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RESEARCH ARTICLE

Identification of Supply Chains Based on Input-Output Data

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Abstract
The paper focuses on supply chain modeling issues, namely how subspace identification techniques can be used to characterize the strength of relations between certain system parameters. This might be useful when no knowledge about the internal workings or inner structure of the system is available, thus only blackbox like approaches can be utilized. Here let us show how supply chains can be identified and modeled by deterministic linear state space models and how the accuracy of the identified model reflects the relation between certain system parameters.

Keywords
Subspace identification, supply chain, loading system, modeling

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1 Introduction

In logistics the improvement of material and information flow plays significant role, thus accurate models of the system are needed in order to predict its behaviour and thus improve its performance. Supply chains encompass the full range of intra-company and inter-company activities beginning with raw material procurement, through manufacturing and distribution (Viswanadham et al., 2001)

The identified models of logistical processes (LP) may be helpful to predict various features related to the modeled system. A framework to promote the better understanding of supply chain performance measurement and metrics can be followed for example in (Gunasekaran et al., 2004).

Depending on the knowledge about the modeled system a broad range of solutions can be utilized. Since complex logistical systems are non-linear MIMO systems and are influenced by many parameters their modeling is not a trivial task. Many methods have been proposed to deal with MIMO systems in the literature. Perhaps the most popular tool in this topic is the linear parameter varying (LPV) structure by which non-linear systems can be modeled and controlled on the basis of linear control theories (Baranyi et al., 2007; Szeidl et al., 2009; Péter et al., 2014).

If there is no knowledge about the inner structure of the system such as for instance the concrete service strategy and other internal mechanisms only black box like solutions (mainly heuristic approaches) might be utilized. In this case the system might be identified based on measured input-output data. In the literature many models (as for instance scheduling, transportation planning, flow-shop sequencing problem) of logistic systems are based on the fuzzy set and fuzzy control theory, statistics or their combination (Harmati et al., 2007), (Orbán et al., 2009), (Jing-Shing et al., 2002) (Sevastjanov et al., 2003). It is difficult to find a proper mathematical model in form of differential equations which would suitable approximate the behaviour of the observed logistical process even if the identification of the system is considered locally. Subspace identification techniques combined with tensor product transformation seem to be promising to model complex logistical processes based on input-output data. In this case there is no need for an explicit model parameterization, which is a rather complicated matter for multi-output linear systems (Overschee et al., 2011).

In this paper let us focus on subspace identification techniques aimed to model supply chains.

The paper is organized as follows: In Section 2 the subspace identification for deterministic case is briefly described, Section 3 shows how supply chains can be modelled on subspace bases. Finally experimental results and conclusions are reported.

2 Overview on Subspace Identification of Linear Time Invariant Systems

Before turning the focus onto logistical processes, let us give a brief description on how subspace identification techniques can be used to identify linear time invariant (LTI) vertex models in the parameter space. Let us assume that the local behavior of the logistical system is deterministic, thus it can be described in the well-known state space form as follows:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \quad (1)$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k \quad (2)$$

where $\mathbf{x}_k \in \mathbb{R}^n$ stands for the state vector, $\mathbf{u}_k$ and $\mathbf{y}_k$ represent the input and output vector respectively at time k. The goal is to find the model matrices **A**, **B**, **C** and **D** based on input-output pairs. As described in (Overschee et al., 2011). let us first arrange the input-output pairs into so called Hankel matrices (reflecting the history of our input-output data):

$$\mathbf{U}_{1|i} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \dots & \mathbf{u}_j \\ \mathbf{u}_2 & \mathbf{u}_3 & \dots & \mathbf{u}_{j-1} \\ \vdots & \vdots & \dots & \vdots \\ \mathbf{u}_i & \mathbf{u}_{i+1} & \dots & \mathbf{u}_{j+i-1} \end{bmatrix} \quad (3)$$

$$\mathbf{Y}_{1|i} = \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \dots & \mathbf{y}_j \\ \mathbf{y}_2 & \mathbf{y}_3 & \dots & \mathbf{y}_{j-1} \\ \vdots & \vdots & \dots & \vdots \\ \mathbf{y}_i & \mathbf{y}_{i+1} & \dots & \mathbf{y}_{j+i-1} \end{bmatrix} \quad (4)$$

and let the history of states (unknown) to be estimated encode as follows:

$$\mathbf{X}_i = \begin{bmatrix} \mathbf{x}_i & \mathbf{x}_{i+1} & \dots & \mathbf{x}_{i+j-1} \end{bmatrix} \quad (5)$$

It can be recognized from (2) that all row vectors in $\mathbf{Y}_{l|i}$ are in the vector space determined by the union of row space of $\mathbf{X}_i$ and $\mathbf{U}_{l|i}$. Let us assume that the intersection of row space of $\mathbf{X}_i$ and $\mathbf{U}_{l|i}$ is empty. The most simple alternative for estimating $\mathbf{X}_i$ (up to a constant multiple C) is to project the row space of $\mathbf{Y}_i$ onto orthogonal complement of the row space of $\mathbf{U}_{l|i}$. The elements of $\mathbf{Y}_i$ can be expressed with the help of the extended observability matrix $\mathbf{\Gamma}_i$ and lower block triangular Toeplitz matrix $\mathbf{H}_i$ form as follows (Overschee et al., 2011):

$$\mathbf{Y}_{l|i} = \mathbf{\Gamma}_i \mathbf{X}_1 + \mathbf{H}_i \mathbf{U}_{l|i}, \quad (6)$$

where

$$\mathbf{\Gamma}_i = \begin{bmatrix} \mathbf{C}\ \mathbf{CA}\ \dots\ \mathbf{CA^{i-1}} \end{bmatrix}^\top \quad (7)$$

and

$$\mathbf{H_i} = \begin{bmatrix} \mathbf{D} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{CB} & \mathbf{D} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{CAB} & \mathbf{CB} & \mathbf{D} & \dots & \mathbf{0} \\ \mathbf{CA^{i-2}B} & \mathbf{CA^{i-3}B} & \dots & \mathbf{CB} & \mathbf{D} \end{bmatrix} \quad (8)$$

By substituting recursively into (1) we can express the state sequence $\mathbf{X}_{i+1}$ as follows:

$$\mathbf{X}_{i+1} = \mathbf{A}^i \mathbf{X}_1 + \Delta_i \mathbf{U}_{1|i}, \quad (9)$$

where

$$\Delta_i = \begin{bmatrix} \mathbf{A}^{i-1}\mathbf{B}\ \mathbf{A}^{i-2}\mathbf{B}\ \dots\ \mathbf{AB}\ \mathbf{B} \end{bmatrix}, \quad (10)$$

stands for the reversed extended controllability matrix (Overschee et al., 2011). From (6) the state sequence $\mathbf{X}_i$ can be expressed as:

$$\mathbf{X}_1 = \mathbf{\Gamma}_i^* \mathbf{Y}_{l|i} - \mathbf{\Gamma}_i^* \mathbf{H}_i \mathbf{U}_{l|i}. \quad (11)$$

By substituting (11) into (9) we obtain:

$$\mathbf{X}_{i+1} = \mathbf{A}^i \mathbf{\Gamma}_i^* \mathbf{Y}_{l|i} - \mathbf{A}^i \mathbf{\Gamma}_i^* \mathbf{H}_i \mathbf{U}_{l|i} + \Delta_i \mathbf{U}_{l|i}, \quad (12)$$

Let us express $\mathbf{X}_{i+1}$ as the sum of two matrices, where one of the matrices contains only the input-output values, i.e.

$$\mathbf{X}_{i+1} = \mathbf{L}_i \mathbf{W}_{1|i}, \quad (13)$$

where

$$\mathbf{L}_i = \begin{bmatrix} \Delta_i - \mathbf{A}^i \mathbf{\Gamma}^*{}_i \mathbf{H}_i & \mathbf{A}^i \mathbf{\Gamma}^*{}_i \end{bmatrix}, \quad (14)$$

and

$$\mathbf{W}_{1|i} = \begin{bmatrix} \mathbf{U}_{1|i} & \mathbf{Y}_{1|i} \end{bmatrix}^\top \quad (15)$$

Since based on (6)

$$\mathbf{Y}_{i+1|2i} = \mathbf{\Gamma}_i \mathbf{X}_{i+1} + \mathbf{H_i} \mathbf{U_{i+1|2i}} = \mathbf{\Gamma_i} \mathbf{L_i} \mathbf{W_{1|i}} + \mathbf{H_i} \mathbf{U_{i+1|2i}}, \quad (16)$$

Let us now project $\mathbf{Y}_{i+1|2i}$ onto orthogonal complement of $\mathbf{U}_{i+1|2i}$. Since the projection of $\mathbf{H}_i \mathbf{U}_{i+1|2i}$ onto its orthogonal complement is empty subspace we obtain (Overschee et al., 2011):

$$\mathbf{Y}_{i+1,2i} / \mathbf{U}^{\perp}_{i+1,2i} = \mathbf{\Gamma}_i \mathbf{L}_i \mathbf{W}_{l|i} / \mathbf{U}^{\perp}_{i+1,2i}, \quad (17)$$

$$(\mathbf{Y}_{i+1,2i} / \mathbf{U}^{\perp}_{i+1,2i})(\mathbf{W}_{l|i} / \mathbf{U}^{\perp}_{i+1,2i})^{-1} = \mathbf{\Gamma}_i \mathbf{L}_i, \quad (18)$$

$$\underbrace{(\mathbf{Y}_{i+1,2i} / \mathbf{U}^{\perp}_{i+1,2i})}_{o_{i+1}} (\mathbf{W}_{l|i} / \mathbf{U}^{\perp}_{i+1,2i})^{-1} \mathbf{W}_{l|i} = \mathbf{\Gamma}_i \underbrace{\mathbf{L}_i \mathbf{W}_{l|i}}_{x_{i+1}}, \quad (19)$$

$$\mathbf{O}_{i+1} = \mathbf{\Gamma}_i \mathbf{X}_{i+1} \quad (20)$$

Let us investigate the structure of $\mathbf{O}_{i+1}$. Based on (7) and (5) it can be expressed as:

$$\mathbf{O}_{i+1} = \begin{bmatrix} \mathbf{C} & \mathbf{CA} & \ldots & \mathbf{CA}^{i-1} \end{bmatrix}^{\top} \begin{bmatrix} \mathbf{x}_{i+1} & \mathbf{x}_{i+2} & \ldots & \mathbf{x}_{i+j} \end{bmatrix} \quad (21)$$

Based on (21) the rank of $\mathbf{O}_{i+1}$ equals to the rank of the state sequence matrix $\mathbf{X}_{i+1}$. Equivalently, the dimensionality of the state vector $\mathbf{x}$ equals to the dimensionality of $\mathbf{O}_{i+1}$. The rank of $\mathbf{O}_{i+1}$ can be determined by singular value decomposition (SVD) as follows (Overschee et al., 2011):

$$\mathbf{O}_{i+1} = \mathbf{U}_1 \mathbf{S}_1 \mathbf{V}_1 \quad (22)$$

$$\Gamma_i \mathbf{X}_{i+1} = \mathbf{U}_1 \mathbf{S}_1^{1/2} \mathbf{T}\mathbf{T}^{-1} \mathbf{S}_1^{1/2} \mathbf{V}_1, \quad (23)$$

where T is an arbitrary invertible square matrix representing a similarity transformation.

$$\mathbf{X}_{i+1} = \mathbf{T}^{-1} \mathbf{S}_1^{1/2} \mathbf{V}_1 \quad (24)$$

$$\tilde{\mathbf{X}}_{i+1} = \mathbf{S}_1^{1/2} \mathbf{V}_1 \quad (25)$$

The system matrix can be estimated in the least squares sense from the following set of equations:

$$\begin{bmatrix} \tilde{\mathbf{X}}_{i+2} \\ \mathbf{Y}_{i+1} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{C}} & \tilde{\mathbf{D}} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}_{i+1} \\ \mathbf{U}_{i+1} \end{bmatrix} \quad (26)$$

where $\mathbf{U}_{i+1}$ and $\mathbf{Y}_{i+1}$ are input and output block Hankel matrices, respectively having one block row.

3 Supply-Chains and Subspace Identification

In logistics the improvement of material and information flow plays significant role, thus accurate models of the system are needed in order to predict its behaviour and thus improve its performance. Supply chains encompass the full range of intra-company and inter-company activities beginning with raw material procurement, through manufacturing and distribution (Viswanadham et al., 2001).

In this section let us show how the relation between certain system parameters can be identified on subspace basis by using measurements only. Let us assume that the inner structure of the system is completely unknown, only measurements are available. As it will be shown later, based on the accuracy of identified models the strength of relations between certain parameters of the system can be characterized, as well. By using this approach the parameters being in strong relation with a given system feature can be detected. We may consider these parameters as factors which directly influence the performance of the observed system. Let us refer to them as system performance factors. In the followings let us focus on loading systems, which are specific types of supply chains. Nevertheless, they can be considered as specific type of queuing systems, as well. During the experiments simulated input-output data have been used, thus the model of the investigated loading system had to be designed. Although there are various ways how to model supply chains or loading systems, e.g. by petri nets (Haoxun et al., 2005), by queuing networks (Viswanadham et al., 2001), etc. Here let us represent our experimental loading system by a queuing network model (see Fig. 1) and use it to generate input-output data pairs. Queuing network model is a collection of services and demands, where services represent resources while demands stand for customers or transactions, etc.

However in the practice we can assume that real measurements are available and as already mentioned no knowledge about the internal workings of the system is needed. It is considered as a black box. In our experiments the generated data will represent the measurements and will be used to identify the relation between various parameters of the system on subspace basis. In Figure 1 the architecture of a simple loading system can be observed. It consist of three main stages, i.e. loading, transfer and unloading. In the system there are five queues, i.e. one for the waiting resources such as loading machines (LMs), one for waiting resources such as unloading (UMs) , one queue for vehicles waiting for unloading after arrival to the destination and finally on the demand side there is one queue for arriving demands, i.e. in this case the incoming vehicles. The number of resources in the system is fixed. The flow of servicing demands is as follows: The arrival of demands is represented by a probability variable with exponential distribution. Each incoming demand (vehicle) is assigned with a loading machine (if available). After assignment the loading is executed which duration is represented by a probability variable with exponential distribution. After the loading is finished the two assigned entities are separated, i.e. the vehicle and the LM, which means that the loading machine at this point can be released and returned to the pool of waiting loading resources. On the other hand the loaded vehicle can proceed to the next stage of servicing, i.e. the loaded goods can be transported to the destination. Depending on the traffic it may take more or less time to reach the destination. The transportation time is represented by a probability variable with exponential distribution. At the destination the goods are unloaded. The arrived vehicles as well as the resources necessary to unload the vehicle are waiting in the corresponding queues. The unloading process similarly to the loading one is represented by a probability variable with exponential distribution. After the unloading is finished the corresponding UM is released and the demand is considered to be completed.

During the identification let us focus on the average lead time, average waiting times in certain queues, arrival rate of demands and servicing times in certain processing blocks (loading, unloading, transport). Among these factors the total average lead time for a demand entering the supply chain is a crucial performance measure (Raghavan et al., 2001). In the following section let us introduce the parameters of the simulated loading system as well as the results of the identification based on generated input-output data.

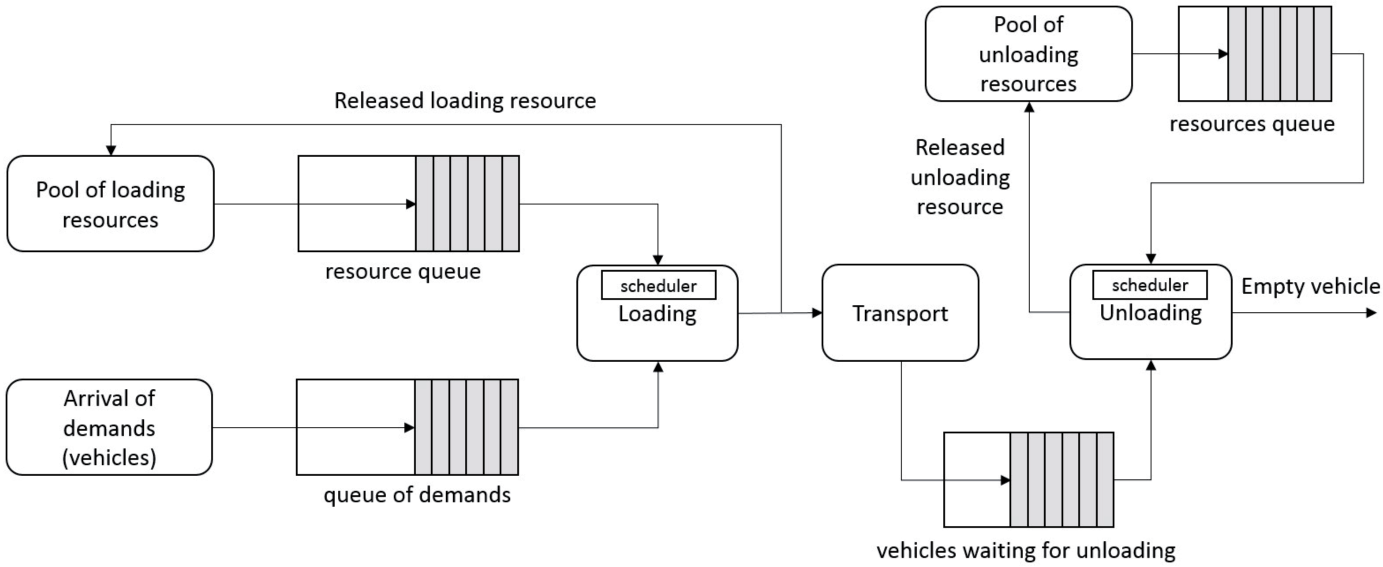

Fig. 1 The architecture of the simulated environment

4 Results

In this section let us show the identified models based on generated data. Although our assumption is that the modeled system is black-box like, let us show the setup of the simulated environment aimed to generate simulation data. Two experiments are reported with different system settings (see Table 1 and 2). In the first experiment the relation between the average waiting time of incoming vehicles (demands) and their servicing stands in the focus. 200 input-output data pairs have been generated by the simulation environment. Among this data the first 100 pairs have been considered for identification. In Fig. 1 and Fig. 2 the input and the response of the system together with the response of the identified model can be followed. In order to validate the identified model all 200 data pairs have been considered. As the results reflect the identified model accurately follows the characteristics of the measurements. In the second experiment as input the arrival rate of demands (see Fig. 5) while as output the number of waiting LMs in the corresponding queue has been considered. In Fig. 6 the response of the identified model and that of the simulated system can be followed. The validation – similarly to the previous example – is based on 200 data pairs (see Fig. 7).

Table 1 System setup for the first experiment

Parameter	Value
Number of LMs	5
Number of UMs	10
E[T_{Load}]; exp. distribution	3
E[$_{Trans}$]; exp. distribution	3
E[T_{Unload}]; exp. distribution	3
E[Arrival of demands]; exp. distribution	0.1

Table 2 System setup for the second experiment

Parameter	Value
Number of LMs	10
Number of UMs	5
E[T_{Load}]; exp. distribution	3
E[T_{Trans}] ; exp. distribution	3
E[T_{Unload}]; exp. distribution	3
E[Arrival of demands]; exp. distribution	0.1

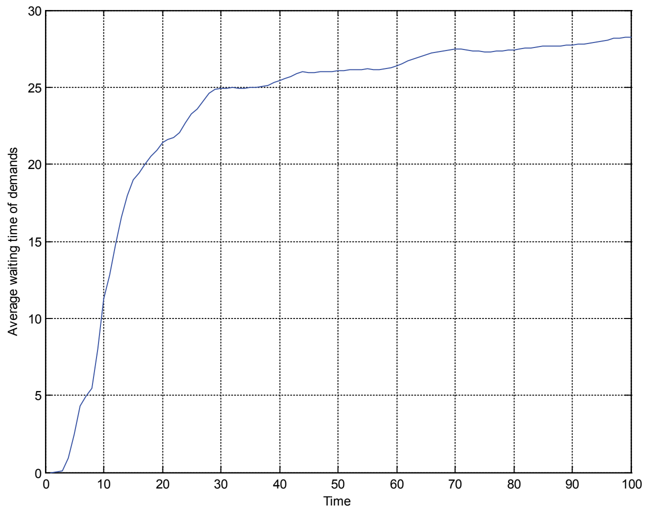

Fig. 2 Model input: average waiting time of demands (100 input-output data pairs).

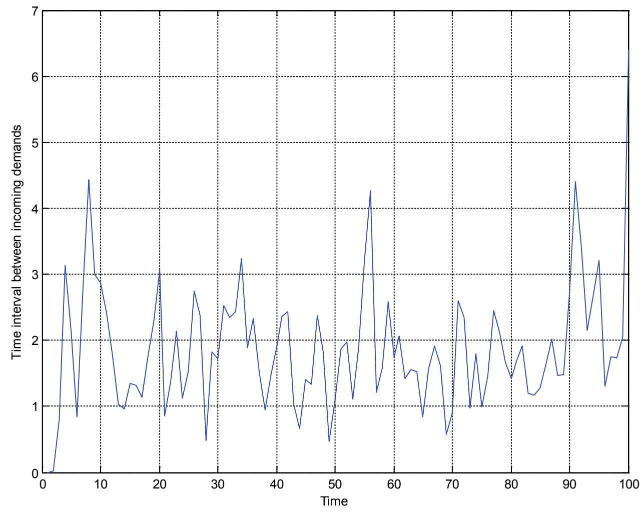

Fig. 5 Arrival rate of demands, i.e. vehicles to transport goods.

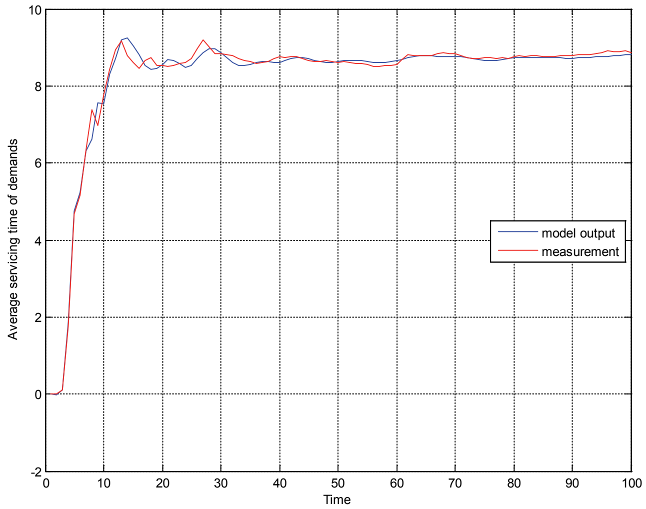

Fig. 3 The response of the estimated model and the real measurements. The estimation is based on 100 input-output data pairs.

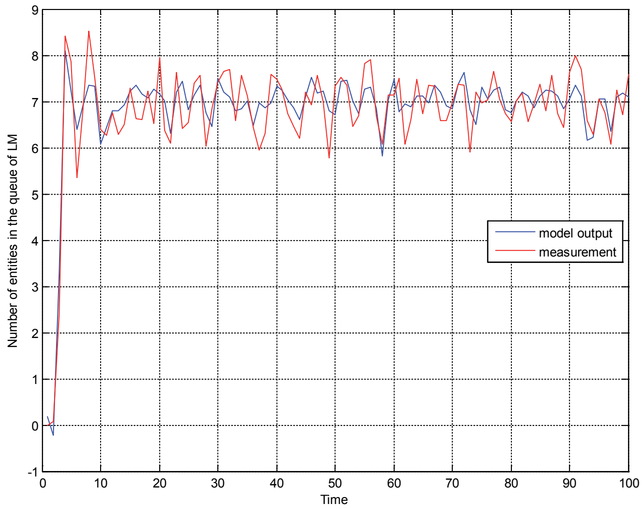

Fig. 6 The response of the estimated model together with the real measurements. The estimation is based on 100 input-output data pairs.

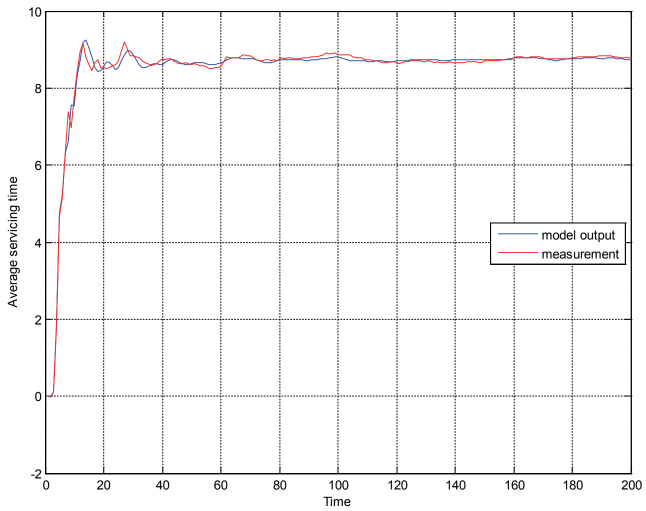

Fig. 4 Model verification: The verification is based on 200 input-output data pairs.

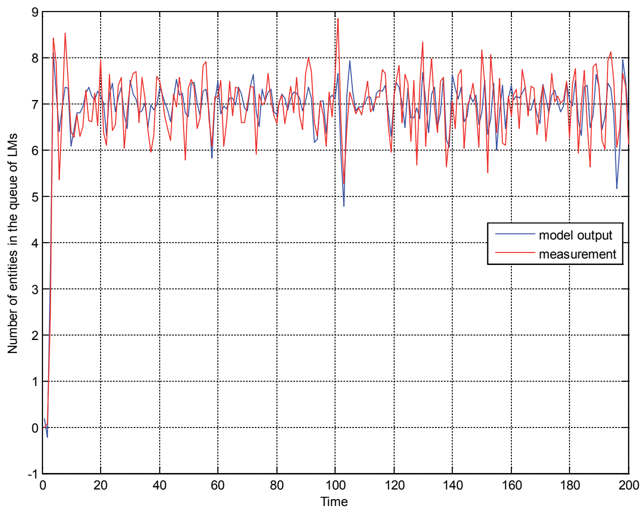

Fig. 7 Model verification: The verification is based on 200 input-output data pairs.

5 Conclusion

The paper describes how subspace identification techniques can be used to identify supply chains based on input-output data. It was assumed that the internal structure of the modelled system is unknown, thus only blackbox like approaches can be considered. It was shown how the relation between system parameters can be identified and modelled by subspace based approach. Although more complex supply chains may reflect strong nonlinearities, in this paper only the linear, deterministic case was considered.

The results clearly reflect that subspace techniques are efficient for identifying supply chains. In addition, the accuracy of the identified models can be used to characterize the strength of relation between observed parameters.

As future work we are planning to perform experiments with various setup of incoming rate of demands as well as to validate the model on real life data. In addition, we are planning to investigate supply chains in connection with linear parameter varying framework, where all nodes will be identified on subspace basis.

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