-9, 2014
DOI:10.3311/PPtr.7055
http://www.pp.bme.hu/tr/article/view/7055

Sándor Hajdu / *Péter* Gáspár

RESEARCH ARTICLE

RECEIVED 30 SEPTEMBER 2013; ACCEPTED 21 JANUARY 2014

Abstract

This paper presents distributed parameter dynamical modeling capabilities of single-mast stacker crane structures. In the frame structure of stacker cranes due to external excitation or inertial forces undesirable structural vibrations may arise. These vibrations reduce the stability and positioning accuracy of stacker crane and causes increasing cycle time of storage/retrieval operation. Thus it is necessary to investigate of these vibrations. In this paper the dynamical behavior of single-mast stacker cranes is approximated by means of distributed parameter models. The first model is a cantilever beam model with uniform material and cross-sectional properties. This model is used to demonstrate fundamental properties of Euler-Bernoulli beam models. The second model is cantilever beam model with variable cross-sectional properties and lumped masses. The eigenfrequencies and mode shapes of this mast-model are determined by means transfer matrix method. In the third model the whole structure of single-mast stacker crane is modeled. Beside the eigenfrequencies and mode shapes of this model the Bode-diagrams of frequency response function is also calculated.

Keywords

distributed parameter dynamic model · Euler-Bernoulli beam · transfer matrix method · stacker crane

Sándor Hajdu
Department of Mechanical Engineering, University of Debrecen,
H-4028 Debrecen, Ótemető u. 2-4., Hungary
e-mail: hajdusandor@eng.unideb.hu

Péter Gáspár
Systems and Control Laboratory,
Computer and Automation Research Institute, Hungarian Academy of Sciences,
H-1111 Budapest, Kende u. 13-17, Hungary
e-mail: gaspar.peter@sztaki.mta.hu

1 Introduction

The advanced stacker cranes in automated storage/retrieval systems (AS/RS) have the requirement of fast working cycles and reliable, economical operation. Today these machines often dispose of 1500 kg pay-load capacity, 40-50 m lifting height, 250 m/min velocity and 2 m/s^2 acceleration in the direction of aisle with 90 m/min hoisting velocity and 0,5 m/s^2 hoisting acceleration. Therefore the dynamical loads, inertial forces on mast structure of stacker cranes are very high, while the stiffness of these structures due to dead-weight reduction is relatively low. Thus undesirable structural vibrations, mast-sway may arise in the frame structure during operation.

These vibrations reduce the stability and positioning accuracy of stacker crane and causes increasing cycle time of storage/retrieval operation. Thus it is necessary to investigate and predict of these vibrations.

Practically the mast structure has two fundamental configurations: the so-called single-mast and twin-mast structures. In our work we analyze single-mast structures since this configuration is more responsive to dynamical excitations. A schematic drawing of single-mast stacker crane with its main components is shown in Fig. 1.

In order to realize the dynamical investigation of structural vibrations several kinds of models can be chosen with different kinds of results, different application areas and different approximation accuracy.

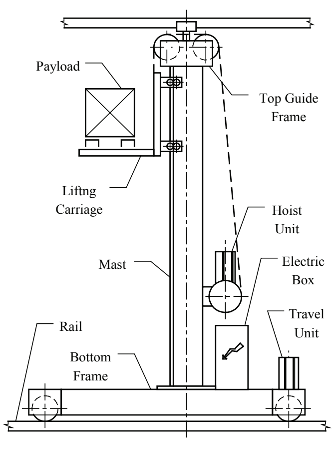

Fig. 1. Single mast stacker crane

In our work the eigenfrequencies, mode shapes and transfer functions of single-mast stacker crane frame is determined by the help of distributed parameter models. The area of distributed parameter dynamic modeling has a very extensive literature in dynamical investigation of engineering structures (Bashash et al., 2008; Aleyaasin et al., 2001; Zollner, Zobory, 2011a; Zollner, Zobory, 2011b) as well as stacker crane frames (Bachmayer et al., 2008; Bachmayer et al., 2009; Bopp, 1993; Dietzel, 1999; Görges et al., 2009; Oser, Kartnig, 1994; Reisinger, 1998; Staudecker et al., 2008).

The aim of this paper is to generate a basic dynamic model with good accuracy. In the further steps of our research this model is applied to verify the accuracy of other simpler models e.g. multi-body models with few degrees of freedom. The main parameters of investigated stacker crane are shown in Table 1.

2 Cantilever prismatic beam model

The simplest mast model of single-mast stacker cranes is the cantilever beam model with uniform material and cross-sectional properties along its length. This model with its main parameters, cross-sectional and material properties is shown in Fig. 2. The deflection function of beam is denoted by $u(y,t)$, A_1 is the cross sectional area, I_{z1} is the area moment of inertia, E is the modulus of elasticity and ρ_{ST} is the mass density.

The governing equation for transversal vibrations of this beam is a fourth order partial differential equation (PDE):

$$I_{z1}E\frac{\partial^4 u(y,t)}{\partial y^4}+A_1\rho_{ST}\frac{\partial^2 u(y,t)}{\partial t^2}=0. \tag{1}$$

This is the so called Euler-Bernoulli beam theory equation for free vibrations. Now let's assume that the solution of equation (1) in case of standing wave solution is separable into time and space domains:

$$u(y,t)=X(y)\cdot T(t), \tag{2}$$

where $X(y)$ denotes the spatial mode shape function and $T(t)$ represents the time-dependent coordinate. Substituting equation (2) into equation (1) yields two separated equations:

$$\frac{d^2T(t)}{dt^2}+\alpha^2T(t)=0, \tag{3}$$

$$\frac{d^4X(y)}{dy^4}-\alpha^2\frac{A_1\rho_{ST}}{I_{z1}E}X(y)=0, \tag{4}$$

where α^2 is a separation constant. With the denotation

$k^4=\alpha^2\frac{A_1\rho_{ST}}{I_{z1}E}$ equation (4) can be simplified as:

$$\frac{d^4X(y)}{dy^4}-k^4X(y)=0. \tag{5}$$

Tab. 1. Main parameters of investigated stacker crane

Denomination	Denotation	Value
Payload:	m_p	1200 kg
Mass of lifting carriage:	m_{lc}	410 kg
Mass of hoist unit:	m_{hd}	470 kg
Mass of top guide frame:	m_{tf}	70 kg
Mass of bottom frame:	m_{sb}	2418 kg
Mass of entire mast:	m_{sm}	8148 kg
Lifted load position:	h_h	1-44 m
Length of sections:	l_1	2,9 m
	l_2	3 m
	l_4	3,5 m
	l_6	11,5 m
	l_8	29 m
	l_9	1 m
Cross-sectional areas:	A_1; A_2	0,03900 m^2
	A_4; A_6	0,02058 m^2
	A_8; A_9	0,01518 m^2
Second moments of areas:	I_{z1}; I_{z2}	0,00152 m^4
	I_{z4}; I_{z6}	0,00177 m^4
	I_{z8}; I_{z9}	0,00106 m^4

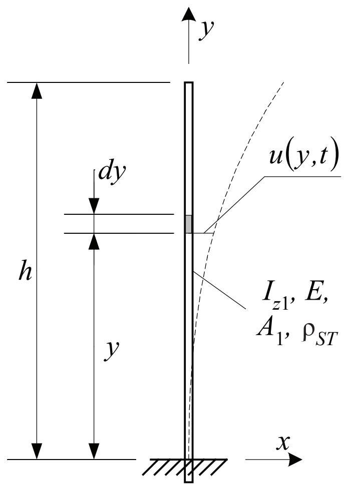

Fig. 2. Cantilever prismatic beam model of stacker crane mast

The general solutions of the two ordinary differential equations (ODE) presented above are

$$T(t)=A\cos(\alpha t)+B\sin(\alpha t), \tag{6}$$

$$X(y)=C\cdot S(ky)+D\cdot T(ky)+E\cdot U(ky)+F\cdot V(ky), \tag{7}$$

respectively, where A, B, C, D, E, F, are constants of integration determined by initial and boundary conditions. The first solution shows that α corresponds to the frequency of vibration, while equation (7) gives the general mode shapes. During determination of (7) we used the $S(.)$, $T(.)$, $U(.)$, $V(.)$ Rayleigh functions. With this form the determination of unknown C, D, E, F constants of mode shapes will be very simple. Rayleigh functions can be expressed as:

$$\begin{aligned} S(ky) &= \frac{1}{2}\left(cosh(ky) + cos(ky)\right) \\ T(ky) &= \frac{1}{2}\left(sinh(ky) + sin(ky)\right) \\ U(ky) &= \frac{1}{2}\left(cosh(ky) - cos(ky)\right) \\ V(ky) &= \frac{1}{2}\left(sinh(ky) - sin(ky)\right). \end{aligned} \tag{8}$$

Some useful properties of Rayleigh functions are:

$$\begin{aligned} &S(0) = 1;\ T(0) = U(0) = T(0) = 0; \\ &\frac{dS(z)}{dz} = V(z);\ \frac{dT(z)}{dz} = S(z); \\ &\frac{dU(z)}{dz} = T(z);\ \frac{dV(z)}{dz} = U(z). \end{aligned}$$

Eigenfrequencies of vibrations can be determined by means of boundary conditions. Boundary conditions regarding to clamped end are

- $X = 0$ (deflection is zero),
- $\phi = \frac{dX}{dy} = 0$ (rotation angle is zero).

Boundary conditions regarding to free end are

- $M = -I_{z1}E\frac{d^2X}{dy^2} = 0$ (bending moment is zero),
- $V = -I_{z1}E\frac{d^3X}{dy^3} = 0$ (shear force is zero).

The general form of deflection function, rotation angle, bending moment and shear force are:

$$\begin{aligned} X(y) &= C\cdot S(ky) + D\cdot T(ky) + E\cdot U(ky) + F\cdot V(ky) \\ \phi(y) &= k\left[C\cdot V(ky) + D\cdot S(ky) + E\cdot T(ky) + F\cdot U(ky)\right] \\ M(y) &= -I_{z1}Ek^2\left[C\cdot U(ky) + D\cdot V(ky) + E\cdot S(ky) + F\cdot T(ky)\right] \\ V(y) &= -I_{z1}Ek^3\left[C\cdot T(ky) + D\cdot U(ky) + E\cdot V(ky) + F\cdot S(ky)\right]. \end{aligned} \tag{9}$$

From the boundary conditions of clamped end:

$$\begin{aligned} X(0) &= C = 0 \\ \phi(0) &= D = 0. \end{aligned} \tag{10}$$

From the boundary conditions of free end:

$$\begin{aligned} -\frac{M(h)}{I_{z1}Ek^2} &= E\cdot S(kh) + F\cdot T(kh) = 0 \\ -\frac{V(h)}{I_{z1}Ek^3} &= E\cdot V(kh) + F\cdot S(kh) = 0. \end{aligned} \tag{12}$$

The nontrivial solution of (12) exists when the determinant of coefficients vanishes. With this the following frequency equation can be determined.

$$\begin{vmatrix} S(kh) & T(kh) \\ V(kh) & S(kh) \end{vmatrix} = S^2(kh) - T(kh)V(kh) = 0 \tag{13}$$

The first three roots of frequency equation are $(kh)_1 = 1{,}875$, $(kh)_2 = 4{,}694$, $(kh)_3 = 7{,}855$. By the help of these roots the eigenfrequencies can be calculated with substitution in the following equation.

$$\alpha_i = (kh)_i^2 \frac{1}{h^2}\sqrt{\frac{I_{z1}E}{A_1\rho_{ST}}} \tag{14}$$

The unknown constants of mode shapes can also be determined with substitution roots into (12) and solution of the resulted system of equations.

3 Cantilever beam model with multiple sections and lumped masses

In our second model (see in Fig. 3.) the mast of stacker crane is modeled as a cantilever beam with variable cross-sectional properties and lumped masses. The position of lifted load can be varying along the mast. During our calculations, without the loss of generality we take the lifted load into consideration in its uppermost position. As can be seen in Fig. 3. the mast is divided into prismatic sections, to solve these kinds of problems in most cases the method of transfer matrix is used (see in Ludvig, 1983).

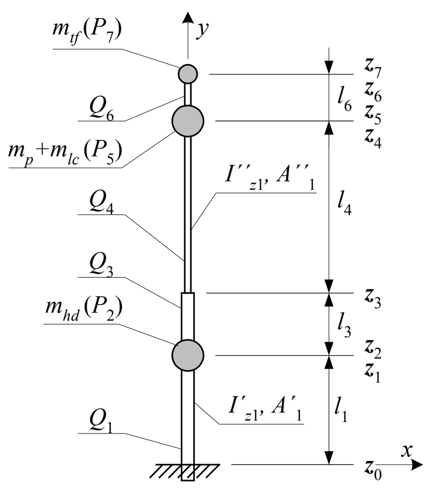

Fig. 3. Cantilever beam model with several cross-sections and lumped masses

The governing equations of section-wise uniform beam model must be generated according to every sections (see in Fig. 4.). During investigations the following assumptions and denotations are applied:

- the cross-sectional properties (A_i, I_{zi}) inside the sections are constant,
- the length of i-th section is denoted by l_i, the position of investigated differential beam element (y) is measured from the initial point of i-th section,
- the deflection at endpoint of i-th section is denoted by X_i, the rotation angle is ϕ_i, the bending moment is M_i and the shear force is V_i.

With the deflection, rotation angle, bending moment and shear force respectively the so called state vector can be defined. This state vector is shown in expression (15).

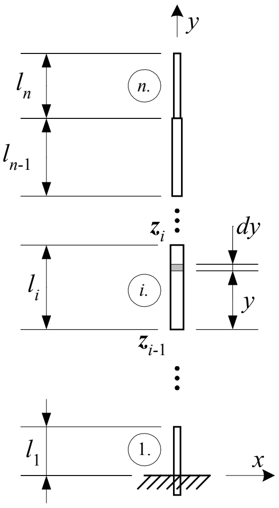

Fig. 4. Beam model with several cross-sections

$$z_i = \begin{bmatrix} X_i \\ \phi_i \\ M_i \\ V_i \end{bmatrix} = \begin{bmatrix} X \\ \phi \\ M \\ V \end{bmatrix}_i \tag{15}$$

Let's apply the next simplifying relations.

$$\left(\frac{\lambda_i}{l_i}\right)^4 = \alpha^2 \frac{A_i \rho_{ST}}{I_{zi} E} \; ; \; \lambda_i = l_i \sqrt[4]{\frac{\alpha^2 A_i \rho_{ST}}{I_{zi} E}} \tag{16}$$

With the denotations shown in (16) the differential equation of mode shapes and its general solution according to i-th section can be expressed as follows.

$$\frac{d^4 X_i(y)}{dy^4} - \left(\frac{\lambda_i}{l_i}\right)^4 X_i(y) = 0 \tag{17}$$

$$X_i(y) = C_i \cdot S\left(\frac{\lambda_i}{l_i} y\right) + D_i \cdot T\left(\frac{\lambda_i}{l_i} y\right) + E_i \cdot U\left(\frac{\lambda_i}{l_i} y\right) + F_i \cdot V\left(\frac{\lambda_i}{l_i} y\right) \tag{18}$$

In order to calculate the eigenfrequencies of the model we have to determine relationship between state vectors according to initial point and endpoint of i-th section. If we know the components of state vector at the initial point of i-th section, then we can determine the unknown coefficients of this section applying the special properties of Rayleigh functions.

$$\begin{aligned} X_i(0) &= C_i = X_{i-1} \\ \phi_i(0) &= \frac{\lambda_i^2}{l_i^2} D_i = \phi_{i-1} \\ M_i(0) &= -I_{zi} E \frac{\lambda_i^2}{l_i^2} E_i = M_{i-1} \\ V_i(0) &= -I_{zi} E \frac{\lambda_i^3}{l_i^3} F_i = V_{i-1} \end{aligned} \tag{19}$$

Now we can define the general relationship between state vectors according to both ends of i-th section. This relationship in matrix form is expressed as follows:

$$\begin{bmatrix} X_i \\ \phi_i \\ M_i \\ V_i \end{bmatrix} = \begin{bmatrix} s(\lambda_i) & l_i t(\lambda_i) & \beta_i u(\lambda_i) & \beta_i l_i v(\lambda_i) \\ \frac{\lambda_i^4}{l_i} v(\lambda_i) & s(\lambda_i) & \frac{\beta_i}{l_i} t(\lambda_i) & \beta_i u(\lambda_i) \\ \frac{\lambda_i^4}{\beta_i} u(\lambda_i) & \frac{\lambda_i^4 l_i}{\beta_i} v(\lambda_i) & s(\lambda_i) & l_i t(\lambda_i) \\ \frac{\lambda_i^4}{\beta_i l_i} t(\lambda_i) & \frac{\lambda_i^4}{\beta_i} u(\lambda_i) & \frac{\lambda_i^4}{l_i} v(\lambda_i) & s(\lambda_i) \end{bmatrix} \begin{bmatrix} X_{i-1} \\ \phi_{i-1} \\ M_{i-1} \\ V_{i-1} \end{bmatrix}, \tag{20}$$

i.e. $z_i = Q_i z_{i-1}$, where:

$$\begin{aligned} -\beta_i &= \frac{l_i^2}{I_{zi} E} \\ s(\lambda) &= S(\lambda) = \frac{1}{2}\left(cosh(\lambda) + cos(\lambda)\right) \\ t(\lambda) &= \frac{T(\lambda)}{\lambda} = \frac{1}{2\lambda}\left(sinh(\lambda) + sin(\lambda)\right) \\ u(\lambda) &= \frac{U(\lambda)}{\lambda^2} = \frac{1}{2\lambda^2}\left(cosh(\lambda) - cos(\lambda)\right) \\ v(\lambda) &= \frac{V(\lambda)}{\lambda^3} = \frac{1}{2\lambda^3}\left(sinh(\lambda) - sin(\lambda)\right) \end{aligned} \tag{21}$$

are simplifying equations. The Q_i matrix is known as the section matrix according to i-th section. When lumped masses are placed on the uniform beam the shear force of beam suddenly changes at the position of these lumped masses. The value of this shear force jump (since the motion of every point of beam is harmonic) is directly proportional to the magnitude of lumped mass, the amplitude of motion and the square of frequency. Because of this the state vector changes at positions of lumped masses therefore beyond these points we have to start a new section. The magnitude of shear force jump is expressed as:

$$V_i = V_{i-1} + \Delta V_i$$
$$V_i = V_{i-1} - m_i \alpha^2 X_{i-1} \tag{22}$$

Thus the relationship between state vector before and after lumped mass in matrix form is:

$$\begin{bmatrix} X_i \\ \phi_i \\ M_i \\ V_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -m_i\alpha^2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{i-1} \\ \phi_{i-1} \\ M_{i-1} \\ V_{i-1} \end{bmatrix}, \tag{23}$$
$$z_i = P_i z_{i-1}.$$

The P_i matrix is known as the point matrix according to lumped mass m_i. The common designation of section and point matrices is transfer matrix.

Now by means of these transfer matrices the eigenfrequencies of model can be determined. The state vector at the bottom of mast consists two unknowns since here the deflection and the rotation angle are zero. Thus the state vector according to the bottom of mast generally, using unit vectors c_0 and d_0 can be expressed as follows.

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ M_0 \\ V_0 \end{bmatrix} = M_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + V_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{24}$$
$$z_0 = M_0 c_0 + V_0 d_0$$

Thus the state vectors in the further connection points of sections by means of transfer matrices are:

$$z_1 = Q_1 z_0 = M_0 Q_1 c_0 + V_0 Q_1 d_0$$
$$z_2 = P_2 z_1 = M_0 P_2 c_0 + V_0 Q_1 d_0. \tag{25}$$
$$\vdots$$

This calculation method can be continued until the last state vector at the tip of mast. If we slip upwards the multiplicand vectors and write the result of multiplication next to the matrix then we get the very useful computation structure shown in Fig. 5. The boundary conditions are also denoted in Fig. 5.

The eigenfrequencies of the model presented above can be calculated by means of the following boundary conditions.

$$M_7 = M_0 c_{73} + V_0 d_{73} = 0$$
$$V_7 = M_0 c_{74} + V_0 d_{74} = 0 \tag{26}$$

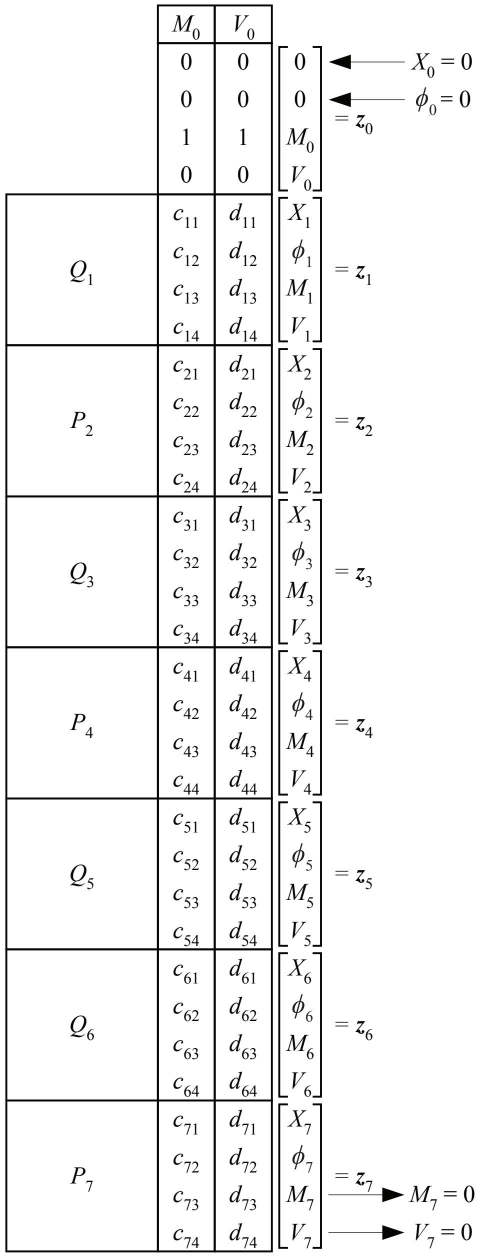

Fig. 5. Eigenfrequency calculation scheme for cantilever beam model

The nontrivial solution of (26) exists when the determinant of coefficients vanishes. Since the actual value of these coefficients depend on the frequency because of the structure of transfer matrices, thus we have to solve the following frequency equation.

$$\Delta(\alpha) = \begin{vmatrix} c_{73} & d_{73} \\ c_{74} & d_{74} \end{vmatrix} = 0 \tag{27}$$

The first three eigenfrequencies in case of our data set are shown in Table 2.

In view of calculated eigenfrequencies the unknown constants of mode shape functions can be determined by the help of boundary and continuity conditions for deflection, rotation angle, bending moment, and shear force of beam. For example from the lower boundary conditions and first few continuity conditions:

$$X_1(0) = C_1 \cdot S(0) = 0 \Rightarrow C_1 = 0$$

$$\phi_1(0) = \frac{\lambda_1}{l_1} D_1 \cdot S(0) = 0 \Rightarrow D_1 = 0$$

$$X_1(l_1) = X_3(0) \Rightarrow E_1 \cdot U(\lambda_1) + F_1 \cdot V(\lambda_1) = C_3 \tag{28}$$

$$\phi_1(l_1) = \phi_3(0) \Rightarrow \frac{\lambda_1}{l_1}\left[E_1 \cdot T(\lambda_1) + F_1 \cdot U(\lambda_1)\right] = \frac{\lambda_3}{l_3} D_3$$

$$\vdots$$

The equations presented above can be summarized into the $A(\alpha) \cdot x_c = 0$ system of equations. The unknown constants of mode shapes according to all sections can be calculated by the help of solution of this equation. The number of resulted mode shape depends on the applied eigenfrequency during the calculation. The first three mode shapes are shown in Fig. 6.

4 Distributed parameter model of single-mast stacker cranes

In our third model (see in Fig. 7.) the whole structure of single-mast stacker crane is modeled. The distributed parameter model of single-mast stacker crane with applied denotations, investigated sections and positions of state vectors are shown in Fig. 7. In this model we take the lifted load into consideration also in its uppermost position.

Since the frame structure of single-mast stacker crane is a branching structure, thus we have to pay special attention to continuity conditions at the connection point of bottom frame and mast. These continuity conditions are:

- Between sections l_1 and l_2 because of the whole mass of mast the shear force suddenly changes. Let's denote the whole mass of mast by m_{sm}, thus the relation between shear forces at this point is expressed as: $V_1 = V'_1 - m_{sm}\alpha^2 Y_1$. This effect is taken into consideration by means of point matrix P_1.
- Because of connecting section l_4 the bending moment at the same point also changes. From the investigation of static equilibrium of this connection point the following expression can be determined between bending moments: $M_1 = M'_1 + M_3$, where M'_1 is the moment before connection point, M_1 is the moment beyond connection point and M_3 is the unknown moment at initial point of vertical section.
- Because of the whole mass of bottom frame at the initial point of vertical section l_4 the shear force suddenly changes. Let's denote the whole mass of bottom frame by m_{sb}, thus the relation according to shear force at this point is expressed as: $V_3 = -m_{sb}\alpha^2 X^3$. This effect is taken into consideration by means of point matrix P_3.

Tab. 2. Eigenfrequencies of cantilever beam model

Eigenfrequencies:
α_1 = 1,920 *rad/s*
α_2 = 13,13 *rad/s*
α_3 = 38,09 *rad/s*

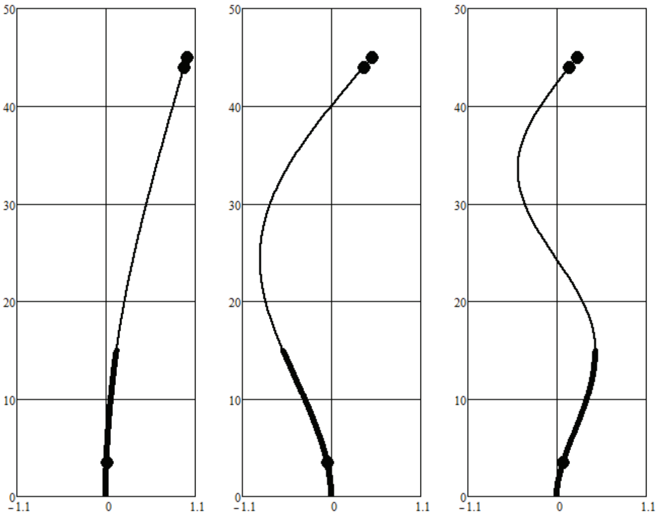

Fig. 6. First three mode shapes of cantilever beam model

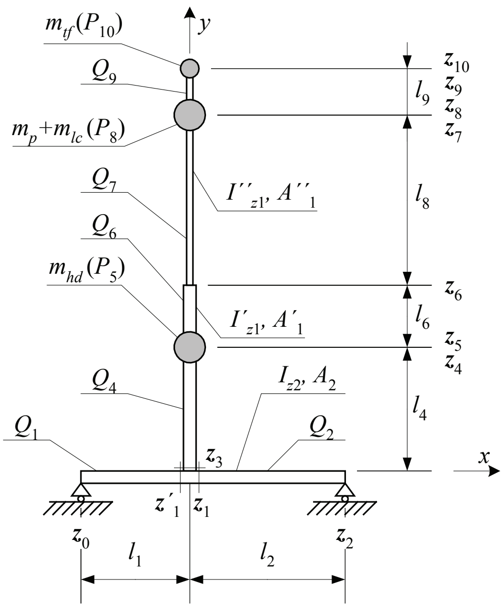

Fig. 7. Distributed parameter model of single-mast stacker cranes

- Because of the rigid connection point the relation between rotation angles here is expressed as: $\phi_3 = -\phi_3$.

In this case at the initial point of first section and at the branching point we have four unknowns. These unknowns are written by means of suitable unit vectors:

$$\phi_0 = \phi_0 b_0 = \phi_0 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}; V_0 = V_0 d_0 = V_0 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix};$$
$$X_3 = X_3 f_3 = X_3 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}; M_3 = M_3 e_3 = M_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}. \tag{29}$$

The calculation scheme with boundary conditions for calculating eigenfrequencies is shown in Fig. 8.

The boundary conditions from Fig. 8. and the frequency equation are:

$$\begin{aligned} Y_2 &= \phi_0 b_{21} + V_0 d_{21} + M_3 e_{21} = 0 \\ M_2 &= \phi_0 b_{23} + V_0 d_{23} + M_3 e_{23} = 0 \\ M_{10} &= \phi_0 b_{103} + V_0 d_{103} + M_3 e_{103} + X_3 f_{103} = 0 \\ V_{10} &= \phi_0 b_{104} + V_0 d_{104} + M_3 e_{104} + X_3 f_{104} = 0 \end{aligned} \tag{30}$$

$$\Delta(\alpha) = \begin{vmatrix} b_{21} & d_{21} & e_{21} & 0 \\ b_{23} & d_{23} & e_{23} & 0 \\ b_{103} & d_{103} & e_{103} & f_{103} \\ b_{104} & d_{104} & e_{104} & f_{104} \end{vmatrix} = 0 \tag{31}$$

The first three eigenfrequencies in case of our data set are shown in Table 3.

The unknown constants of mode shape functions can be determined by the help of boundary and continuity conditions in the same way than in case of our previous model. The first three mode shapes are shown in Fig. 9.

As can be seen in Fig. 9. unlike our previous model this model is free i.e. it has capability of rigid body motion. Thus investigation of excited vibrations can be performed in two ways. On the one hand we can prescribe the horizontal motion law of initial point of mast:

$$x(y,t)\big|_{y=0} = x(0,t) = r_0 \sin(\omega t). \tag{32}$$

This kind of excitation is known as displacement excitation. On the other hand we can also prescribe the time function of force acting on the lowest point of mast:

$$F_g = F_0 \sin(\omega t). \tag{33}$$

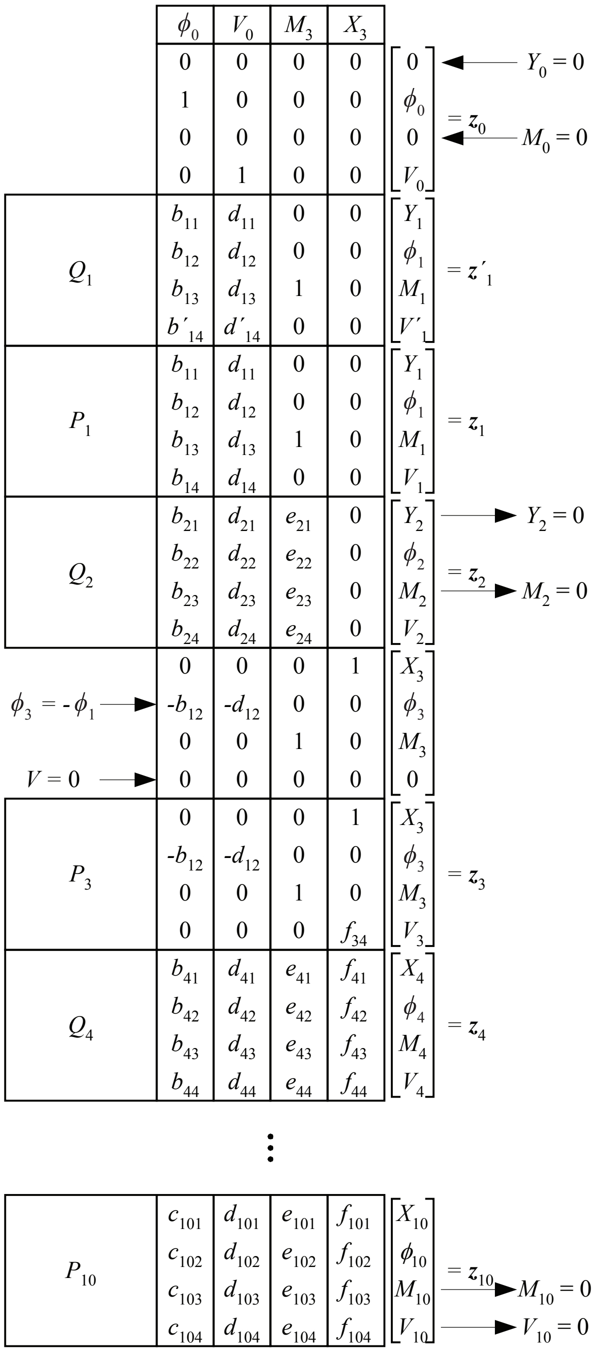

Fig. 8. Eigenfrequency calculation scheme for whole stacker crane model

Tab. 3. Eigenfrequencies of whole stacker crane model

Eigenfrequencies:
α_1 = 2,525 *rad/s*
α_2 = 14,84 *rad/s*
α_3 = 40,09 *rad/s*

This is the so called force excitation.

In both cases of excitation the unknown constants of mode shape functions can be determined by means of boundary and continuity conditions in the same way than in case of eigenfrequency calculations. However, in the systems of equations for boundary and continuity conditions in both cases we have to replace one equation with the following formulas. In case of displacement excitation we have to change the equation according to horizontal position of mast lowest point to the following expression:

$$X_4(0) = C_4 \cdot S(0) = r_0 \Rightarrow C_4 = r_0. \tag{34}$$

In case of force excitation we have to change the equation according to shear force of mast lowest point to the following expression:

$$\begin{gathered} V_4(0) + F_0 = -m_{sb}\alpha^2 X_4(0) \\ \Downarrow \\ -m_{sb}\alpha^2 C_4 + I'_{z1} E \frac{\lambda_4^3}{l_4^3} F_4 = F_0. \end{gathered} \tag{35}$$

Solving the resulted inhomogeneous systems of equations (with substitution of arbitrary ω angular frequency constants of mode shape functions can be calculated. calculations are performed with substitution $r_0 = 1$ or $F_0 = 1$ then the resulted magnitude of deflection at arbitrary point of structure equals to the magnitude of frequen function according to the same point. The Bode-diagrams of these frequency response functions according to mast tip are shown in the following figures.

Fig. 9. First three mode shapes of whole stacker crane model

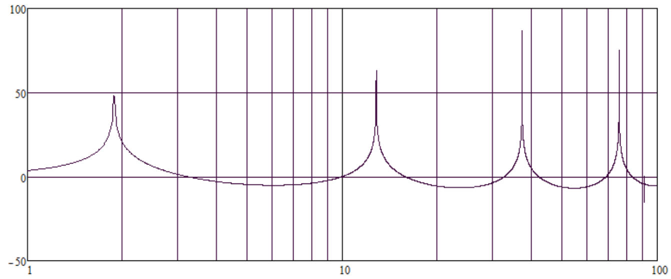

Fig. 10. Bode-diagram (displacement excitation)

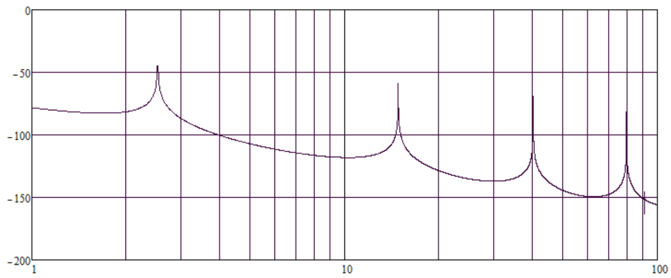

Fig. 11. Bode-diagram (force excitation)

5 Summary

In our paper we introduced a modeling technique based on distributed modeling approach. Three models according to Euler-Bernoulli beam theory are investigated. The first model is a cantilever beam model with uniform material and cross-sectional properties i.e. prismatic beam. The second model is cantilever beam model with variable cross-sectional properties and lumped masses. The eigenfrequencies and mode shapes of this mast-model are determined by means transfer matrix method. In the third model the whole structure of single-mast stacker crane is modeled. Beside the eigenfrequencies and mode shapes of this model the Bode-diagrams of frequency response function is also calculated with the third model. The result of modeling presented in this paper can be useful to verify the accuracy of other simpler models e.g. multi-body models with few degrees of freedom.

References

1 **Bachmayer M., Thümmel T., Ulbrich H., Schipplick M., Kessler S., Günthner W. A.,** *Nachschwingungsfreie Positionierung elastischer Roboter durch numerische und analytische Trajektorienplanung am Beispiel Regalbediengerät.* in 'Elektrisch-mechanische Antriebssysteme : Innovation - Trends - Mechatronik : Vorträge der 3. VDE/VDI-Tagung vom 23. bis 24. September 2008 in Böblingen' VDE-Verl., Berlin, 199-206 (2008).

2 **Bachmayer M., Zander R., Ulbrich H.,** *Numerical approaches for residual vibration free positioning of elastic robots*. Materialwissenschaft und Werkstofftechnik, 40(3), 161-168 (2009). DOI: 10.1002/mawe.200900422

3 **Bopp W.,** *Untersuchung der statischen und dynamischen Positionsgenauigkeit von Einmast-Regalbediengeräten*. Dissertation Institut für Fördertechnik Karlsruhe (1993).

4 **Dietzel M.**, *Beeinflussung des Schwingungsverhaltens von Regalbediengeräten durch Regelung des Fahrantriebs*. Dissertation Institut für Fördertechnik Karlsruhe (1999).

5 **Görges D., Kroneis J., Liu S.**, *Active vibration control of storage and retrieval machines, 2008*. in 'Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, DETC 2008.', 2 (Issue PART B), 1037-1046 (2009).
DOI: 10.1115/DETC2008-49756

6 **Oser J., Kartnig G.**, *Das Schwingungsverhalten eines zugmittelgeführten Leichtregalbediengerätes*. Deutsche Hebe- und Fördertechnik, 4, 66-73 (1994).

7 **Reisinger K. H.**, *Schwingungssimulation von Regalförderzeugen*. Dissertation Technische Universität Graz (1998).

8 **Staudecker M., Schlacher K., Hansl R.**, *Passivity based control and time optimal trajectory planning of a single mast stacker crane*. in 'Proceedings of the 17th World Congress IFAC, Seoul, Korea' 17(1), 875-880 (2008).
DOI: 10.3182/20080706-5-KR-1001.00150

9 **Bashash S., Salehi-Khojin A., Jalili N.**, *Forced Vibration Analysis of Flexible Euler-Bernoulli Beams with Geometrical Discontinuities*. in 'Proceedings of the 2008 American Control Conference : The Westin Seattle, Seattle, WA, USA, June 11-13, 2008, 4029-4034 (2008).
DOI: 10.1109/ACC.2008.4587123

10 **Aleyaasin M., Ebrahimi M., Whalley R.**, *Flexural vibration of rotating shafts by frequency domain hybrid modelling*. Computers and Structures, 79(3), 319-331 (2001).
DOI: 10.1016/S0045-7949(00)00133-4

11 **Zollner V., Zobory I.**, *On dynamics of the track/vehicle system in presence of inhomogeneous rail supporting parameters*. Periodica Polytechnica Transportation Engineering, 39 (2), 83-85 (2011a).
DOI: 10.3311/pp.tr.2011-2.06

12 **Zollner V., Zobory I.**, *Railway track dynamics with periodically varying stiffness and damping in the Winkler foundation*. Periodica Polytechnica Transportation Engineering, 39 (2), 87-90 (2011b).
DOI: 10.3311/pp.tr.2011-2.07

13 **Ludvig Gy.**, *Gépek dinamikája*. Műszaki könyvkiadó, Budapest (1983).