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RESEARCH ARTICLE

Comparison of Numerical Schemes for LWR Model under Heterogeneous Traffic Conditions

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Abstract

First order macroscopic model like Lighthill-Whitham-Richards (LWR) has been extensively studied and applied for various homogeneous traffic problems. Applicability and adaptability of LWR models to various heterogeneous traffic conditions is still under exploration. Finding solutions for the macroscopic models using analytical methods is a complicated task, numerical approximations are used. The present study attempts to understand the suitability of different numerical schemes for a traffic conditions in detail. Various first order and second order numerical schemes were chosen for numerical integration. Derivation of the numerical scheme, several important issues like accuracy, stability and convergence of each scheme were discussed. Simulated variables like flow, density and speeds were compared with the original data collected from the two different urban arterials with and without bottlenecks in Delhi, India. The comparison of the results of various numerical schemes shows that Lax-Friedrichs and MacCormack schemes produced better results and more stable than the other schemes.

Keywords

LWR model, numerical schemes, v-k *relationship*

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1 Introduction

Macroscopic traffic flow modelling represent how the behaviour of one characteristic of traffic (traffic flow, speed and density) changes with respect to other traffic characteristic. Macroscopic models mainly focus on overall traffic system features like congestion, delay and queue formation but not on the behaviour of a specific vehicle. The kind of problem may be extended from estimation of macroscopic traffic flow variables to study of traffic operations or implementation of alternative route networks based on travel time and delay; study of bottlenecks, Freeway traffic flow problems and other. Selection of a model mainly depends on simplicity, accuracy, efficiency and extensibility.

Lighthill and Whitham model was the first major step in the macroscopic modelling with papers, 'traffic flow on long crowded roads' and 'flood movements in long rivers' (Lighthill and Whitham, 1955), Subsequently, Richards extended the Lighthill and Whitham idea with the introduction of 'shockwaves on the highway' in an identical approach (Richards, 1956). Thus Lighthill-Whitham-Richards (LWR) model came into existence. LWR model was successful in producing shockwaves and identifying traffic jams but has limitation in reproducing nonequilibrium traffic flow situations like stop and go pattern, queue dissipation and hysteresis etc (Škrinjar et al., 2015). These drawbacks have led to the formulation of higher order models.

Even though Payne type higher order models are good at improving accuracy over first order models but they are mathematically complicated and also it is difficult to understand and solve analytically, whereas first order models are easy to understand, implement and analyse. This makes simulation of traffic flow by using LWR model more attractive (Leo and Pretty, 1990). Thus, the basic objective of this study is to check the applicability of the LWR model to two different types of traffic situations. One is with uniform lane conditions with free flow and second one is with bottleneck situation with traffic flow is at capacity.

However the existing research is mainly focused on homogenous traffic flow condition and predominantly it is restricted to cars only traffic (Bhavathrathan and Mallikarjuna, 2012; Mohan and Gitakrishnan, 2013). The heterogeneous traffic environment consists of different operational and performance characteristics of vehicles and vehicles are categorised as motorized and non-motorized vehicles. The Motorised vehicles comprises of cars, busses, trucks, two-wheelers, LCVs, auto rickshaws and others. Non-Motorised vehicles comprises of bicycles, cycle rickshaws and others. There is a high degree of variation in physical and dynamical characteristics of vehicles ply on the road and especially two wheelers have high manoeuvrability. Vehicles on the road use same right of way. Complexity is increased by presence of considerable amount of pedestrians, encroachment, street parking and others. Altogether it makes modelling difficult and thus there is a need for special methods to treat this traffic environment (Khan and Maini, 1999). Few attempts were made to incorporate heterogeneity by using higher order models (Ajitha et al., 2014; Mallikarjuna and Bhavathrathan, 2014). It is evident that a comparative study with first order LWR is not found. This study attempts to understand the suitability of LWR model for heterogeneous traffic flow conditions.

In macroscopic model while simulating the traffic flow, it is crucial to use proper numerical schemes. Macroscopic models are solved either by using analytical methods or numerical approximation schemes. Though analytical methods are more accurate they are arduous to implement in all kinds of traffic situations. On the other hand implementation of numerical schemes is easy. Partial differential equations can be solved by using first order and second order explicit numerical schemes. It is always preferred to implement different numerical schemes and compare their simulation results (Helbing and Treiber, 1999). Second objective of this study is to implement different numerical schemes to traffic flow problems and find their accuracy in reproducing traffic behaviour.

Rest of the paper is organised as follows. The next section explains the basic structure of the LWR model. Third section describes about various numerical schemes adopted in the present study, their accuracy and stability issues, fourth section describes the data collection, while the fifth section presents the formulation of MATLAB® programming, sixth section presents the results and discussion on the findings in this study.

2 LWR model

In LWR model relationships among macroscopic variables flow, density or concentration, position or location of entities are modelled. The LWR model is a simple continuum model which consists of conservation or continuity equation, fundamental flow equation, equation of state or equilibrium speed-density relationship. Mathematical structure of the model is given in Eqs. (1), (2) and (3).

Conservation of mass equation:

$$\frac{\partial k}{\partial t}+\left(\frac{dq}{dk}\right)\left(\frac{\partial k}{\partial x}\right)=0 \tag{1}$$

Fundamental equation:

$$q = k * v \tag{2}$$

Equilibrium equation:

$$v = v_e\left(k\right) \tag{3}$$

In the present study, numerical solutions are adopted over analytical solutions as a solution method for LWR model. Finite difference method, finite volume method and finite volume methods are used for finding solutions for any kind of partial differential equations. In this study finite difference numerical schemes are used as solution methods.

3 Numerical solutions: Finite Difference Methods (FDMs)

The LWR model, i.e. continuity equation coupled with steady state equation needs to be solved numerically (Treiber and Kesting, 2013). In many of the FDMs, derivatives are replaced by appropriate finite difference approximations. Two types of FDMs are implicit FDMs and explicit FDMs. In this paper, explicit methods are used because they are useful for varying boundary conditions according to realistic traffic flow simulations. In explicit FDMs, derivatives are replaced by backward, forward, central differences etc.

3.1 Explicit methods

3.1.1 First order numerical schemes

1. Upwind scheme

$$K_i^{(n+1)} = k_i^n - \left(\Delta t/\Delta x\right)\left(q_i^n\left(k\right) - q_{(i-1)}^n\left(k\right)\right) \tag{4}$$

$$q_i^n\left(k\right) = v_i^n - k_i^n \tag{5}$$

$$v_i^n = v_f\left(1-\left(k_i^n\right)/\left(k_{jam}^n\right)\right)\ldots\text{Greenshield} \tag{6}$$

$$v_i^n = v_f \exp\left(\left(-k_i^n\right)/\left(k_0\right)\right)\ldots\text{Underwood} \tag{7}$$

2. Central difference scheme

$$k_i^{(n+1)} = k_i^n + 0.5\left(\Delta t/\Delta x\right)\left(q_{(i-1)}^n\left(k\right) - q_{(i+1)}^n\left(k\right)\right) \tag{8}$$

3. Downwind scheme

$$k_i^{(n+1)} = k_i^n + \left(\Delta t/\Delta x\right)\left(q_i^n\left(k\right) - q_{(i+1)}^n\left(k\right)\right) \tag{9}$$

4. Lax-Friedrichs scheme

5. $$k_i^{(n+1)} = 0.5\left(k_{(i+1)}^n + k_{(i-1)}^n\right) + 0.5\left(\Delta t/\Delta x\right)\left(q_{(i-1)}^n\left(k\right) - q_{(i+1)}^n\left(k\right)\right) \tag{10}$$

Leap-Frog scheme

$$k_i^{(n+1)} = k_i^{(n-1)} + \left(\Delta t/\Delta x\right)\left(q_{(i-1)}^n\left(k\right) - q_{(i+1)}^n\left(k\right)\right) \tag{11}$$

3.1.2 Second order numerical schemes

1. Lax-Wendroff scheme

$$k_{(i+0.5)}^{(n+0.5)} = 0.5\left(k_i^n + k_{(i+1)}^n\right) - 0.5(\Delta t/\Delta x)\left(q_{(i+1)}^n(k) - q_i^n(k)\right) \quad (12)$$

… Predictor

$$k_i^{(n+1)} = k_i^n - (\Delta t/\Delta x)\left(q_{(i+0.5)}^{(n+0.5)}(k) - q_{(i-0.5)}^{(n+0.5)}(k)\right)$$

… Corrector (12)

2. MacCormack scheme

$$k_i^* = k_i^n - (\Delta t/\Delta x)\left(q_{(i+1)}^n(k) - q_i^n(k)\right) \quad (13)$$

… Predictor

$$k_i^{(n+1)} = 0.5\left(k_i^n + k_i^*\right) - 0.5(\Delta t/\Delta x)\left(q_i^*(k) - q_{(i-1)}^*(k)\right)$$

… Corrector (14)

Eqs. (5), (6) and (7) are common for all other schemes.

3.2 Consistency and accuracy of numerical scheme

Consistency deals with the extent to which FDMs approximate the PDEs.

$$e_h = f(A) - f(N) \ldots \text{Truncation error} \quad (16)$$

Where
$f(A)$ Analytical Solution and
$f(N)$ Numerical Solution

The numerical scheme is said to be consistent if the truncation error (e_h) satisfies

$$\lim_{\Delta t, \Delta x \to 0} e_h = 0 \quad (17)$$

The numerical scheme is said to be constituent of order (a, b) accurate or simply of order (a, b) where

$$e_h = o\left((\Delta t)^a + (\Delta x)^b\right) \quad (18)$$

3.3 Stability

A numerical scheme is said to be stable if truncation error, round-off error etc. are not allowed to increase as the calculation proceeds from one step to next step. CFL condition is a mandatory condition that should be satisfied for the stability of numerical scheme. However CFL condition is necessary condition but not a sufficient condition for the stability of numerical scheme.

3.3.1 Courant-Friedrichs-Lewy (CFL) condition

$$C \text{ or } C_{\text{CFL}} \le v_f(\Delta t/\Delta x) \ldots \text{Courant number} \quad (18)$$

CFL condition is that $\Delta x/\Delta t$ should be greater than v_f. $\Delta \mathbf{x}$ and Δt are chosen in accordance with the CFL condition.

Table 1 Explicit finite difference methods (LeVeque, 1992)

Numerical scheme	CFL Condition	Stability
Upwind	Satisfied if $\Delta t \le \Delta x/ \lvert a \rvert$	Stable
Central difference	Satisfied if $\Delta t \le \Delta x/ \lvert a \rvert$	Unstable
Downwind	Not satisfied	Unstable
Lax-Friedrich	Satisfied if $\Delta t \le \Delta x/ \lvert a \rvert$	Stable
Leap-Frog	Satisfied if $\Delta t \le \Delta x/ \lvert a \rvert$	Stable
Lax-Wendroff	Satisfied if $\Delta t \le \Delta x/ \lvert a$	Stable
MacCormack	Satisfied if $\Delta t \le \Delta x/ \lvert a$	Stable

4 Data collection

The traffic data on Aruna Asaf Ali Marg (near IIT Delhi, N Delhi, India) and Sri Aurobindo Marg (near INA metro station, N Delhi, India) collected during March 2015. Three video cameras are placed at entry point, near midpoint and exit point as shown in Fig. 1 and Fig. 2. Spot speeds of vehicles collected by using speed guns. Space mean speed is calculated from spot speed data. Car, two-wheeler, three-wheeler and truck (or bus) count for every 5 seconds manually extracted from traffic video footage. Total hourly traffic volume at Aruna Asaf Ali Marg is 1533 veh/hr and Total hourly traffic volume at Sri Aurobindo Marg is 3085 veh/hr. Traffic volume is converted from veh/hr to pcus/hr using Static PCU values given in Table 2 (IRC, 1990).

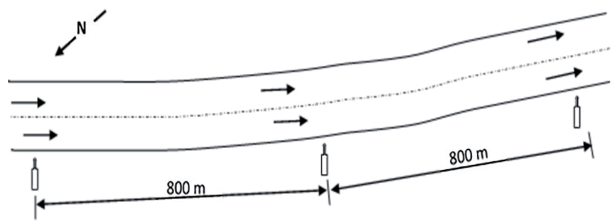

Fig. 1 Aruna Asaf Ali Marg , New Delhi (India)

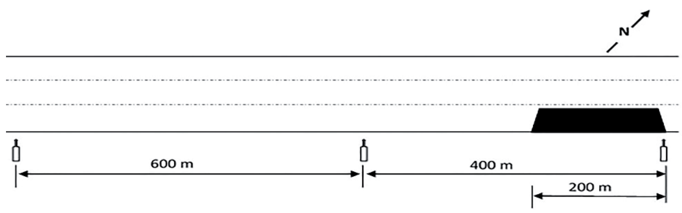

Fig. 2 Sri Aurobindo Marg, New Delhi (India)

Table 2 Recommended Static PCU values on urban roads. By IRC-106 (1990)

Vehicle type	Equivalent PCU Factors for percentage composition of Vehicle type in traffic stream	
	5%	10% and above
Two wheeler	0.5	0.75
Passenger car	1.0	1.0
Three wheeler	1.2	2
Bus	2.2	3.7

4.1 Fitting v-k relationship

As shape of v-k graph control the performance of mathematical model, attempts are made to get reliable v-k relation. v-k graph is drawn by taking speed and density values at each minute from three camera locations. The v-k plot for Arun Asaf Ali Marg is shown in Fig. 3 and for Sri Aurobindo Marg in Fig. 4 and 5. Here the v-k relationships are summarized in Table 3. For the present study Greenshiled Linear relationship and Underwood exponential relationship were taken into consideration. Extensive review on macroscopic v-k relationships is given in Wang et al. (2010).

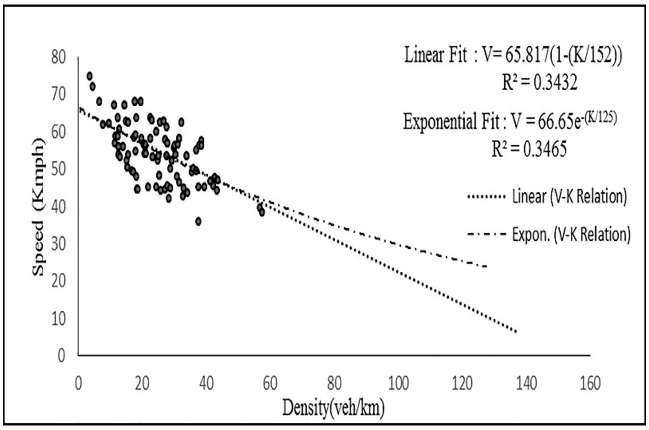

Fig. 3 v-k graph for Aruna Asaf Ali Marg

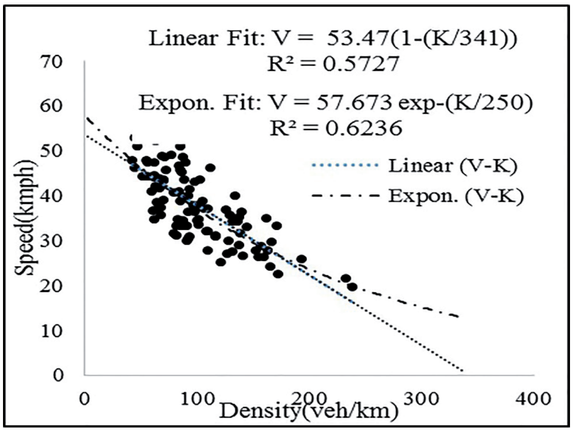

Fig. 4 v-k graph for Sri Aurobindo Marg (Whole stretch)

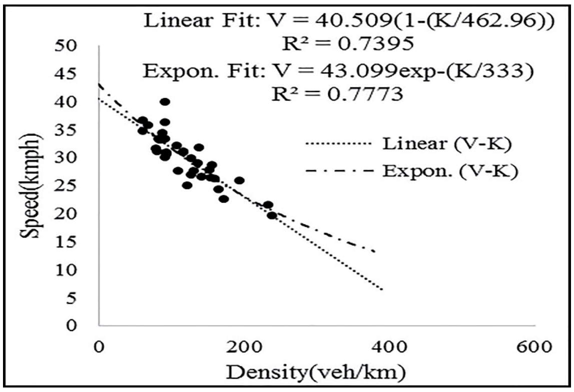

Fig. 5 v-k graph for Sri Aurobindo Marg (Within the bottleneck)

5 Formulation of MATLAB® programming

The estimation of traffic flow variables using numerical approximation is done in MATLAB® environment. The steps involved in numerical simulation programme are discussed below.

Step1: Discretization: Solution for the continuum models are estimated by discretizing the space (road) and time. Δx and Δt are spatial step and time step respectively. Macro variables k, v, q are estimated at each of the nodes ($j\ \Delta x$, n Δt) with j, n ∈ {0, 1, 2 …..n}. Here $K^n_j = K(j\ \Delta x, n\ \Delta t)$.

Step2: Initial conditions: Initial conditions on the road has to be specified in the simulation program for evaluating traffic flow variables. Uniform traffic flow conditions are assumed for Aruna Asaf Ali Marg traffic flow problem and Square wave problem has chosen to specify the initial condition for Sri Aurobindo Marg traffic flow problem. The values that chosen for two different traffic situation are given below

For Aruna Asaf Ali Marg: $K_0(x) = 2$ if $0 \le x \le 1600$

For Sri Aurobindo Marg $\begin{cases} K_0(x) = 15 \text{ if } 300 \le x \le 600 \\ K_0(x) = 10 \text{ otherwise} \end{cases}$

The initial conditions affect the outcomes of the simulation program only for short duration in open boundary case. This is because errors due to specification of initial conditions are propagated very quickly outside the simulation section (Helbing and Treiber, 1999).

Table 3 Parameters for the numerical programming implementation

Location	v-k Relationships	Parameters
Aruna Asaf Ali Marg	Linear Fit : V= 65.817(1-(K/152)) R^2 = 0.3432	V_f=65.817, K_j=152
	Exponential Fit : V = 66.65exp-(K/125) R^2 = 0.3465	V_f=66.65, K_0=125
Sri Aurobindo Marg (whole stretch)	Linear Fit: V = 53.47(1-(K/341)) R^2 = 0.5727	V_f=53.47, K_j=341
	Exponential Fit: V = 57.673exp-(K/250) R^2 = 0.6236	V_f=57.673, K_0=250
Sri Aurobindo Marg (with in the bottleneck)	Linear Fit: V = 40.509(1-(K/462.96)) R^2 = 0.7395	V_f=40.509, K_j=463
	Exponential Fit: V = 43.099exp-(K/333.33) R^2 = 0.7773	V_f=43.099, K_0=333

Boundary Conditions: Dirichlet (or fixed) boundary conditions are considered for the present problem. Dirichlet conditions are empirically measured density values and continuously fed to the simulation program. Dirichlet boundary conditions are specified by k (0, t), k (L, t) and are time dependent and they represent boundary conditions at entry and exit point respectively.

Step 3: By specifying initial and boundary conditions for the road, traffic flow variables q, k, v are estimated for the next time step for the sections $i = 2$ to $nx - 1$($i = 1$ and nx are defined as source and sink cells for the program). If t = final time step then the program will be terminated.

5.1 For uniform lane condition

First traffic situation considered in this study is uniform lane conditions at Aruna Asaf Ali Marg in New Delhi, traffic data collected at this location shows free flow conditions and has lot of fluctuations due to presence of signals. Descriptions of the parameters used in the modelling are given below.

Length of the road section under consideration is 1600 m.

Total length of the simulation time: 45 minutes (2700 sec)

$\Delta x = 200$ m, $\Delta t = 5$ sec, initial conditions and boundary conditions as specified in step 3 of Section 5.

v-k relationships developed for the entire section and parameter values used in the model K_{jam}, V_f, K_0 are taken as mentioned in Table 3.

5.2 For lane drop condition

For this second situation, arterial road with lane drop (work zone) i.e., Aurobindo Marg, New Delhi is chosen for traffic data collection. Details about the road cross section and data collection is discussed in section 4. On this arterial road traffic flow is continuous and higher than the earlier mentioned uniform lane situation. Traffic behaviour is complicated and speed reductions, high density variations can be expected at bottleneck if the traffic flow exceeds the capacity. Various numerical schemes mentioned earlier are applied to check the efficiency in reproducing this situation. Descriptions of the parameters used in the modelling mentioned below.

Length of the road section under consideration is 1000 m.

Length of the work zone is 200 m.

Total length of the simulation time: 35 minutes (2100 sec)

Δx=200 m, Δt=5 sec, initial conditions and boundary conditions as specified in Section 5.

Greenshield and Underwood steady state equations fitted for whole stretch and the work zone separately. Parameter values K_{jam}, V_f, K_0 are taken as mentioned in Table 3.

Various numerical schemes applied to these traffic conditions for finding solution are explained in the previous section. Outcomes and performance of the each scheme will be discussed in the next section.

6 Results and analysis

The performance of various numerical schemes studied in estimating macroscopic flow variables by simulating two different field situations. First order finite difference schemes like upwind scheme, Lax-Friedrichs (LaxFr.) scheme and second order finite difference schemes like Lax-Wendroff (LaxW.) method, MacCormack (MacC.) scheme are used. We dropped central difference, downwind and Leap Frog schemes because of their instability while simulation. Traffic evolution captured for every 5 seconds in terms of density in pcu/km, speed in km/h, and flow in pcu/h. Traffic evolution on Aruna Asaf Ali Marg and Sri Aurobindo Marg are simulated by using Greenshield and Underwood equilibrium speed-density relationships separately.

6.1 For Uniform Lane Condition

Figures 6 and 7 show the space–time evolution of the density for different v-k relationship with different numerical schemes.

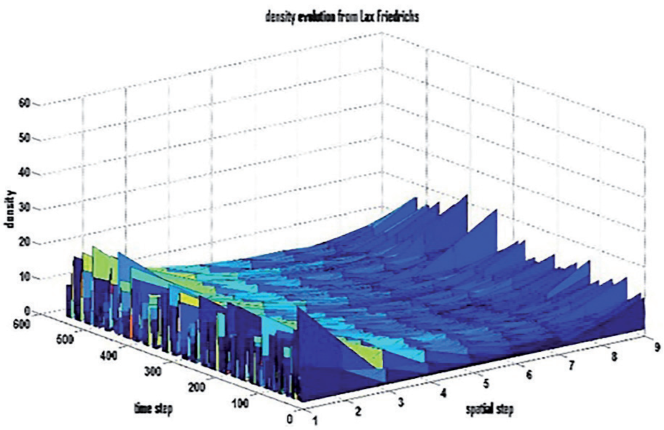

Fig. 6 Density evolution with respect to space and time for Greenshield v-k relationship with Lax Friedrichs scheme

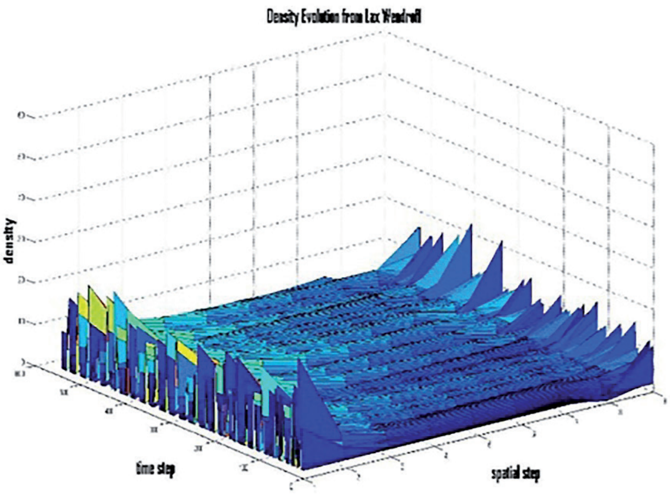

Fig. 7 Density evolution with respect to space and time for Underwood V-K relationship with Laxwendroff scheme

Density values are estimated for 5, 10, 15, 20, and 30 seconds. Error in estimating density is very high for small time step values and it gradually decreases with increasing time step value. For Greenshield steady state equation, at lower time step values MacCormack scheme works well whereas at higher time steps values Lax-Friedrichs scheme and MacCormack scheme give good results. For Underwood steady state equation there is a clear distinction between upwind, Lax-Wendroff schemes and other schemes. They prove accurate compared to other schemes. The simulation results for both steady state equations shows that Lax-Wendroff scheme with underwood steady state equation yield good results for higher time step.

Mean Absolute Percentage Error (MAPE) values for various schemes are shown in Table 4 and Table 5.

Table 4 MAPE values for various time steps for Aruna Asaf Ali Marg

Time (Sec)	MAPE(%) for Greenshield			
	LaxFr.	Upwind	LaxW.	MacC.
5	58.92	38.10	46.42	33.79
10	37.95	25.80	30.71	24.50
15	18.22	15.30	14.84	15.35
20	11.83	12.84	11.93	12.79
30	10.79	12.66	12.17	11.86

Table 5 MAPE values for various time steps for Aruna Asaf Ali Marg

Time (Sec)	MAPE(%) for Underwood			
	LaxFr.	Upwind	LaxW.	MacC.
5	60.83	41.86	51.84	46.11
10	39.39	29.50	35.28	31.17
15	19.27	15.83	16.84	18.48
20	12.98	11.15	11.50	14.17
30	12.30	9.49	8.91	12.30

Figure 8 and Fig. 9 gives a visual idea of variation of MAPE versus different time step values. Efficiency of all numerical schemes is very low at lower time steps.

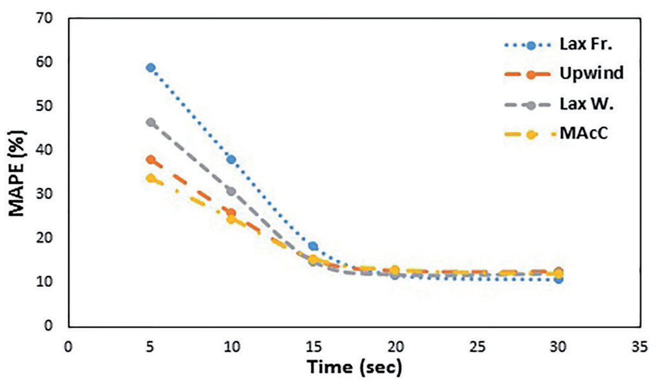

Fig. 8 Variation of MAPE value for various time steps using Greenshield v-k relation

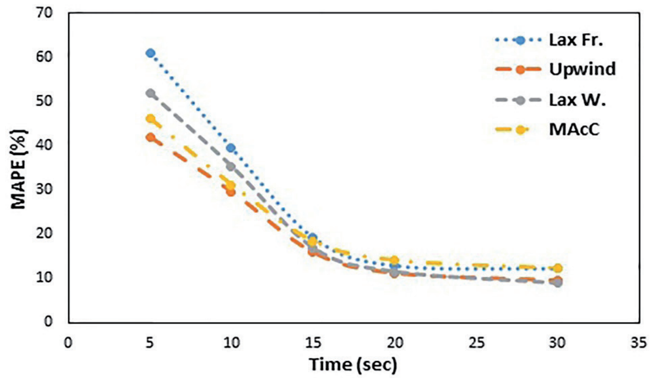

Fig. 9 Variation of MAPE value for various time steps using Underwood v-k relation

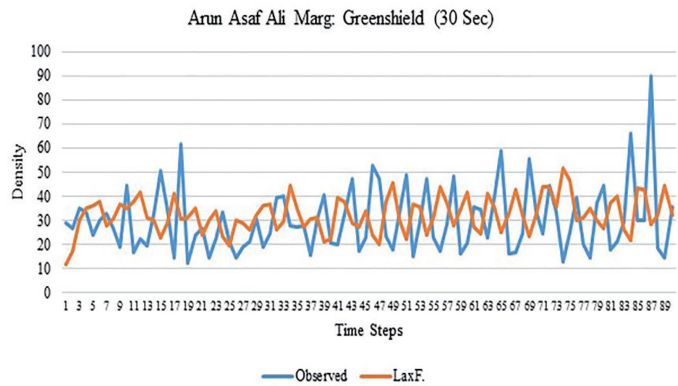

Fig. 10 Density variation with respect to time step using Lax –Friedrichs method

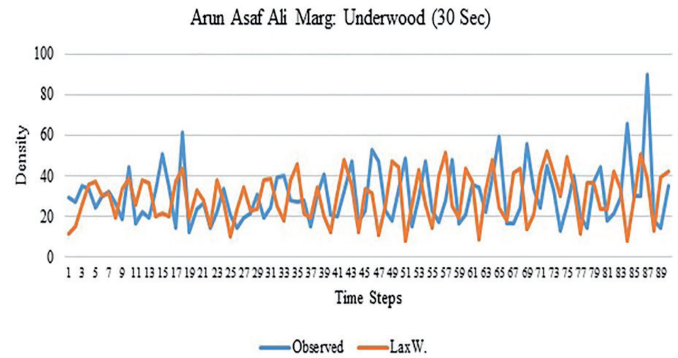

Fig. 11 Density variation with respect to time step using Lax –Wendroff method

From this study it is understood that LWR model is unsuitable for low traffic volume situations where interaction between vehicles is less and steady state equation fails in capturing speed fluctuations around equilibrium flow.

6.2 For lane drop condition

Density evolution with respect to space and time is shown in Fig. 12 and Fig. 13.

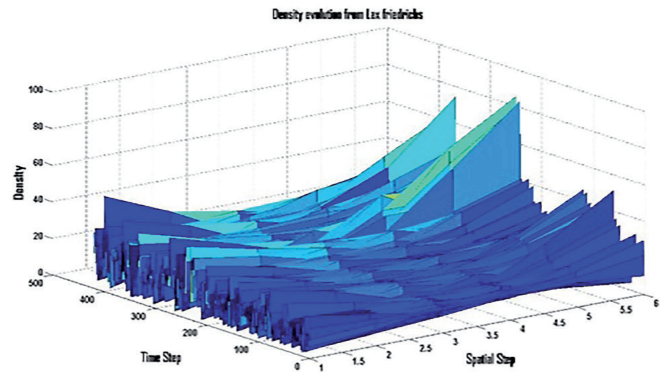

Fig. 12 Density evolution with respect to space and time for Greenshield v-k relationship with Lax-Friedrichs scheme

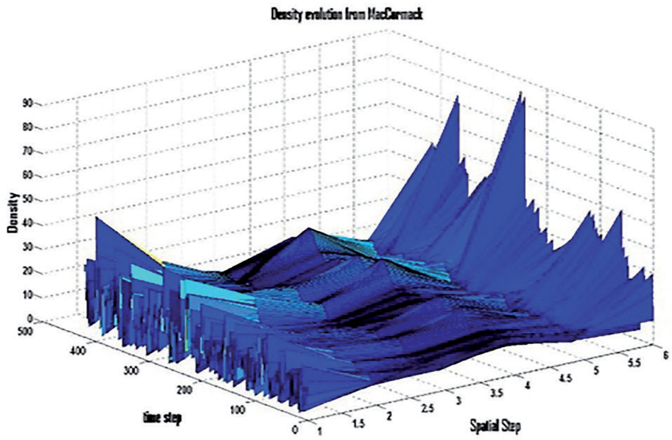

Fig. 13 Density evolution with respect to space and time for Underwood v-k relationship with MacCormack Scheme

High density region is observed before commencing of the work zone that means traffic flow exceeded the capacity of the work zone. From the numerical analysis it is observed that Lax-Friedrichs and MacCormack schemes are outperforming other methods. Table 6 and Table 7 shows MAPE values concerned to various time steps.

Table 6 MAPE values obtained for various time steps for Sri Aurobindo Marg

Time (Sec)	MAPE(%) for Greenshield			
	LaxFr.	Upwind	LaxW.	MacC.
5	24.84	25.61	22.22	25.97
10	17.32	24.29	18.46	19.89
15	10.95	24.76	17.30	15.20
20	8.05	23.75	16.36	11.31
30	6.81	26.13	17.92	8.19

Table 7 MAPE values obtained for various time steps for Sri Aurobindo Marg

Time (Sec)	MAPE(%) for Underwood			
	LaxFr.	Upwind	LaxW.	MacC.
5	25.39	25.92	20.26	23.67
10	18.10	23.97	16.04	17.24
15	11.21	23.95	15.59	12.93
20	9.05	22.92	14.82	9.99
30	6.69	25.23	15.71	7.26

Lax Friedrichs and MacCormack schemes are very consistent at all levels. Efficiency of estimating macroscopic traffic variables with Greenshield and Underwood steady state equation is very high in Lax Friedrichs case. Visual representation of variation of MAPE values versus time step are shown in Fig. 14 and Fig. 15.

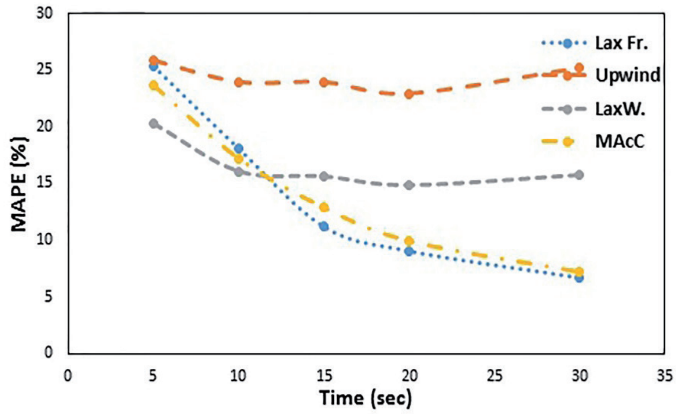

Fig. 14 Variation of MAPE value for various time steps using Greenshield v-k relation

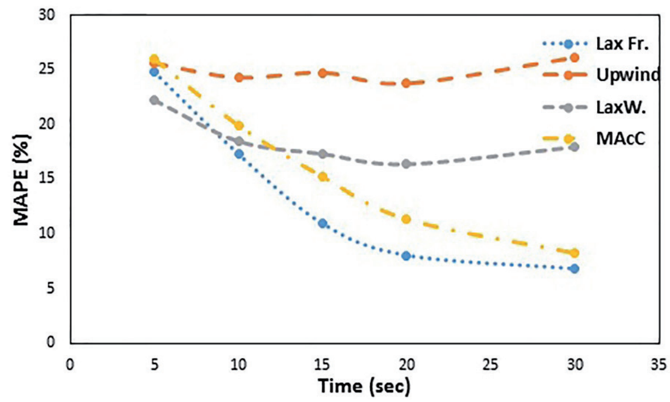

Fig. 15 Variation of MAPE value for various time steps using Underwood v-k relation

Figure 16 and Fig. 17 shows the comparison between observed and estimated density values for 30 sec time step.

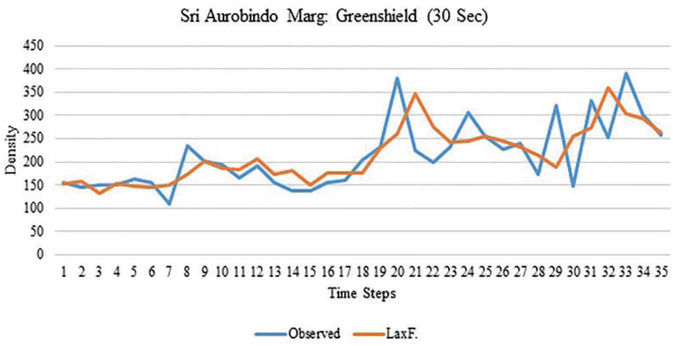

Fig. 16 Density variation with respect to time step using Lax –Friedrichs method

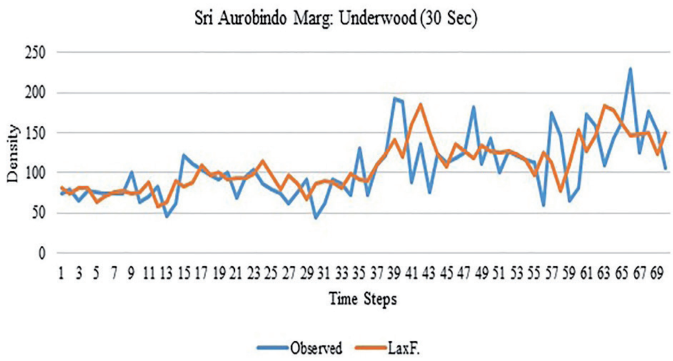

Fig. 17 Density variation with respect to time step using Lax –Friedrichs method

MacCormack scheme is successful in preventing vehicles moving backward from the jam region to the empty region (Ngoduy et al., 2004). MacCormack scheme produces numerical dispersion, leads to oscillations in large density gradient regions and this scheme is more sensitive to the nonlinear instabilities compared to first order schemes like Lax-Friedrichs and upwind scheme. MacCormack scheme is efficiently reproduce traffic conditions in congested traffic situation. In bottleneck situation traffic flow is continuous and there is interaction among the vehicles. This kind of situation has profound influence on performance of first order macroscopic models and the results replicate the same. When compared to uniform lane situation accuracy is improved in lane drop situation.

7 Discussion

The main aim of the present study was to examine the performance of first order macroscopic models in representing the traffic stream behaviour. For that two different traffic situations have chosen to implement numerical simulation approach. Based on stability, accuracy and convergence point of view four numerical schemes have finalised and they are upwind, Lax-Friedrichs, Lax-Wendroff and MacCormack schemes. Numerical simulation was implemented for two types of v-k relationships namely Greenshield and Underwood. From this study it was found that efficiency of the numerical schemes at smaller time steps (5 sec) is very low in free flow traffic behaviour observed at Aruna Asaf Ali Marg. At higher time steps (30 sec) Lax-Wendroff scheme with Underwood steady state relationship works well for Aruna Asaf Ali Marg. Accuracy of the simple LWR model is affected in capturing fluctuations in traffic flow due to static fundamental relationship used in the model. Whereas the efficiency of numerical schemes in lane drop situation was impressive. Interestingly Lax-Friedrichs scheme estimates traffic flow variables more accurately with both Greenshield and Underwood, followed by MacCormack scheme. The traffic flow observed at Sri Aurobindo Marg (lane drop situation) almost behaves like continuous fluid flow that means there is a continuous interaction among the vehicles. This could be one of the reasons behind the success of LWR model and if one observes the v-k relationship for Sri Aurobindo Marg there is a little fluctuation of speeds around the mean. Lax-Friedrichs scheme has been proved to be stable approximation with different v-k relationships.

Selection of a macroscopic model for a particular problem depends on its strength and weaknesses of the model. From the literature it is understood that the first order models fail in reproducing non-equilibrium traffic phenomenon observed on the road like stop and go pattern, queue pattern, formation of clusters and hysteresis (Daganzo, 1995; Zhang, 1998). These draw backs emerge from the fact that the steady state equation does not allow the speed to fluctuate around equilibrium speed. There is only one speed for a given parameters. This problem also persists in the two traffic conditions observed in the present study. Accuracy of the model may improve by replacing steady state equation with dynamic velocity equation. Numerical Stability of the explicit finite difference schemes is depend on CFL condition i.e., $v_f(\Delta t/\Delta x) \leq 1$ and Δx, Δt values chosen accordingly. Accuracy of the scheme is greatly influenced by CFL condition and it is one of the drawbacks of the explicit finite difference schemes. Implicit finite difference scheme can be used if stability is the issue. This numerical study can be further extended to represent traffic heterogeneity by incorporating the area occupancy in place of linear density (Mallikarjuna and Rao, 2006).

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