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RESEARCH ARTICLE

Prediction of Road Traffic Accidents Using a Combined Model Based on IOWGA Operator

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Abstract

Traffic accident prediction plays a important role in reducing the likelihood of traffic accidents and improving the management levels of traffic safety. A new combined prediction model based on the induced ordered weighted geometric average (IOWGA) operator was proposed. This new model combines the GM(1,1) model and the Verhulst model with changeable weight coefficients of each single model. A combined model based on the optimal weighted(OW) method is also presented for comparison. An example is given with the number of deaths by road traffic accidents in China from 2003 to 2008. The results indicate that the proposed combined model is better than the other three models.

Keywords

Combination model, accident prediction, Verhulst model, GM (1,1), model, IOWGA operator

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1 Introduction

With the rapid development of urbanization and motorization across China, traffic jams have become the most important feature in modern urban transportation systems. It induces travel delay, air pollution, traffic accidents and the other social problems. According to the Ministry of Public Security Traffic Management Bureau of China, more than one hundred thousand people have died every year since 2001 because of road accidents in China. Until 2004, China experienced a decrease in deaths number from road accidents, which is largely attributed to improvements in traffic safety management.

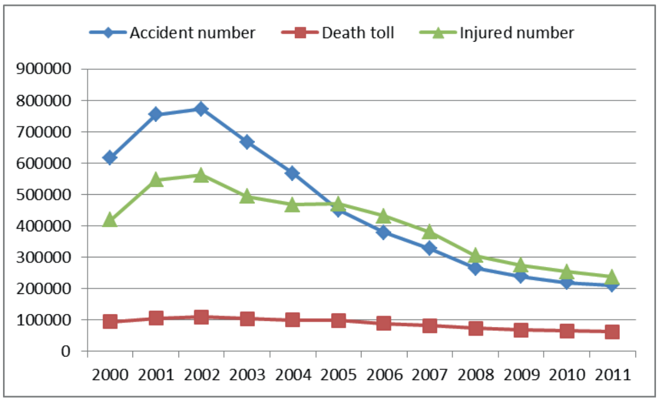

Fig. 1 Statistical data of road accidents in China from 2000-2001
(Note: Data in this figure is obtained from Ministry of Public Security Traffic Management Bureau of China).

There are tremendous losses, both economically and socially, associated with traffic accidents, which has led researchers to seek effective measures that aim to reduce accidents (Polat and Durduran, 2012; Sokolovskij and Prentkovskis, 2013; Çelik, 2014; Török 2015). Accident analyses and prediction are the most important issues in terms of traffic safety management. Accordingly, effective accident prediction would greatly contribute to reasonable road networks planning and the improvement management of road safety management (Beke et al., 2014).

Many approaches have been developed to predict road accidents. Regression models and time series analysis techniques are widely used, as suggested by many previous studies. For example, Lord, D. et al. (2008) presented a model to describe motor vehicle crashes with a generalized Conway-Maxwell-Poisson linear model. The results of this study indicated that the proposed method is better than the traditional negative binomial model. Ramirez, B. A. et al. (2009) applied negative binomial models to analyze the influence of traffic volume and density on road accidents in Spain. Commandeur, J. J. F. et al. (2013) used time series analysis techniques to describe the development of accidents into the near future. This study also presented practical guidelines for further application of the time series forecasting model. Zhang Jie et al. (2007) analyzed the fatality rate of traffic accidents in Beijing using the ARIMA model. The results of this study indicated that the ARIMA model is suitable for seasonal and non-seasonal time series alike.

Due to a shortage in comprehensive statistics regarding the number of accidents, grey models have been commonly used in practice to predict road accidents. Wang Fu-jian, et al. (2006) described the properties of GM (1, 1) model as well as the Verhulst model, which are both used for predicting the number of deaths as a result of road accidents in China. Twala Bhekisipho (2013) developed a grey relational system used to predict traffic accidents from incomplete data. The results from this study detail the efficiency and robustness of the grey relational analysis method.

In recent years, soft computing approaches such as Fuzzy Logic, Artificial Neural Network (ANN), Particle Swarm Optimization, and their hybrid models have been commonly used to predict road accidents (Kalyoncuoglu and Tigdemir, 2004; Delen et al., 2006). A prediction model of highway tunnel traffic accidents in China based on a BP neural network is presented by Zhao, Jian-you, et al. (2010). Chiou, Y C (2006) developed an artificial neural network-based expert system to appraise the influential variables involved in two-car crash accidents. Wang Hao, et al. (2011) proposed a traffic accidents prediction model based on fuzzy logic and the test results reveal that predictions of traffic accidents by fuzzy logic is a viable method. Akgungor and Dogan (2009) presented a Genetic Algorithm and an ANN model to forecast the number of traffic accidents in Ankara, Turkey. The results of this study show that the performance of the ANN model outperformed the GA model. Besides the above-mentioned models, there are still others that are also presented by researchers to forecast traffic accidents (de Ona et al. 2011; Wang et al. 2011; El-Basyouny and Sayed, 2010).

Some researchers presented hybrid models involving a combination of the ANN and fuzzy techniques (Polat, and Durduran, 2012; Hosseinpour et al., 2013). Still other hybrid models of traffic accidents were also developed. Ren Gang and Zhou Zhuping (2011) proposed a novel approach to evaluate the development tendencies of traffic accidents. This study combined the support vector machine and particle swarm optimization (PSO-SVM), the results of which indicate that traffic accident prediction using a PSO-SVM model is better than using a BP neural network.

Although researchers have developed a large number of forecasting models for traffic accidents, each single forecasting model has its own limitations and applied usage. Thus, an increasing number of combination models have been proposed for predicting traffic accidents, all of which deal with the prediction results of several single models, by distributing the weight average coefficients of each single model. Therefore, the combination forecasting methods effectively improve the accuracy of the results. Since Bates and Granger (1969) developed combination forecasting models in 1969, combination forecasting models have been widely applied in the field. Zheng Jianhu (2009) proposed a combination prediction model for road traffic accidents based on an optimal weighted method, which combined the results of the GM(1,1) model as well as the Verhulst model. The results indicated that the forecasting accuracy of the combination model is better than that of the GM(1,1) model and the Verhulst model. Zhou Deqiang (2010) discussed a combination model that took the Verhulst model and the support vector machine into account, thus showing the superiority of the combination model. For most of the existing combination models, the weighted average coefficients of the single method remained unchanged at different points. However, the prediction error of each single method will different at different point, so unchanged weighted average coefficients is inconsistent with the real condition. Jiang, L-H. and Chen, H-Y. (2010) proposed a combined predictive model based on induced ordered weighted geometric average (IOWGA) operator, which gives the changed weighted average coefficients of each single model according to the prediction accuracy at different point, thus a new kind model for combination forecasting was proposed.

In order to effectively utilize the information that was provided at different points by each single forecasting model, a model that combined the GM (1,1) and the Verhulst model was constructed based on ad IOWGA operator, which aims to improve the forecasting accuracy of traffic accidents. For the sake of comparison, a combined forecasting model based on the optimal weighted (OW) method was given. An example was illustrated to test the performance of the proposed model, using the number of deaths caused by traffic accidents in China from 2003 to 2008 as the original data. The remainder of the paper is organized as follows. The next section describes two single forecasting models: the GM (1,1) and the Verhulst model. A combined forecasting model based on the optimal weighted (OW) method is discussed in section 3. A proposed combination forecasting model based on an IOWGA is presented in section 4. Section 5 provides a case study, followed by the conclusion in the final section.

2 Grey forecasting model

Grey prediction has been widely used for solving the problems under discrete data and incomplete information (Li et al., 2005). The grey dynamic model GM (1,1) has the advantages of developing a model with a limited amount of data, which uses generating approaches to reduce the variation within the original data series. Among the generating approaches, the accumulative generation operation (AGO) is the most commonly used. The AGO could reduce the randomness of the data and enhance the regularity of the data series (Ren and Zhou, 2011)

2.1 GM (1,1) model

Assume that $X^{(0)}=\left\{x_1^{(0)},x_2^{(0)},\cdots,x_n^{(0)}\right\}$ is the original data sequence of the given traffic accidents. Where n is the size of the data sequence, and the data sequence $X^{(0)}$ is subjected to AGO, the following 1-AGO sequence is obtained.

$$X^{(1)}=\left\{x_1^{(1)},x_2^{(1)},\cdots x_n^{(1)}\right\} \tag{1}$$

Where $x_1^{(1)}=x_1^{(0)}$, $x_k^{(1)}=\sum_{j=1}^{k}x_j^{(0)}$ and k = 2, 3, ..., n.

It is evident that the origin data $x_i^{(0)}$ can be obtained from:

$$x_i^{(0)}=x_i^{(1)}-x_{i-1}^{(1)} \tag{2}$$

Where $x_1^{(0)}=x_1^{(1)}$ and $x_i^{(1)}\in x_1^{(1)}$. The process of (2) is called the inverse AGO.

The GM (1,1) model is formulated by establishing a first-order differential equation for $X^{(1)}$ as is seen below:

$$\frac{dX^{(1)}}{dt}+\alpha X^{(1)}=u \tag{3}$$

Where α and μ are the undetermined coefficients which can be obtained by using the least square method as seen below (Ren and Zhou, 2011):

$$\hat{\alpha}=\left[\alpha \quad u\right]^T=\left(B^TB\right)^{-1}\left(B^Ty_n\right) \tag{4}$$

In which: $y_n=\left[x_2^0,x_3^0,\cdots,x_n^0\right]^T$

and $B=\begin{bmatrix}-\left(x_2^{(1)}+x_1^{(1)}\right)/2 & 1\\ \vdots & \vdots\\ -\left(x_n^{(1)}+x_{n-1}^{(1)}\right)/2 & 1\end{bmatrix}$

And then, coefficients α and μ are substituted into (3), the solution of $\hat{x}_{k+1}^{(1)}$ is

$$\begin{cases}\hat{x}_{k+1}^{(1)}=\left(x_1^{(1)}-u/\alpha\right)e^{-\alpha k}+u/\alpha,\left(k=2,3,\cdots n\right)\\ \hat{x}_1^{(1)}=x_1^{(0)}\end{cases} \tag{5}$$

Using the inverse AGO for (5), the predicted value of the original dada at moment k is obtained as follow:

$$\hat{x}_k^{(0)}=\left(x_1^{(0)}-u/\alpha\right)\left(1-e^{-\alpha}\right)e^{-\alpha(k-1)},\left(k=2,3,\cdots n\right) \tag{6}$$

The characteristics of the GM (1,1) model are represented by discrete data, and the minimum number of the original data is $n \geq 4$.

2.2 Verhulst model

The Verhulst model was proposed by the Germany biologist Verhulst, and can be established by using a first order differential equation. The grey Verhulst model is describes as follows (Kayacan et al., 2010; Ming et al., 2013):

Assume that $X^{(0)}=\left\{x_1^{(0)},x_2^{(0)},\cdots,x_n^{(0)}\right\}$ is the original data sequence of traffic accidents, where n is size of the data sequence. When the data sequence $X^{(0)}$ is subjected to AGO, the following generation sequence is obtained,

$$X^{(1)}=\left(x_1^1,x_2^1,\cdots,x_n^1\right) \tag{7}$$

Where $x_k^{(1)}=x_k^{(0)}-x_{k-1}^{(0)}$ and k = 1, 2, ..., n.

Then, the average value of the adjacent number for value $X^{(0)}$ is calculated, from which the following generation sequence is obtained,

$$Z^{(1)}=\left(z_2^{(1)},z_3^{(1)},z_4^{(1)},\cdots,z_n^{(1)}\right) \tag{8}$$

Where $z_k^{(1)}=\left(x_k^{(0)}+x_{k-1}^{(0)}\right)/2$ and k = 2,3, ⋯ n.

Accordingly, we name the following equation the Verhulst model:

$$X^{(0)}+aZ^{(1)}=b\left(Z^{(1)}\right)^2 \tag{9}$$

Where a, b are undetermined coefficients. The whitening equation of Verhulst model can be written as:

$$\frac{dx^{(0)}}{dt}+ax^{(0)}=b\left(x^{(0)}\right)^2 \tag{10}$$

Similar to GM(1,1), the coefficients α and b can be obtained by the least-squares method as follows:

$$\hat{a}=\left[a \quad b\right]^T=\left(B^TB\right)^{-1}B^TY \tag{11}$$

In the above equation,

$$Y=\left[x_2^{(1)} \quad x_3^{(1)} \quad \cdots \quad x_n^{(1)}\right]^T$$

and $B=\begin{bmatrix}-z_2^{(1)} & \left(z_2^{(1)}\right)^2\\ -z_3^{(1)} & \left(z_3^{(1)}\right)^2\\ \vdots & \vdots\\ -z_n^{(1)} & \left(z_n^{(1)}\right)^2\end{bmatrix}$.

The solution of Eq. (11) at the moment t can be expressed as:

$$x^{(1)}\left(t\right)=\frac{ax_1^{(0)}}{bx_1^{(0)}+\left(a-bx_1^{(0)}\right)e^{at}} \tag{12}$$

Through applying the inverse AGO for (12), the grey prediction model is obtained, which is expressed as:

$$\hat{x}_{k+1}^{(0)}=\frac{ax_1^{(0)}}{bx_1^{(0)}+\left(a-bx_1^{(0)}\right)e^{ak}},k=0,1,\cdots,n-1 \tag{13}$$

The Verhulst model requires sample data that is no less than 4.

3 Combined model prediction based on the OW method

The combined model based on the optimal weight (OW) method involves a linear combination method, which is simpler than nonlinear methods. Not surprisingly, then, this method has been widely applied in the real world. The combination is constructed by minimizing the mean square error of each single forecasting method. Thus, the process of the combination model based on OW can be described as follows (Li et al., 2012):

Let $X^{(0)} = \{x_1^{(0)}, x_2^{(0)}, \cdots, x_n^{(0)}\}$ be the original data of traffic accidents. $\hat{Y}_i (i = 1,2,\cdots,m)$ denotes the forecasted value of every single model, and $W = [w_1, w_2, \cdots, w_m]^T$ is the weight coefficients of the value m from the single model. Finally, $\sum_{i=1}^{m} w_i = 1$. The combined forecasting model can then be described as:

$$Y = w_1 \hat{y}_1 + w_2 \hat{y}_2 + \cdots + w_m \hat{y}_m = \sum_{i=1}^{m} w_i \hat{y}_i \tag{14}$$

Where the assumed prediction error sequence of the single method is:

$$e_{it} = y_{it} - \hat{y}_{it} \qquad (i = 1,2,\cdots,m; t = 1,2,\cdots,n) \tag{15}$$

Then the prediction error matrix of the single method can be written as:

$$E = \begin{bmatrix} \sum_{t=1}^{n} e_{1t}^2 & \sum_{t=1}^{n} e_{1t}e_{2t} & \cdots & \sum_{t=1}^{n} e_{1t}e_{mt} \\ \sum_{t=1}^{n} e_{2t}e_{1t} & \sum_{t=1}^{n} e_{2t}^2 & \cdots & \sum_{t=1}^{n} e_{2t}e_{mt} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{t=1}^{n} e_{mt}e_{1t} & \sum_{t=1}^{n} e_{mt}e_{2t} & \vdots & \sum_{t=1}^{n} e_{mt}^2 \end{bmatrix}. \tag{16}$$

The optimal weight of the combined model can be obtained by solving the following mathematical programming problem:

$$\begin{cases} \min Q = \sum_{t=1}^{n} e_t^2 \\ s.t. \quad \sum_{i=1}^{m} w_i = 1 \end{cases} \tag{17}$$

Let $R = [1,1,\cdots,1]^T$ we have

$$\begin{cases} \min Q = \sum_{t=1}^{n} e_t^2 = W^T E W \\ s.t. \quad \sum_{i=1}^{m} R^T W = 1 \end{cases} \tag{18}$$

Through using the Lagrange Multiplier method to solve the problem model in (18), the following equation is obtained:

$$W = \frac{E^{-1}R}{R^T E^{-1} R} \tag{19}$$

The optimal objective function is written as:

$$\min Q = \frac{1}{R^T E^{-1} R} \tag{20}$$

The combined forecasting model based on the OW has no special requirement for the number of original data, and its prediction accuracy depends on the accuracy of each single method.

4 Combination prediction model based on IOWGA operator

Yager R. R. (1998) proposed the use of an ordered weighted averaging operator (OWA) to aggregate the information based on OWA. A few years later, Xu Z. S. et al. (2002) proposed further the use of an induced ordered weighted averaging (IOWGA) operator, which provides the changeable weight of every single method at different points. Because it effectively utilizes the information of every method, the IOWGA operator has been widely applied in the field of decision making (Liu, 2011). There are only a few research findings available regarding the combined prediction-model-based IOWGA operators. However, one such example is Jiang, L. and Chen, H. (2010), who presented a combined forecasting model based on an IOWGA operator. The results from this experiment show the superiority of this combination method. The combinated forecasting model based on IOWGA operator is described as below.

Let $X^{(0)} = \left(x_1^{(0)}, x_2^{(0)}, \cdots\cdots, x_n^{(0)}\right)$ be the actual number of death by traffic accidents, m the single forecasting methods, x_{it} the forecasting value at the moment t of the i forecasting method, and $i = 1, 2, ..., m$. Assuming that a_{it} is the predictive accuracy of value i, the forecasting method at t moment, we have:

$$a_{it} = \begin{cases} 1 - \left|\left(x_t - x_{it}\right)/x_t\right|, & \left|\left(x_t - x_{it}\right)/x_t\right| < 1 \\ 0, & \left|\left(x_t - x_{it}\right)/x_t\right| \geq 1 \end{cases} \tag{21}$$

Where $i = 1, 2, \cdots, m$, and $t = 1, 2, \cdots, n$.

Taking a_{it} as the induced values of forecasting the value x_{it}, a two dimension array is constructed as below:

$$\langle a_{1t}, x_{1t} \rangle, \langle a_{2t}, x_{2t} \rangle, \cdots, \langle a_{mt}, x_{mt} \rangle$$

This array ranks the forecasting accuracy series of each single method in decreasing order, and ρ_{it} is the i^{th} bigger accuracy. The following equation is the combined predicted value produced by the induced ordered weighted geometric averaging.

$$\begin{aligned} IOWGA &= \left(\langle a_{1t}, x_{1t} \rangle, \langle a_{2t}, x_{2t} \rangle, \cdots, \langle a_{mt}, x_{mt} \rangle\right) \\ &= \prod_{i=1}^{m} x_{a-index(it)}^{l_i} \end{aligned} \tag{22}$$

Let $e_{a-index(it)} = \ln x_i - \ln x_{a-index(it)}$, and we have:

$$S=\sum_{i=1}^{n}\left(\ln x_i-\prod_{i=1}^{m}x_{a-index(it)}^{l_i}\right)^2$$
$$=\sum_{i=1}^{n}\left(\ln x_i-\sum_{i=1}^{m}l_i\ln x_{a-index(it)}\right)^2 \quad (23)$$
$$=\sum_{i=1}^{m}\sum_{j=1}^{m}l_il_j\left(\sum_{t=1}^{n}e_{a-index(it)}e_{a-index(jt)}\right)$$

Therefore, the combined prediction model based on the IOWGA can be expressed by the following optimal model:

$$\min D(L)=\sum_{i=1}^{m}\sum_{j=1}^{m}l_il_j\left(\sum_{t=1}^{n}e_{a-index(it)}e_{a-index(jt)}\right)$$
$$s.t.\begin{cases}\sum_{i=1}^{m}l_i=1\\ l_i\ge 0, i=1,2,\cdots m\end{cases} \quad (24)$$

The above model takes the mean log square error as the optimal criteria, which generates a larger weight coefficient l_i for a small error in the single model at different points.

5 Example and results

To illustrate the effectiveness of the proposed combination forecasting model based on the IOWGA operator, an experiment was conducted to predict the number of death caused by road traffic accidents from 2003-2008 in China. Table 1, below, presents the original data.

Table 1 Death number of road traffic accidents from 2003-2008 in China

Year	Deaths number
2004	99977
2005	98938
2006	89455
2007	81649
2008	73484
2009	67759
2010	65225
2011	62387

5.1 Construction of two single models

According to the above model, the construction process of a single forecasting method, GM (1,1) model and Verhulst model were established first. The two single forecasting models were obtained respectively as shown below:

GM (1, 1) model: $\hat{x}_k^{(0)}=-102980e^{-0.0779(k\text{-}1)}$

Verhulst model : $\hat{x}_{k+1}^{(0)}=\dfrac{39199}{0.3472+0.0285e^{0.3757k}}$

5.2 Construction of the combined model based on the OW method

Let $\hat{y}_1$ be the prediction value of the GM (1,1) model and $\hat{y}_2$ be the prediction value of the Verhulst model. By applying the above combination model based on the OW method, a fitting error matrix was achieved according to (16):

$$E=\begin{bmatrix}26763983 & 13767108\\ 13767108 & 9492015\end{bmatrix}$$

By appling (19), then we get:

$$W=\begin{bmatrix}0.365\\ 0.635\end{bmatrix}$$

Therefore, the combination forecasting model based on the OW method was suggested as below:

$$Y=0.365\hat{y}_1+0.635\hat{y}_2$$

5.3 Combined model construction based on the IOWGA operator

The predicted accuracy, a_{it}, was calculated first according to (21). Accordingly, the results regarding the predicted accuracy for the GM (1,1) model as well as the Verhulst model can be seen in Table 2.

Through ranking the predicted accuracy in decreasing order, we achieved:

$$\hat{x}_1=IOWGA(\langle a_{11},x_{11}\rangle,\langle a_{21},,x_{21}\rangle)$$
$$=104327l_1104327l_2$$

$$\hat{x}_2=IOWGA(\langle a_{12},x_{12}\rangle,\langle a_{22},,x_{22}\rangle)$$
$$=100840l_l102980l_2$$

$$\hat{x}_3=IOWGA(\langle a_{13},x_{33}\rangle,\langle a_{23},,x_{23}\rangle)$$
$$=96167l_195236l_2$$

$$\hat{x}_4=IOWGA(\langle a_{14},x_{14}\rangle,\langle a_{24},,x_{24}\rangle)$$
$$=90077l_188123l_2$$

$$\hat{x}_5=IOWGA(\langle a_{15},x_{15}\rangle,\langle a_{25},,x_{25}\rangle)$$
$$=81519l_182474l_2$$

$$\hat{x}_6=IOWGA(\langle a_{16},x_{16}\rangle,\langle a_{26},,x_{26}\rangle)$$
$$=73448l_175409l_2$$

The following optimal equation is constructed from (24).

$$\min D(L)=0.0032l_1^2+0.0024l_1l_2+1.0302l_2^2$$
$$s.t.\begin{cases}l_1+l_2=1\\ l_1>0, l_2>0\end{cases}$$

Solving the above equation with the optimal tool of MATLAB, we achieved $l_1 = 0{,}998$ and $l_2 = 0{,}002$.

Table 2 Results of predicted accuracy of the GM (1,1) model and the Verhulst model

Year	Actual value	Predicted value		Predicted accuracy	
		GM(1,1)	Verhulst	GM(1,1)	Verhulst
2003	104327	104327	104327	1.0000	1.0000
2004	99977	102980	100840	0.9700	0.9914
2005	98938	95263	96167	0.9629	0.9720
2006	89455	88123	90077	0.9851	0.9930
2007	81649	81519	82474	0.9984	0.9899
2008	73484	73448	75409	0.9738	0.9995
2009	67759				
2010	65225				
2011	62387				

5.4 Analysis of the predicted results

The predicted results of four models proposed by this paper are show in Table 2 and Fig. 1. Figure 1 shows that the combined model based on the IOWGA fits the actual curve best, and the Verhulst model is second.

Table 3 Results of predicted value of four proposed forecasting model.

Year	Actual value	Predicted value			
		GM(1,1) model	Verhulst Model	OW model	IOWGA model
2003	104327	104327	104327	104327	104327
2004	99977	102980	100840	101621	100844
2005	98938	95263	96167	95837	96165
2006	89455	88123	90077	89364	90073
2007	81649	81519	82474	82125	81521
2008	73484	75409	73448	74164	73452

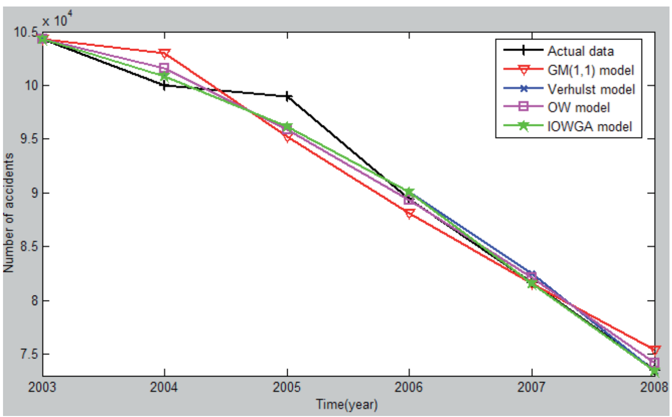

Fig. 2 The curve of the predicted value and actual data of deaths number from 2003-2008 in China

Five statistical error indices were used to test the performance of the proposed combined model based on the IOWGA operator compared to the other models. These indices include the mean square error (MSE), the mean absolute error (MAE), the root mean square error (RMSE) and the mean relative error (MRE).

The formulas for these error indices are defined respectively as follows:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2 \tag{25}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \hat{y}_i\right| \tag{26}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2} \tag{27}$$

$$MRE(\%) = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \tag{28}$$

Where $\hat{y}_i$ is the predicted value, y_i is the real value, and n is the number of samples. Table 4 summarizes the results regarding the error indices of the proposed models, according to the formulas above. From Table 4, the MSE, MAE, RMSE and MRE of the combined model based on the IOWGA operator reach up to 1473425, 736, 1214 and 0.76%, respectively. The table clearly indicates that the performance of the proposed combined model based on the IOWGA operator is better than that of the other three models in the study.

Table 4 Comparison of the error index of four proposed forecasting model.

	MSE	MAE	RMSE	MRE (%)
GM(1,1) model	1336730	1678	2161	1.83
Verhulst model	1582002	853	1258	0.91
OW model	2169365	999	1473	1.06
IOWGA model	1473425	736	1214	0.76

6 Conclusions

A large number of models have been developed to predict road traffic accidents, and every model has its advantages in certain application situations. Thus, the proposed combination model is a promising method for predicting traffic accidents in the future. Some of the conclusions can be sum up as follow:

1) Grey prediction is suitable for problems with small samples, such as the examples used in this paper. Only six traffic accidents were used to construction the prediction model, yet the predicted value still fit the actual number well, which can be seen on Fig. 1.

2) The combined model based on the IOWGA operator proposed in this paper, which gives the changeable weight coefficients of each single model according to the prediction accuracy at different points. Accordingly, the model is consistent with real world accident tendencies. As a result, the performance of the combined model based on the IOWGA operator is better than that of many others.

There are, however, some limitations to the proposed combined model, which requires further research. On the one hand, how to best select suitable single methods with respect to the real application problem is not addressed here. On the other hand, only the construction process of the combined model based on the IOWGA operator effectively utilize the information at different points provided by each single forecasting model. Once the weight coefficient of a single model is determined, it will never change in the future prediction, which limits its utility.

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