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RESEARCH ARTICLE

A Review Analysis of Optimal Velocity Models

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Abstract
To understand traffic behavior, we require a thorough knowledge of traffic stream parameters and their mutual relationships. This relationship between the traffic parameters results many researches yielded many mathematical models named Traffic flow models. This paper presents an overview of two great approaches of traffic flow: macroscopic and microscopic models. We aim to provide an historical overview of the development of microscopic models, particularly car-following models which are fundamental in the replication of traffic flow and thus they have received considerable attention. In this work, we present a survey of recent researches based on the optimal velocity model proposed by Bando and we discuss the capability of these models, their strong points and also their weakness.

Keywords
Car-following models, Traffic flow, Microscopic models, Macroscopic models, Optimal velocity model

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1 Introduction

With the rapid quantitative increase of cars, the traffic jam becomes more and more serious. To solve these problems, the researcher's activity had its beginnings from the 1920's, to describing the propagation of traffic flows by means of dynamic macroscopic and microscopic models. Previous state-of –art and review aim to take one step further back and give an historical overview of the highlights in traffic flow modelling (Papageorgiou, 1998; Brackstone and McDonald, 1999; Hoogendoorn and Bov, 2001; Darbha et al., 2008; Orosz et al., 2010; Bellomo and Dogbe, 2011; Wilson and Ward, 2011; Aghabayk et al., 2015). In this paper, we present a review of macroscopic and microscopic traffic flow models focused on the optimal velocity model (Bando et al., 1995). The optimal velocity model has not the ability to explain only individual behavior of a vehicle, but also its connectivity to some macroscopic values such as traffic flow and density (Nugrahani, 2013). As mentioned, there are two major approaches to describe the traffic flow problem. Macroscopic traffic flow models make use of the picture of traffic flow as a physical flow of a fluid. They describe the traffic dynamics in terms of aggregated macroscopic quantities such as the traffic density, traffic flow or the average velocity as a function of space and time corresponding to partial differential equations. By way of contrast, microscopic traffic models describe the motion of each individual vehicle. They model the action, such as accelerations, decelerations and lane changes of each driver as a response to the surrounding traffic (Kesting et al., 2008) (Fig. 1).

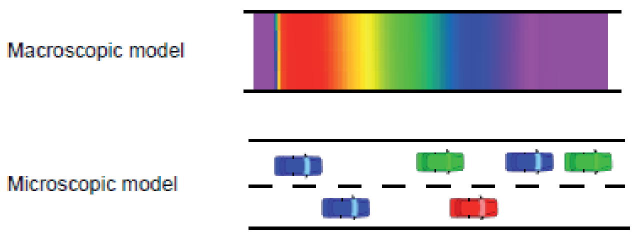

Fig. 1 Illustration of different traffic modeling approaches

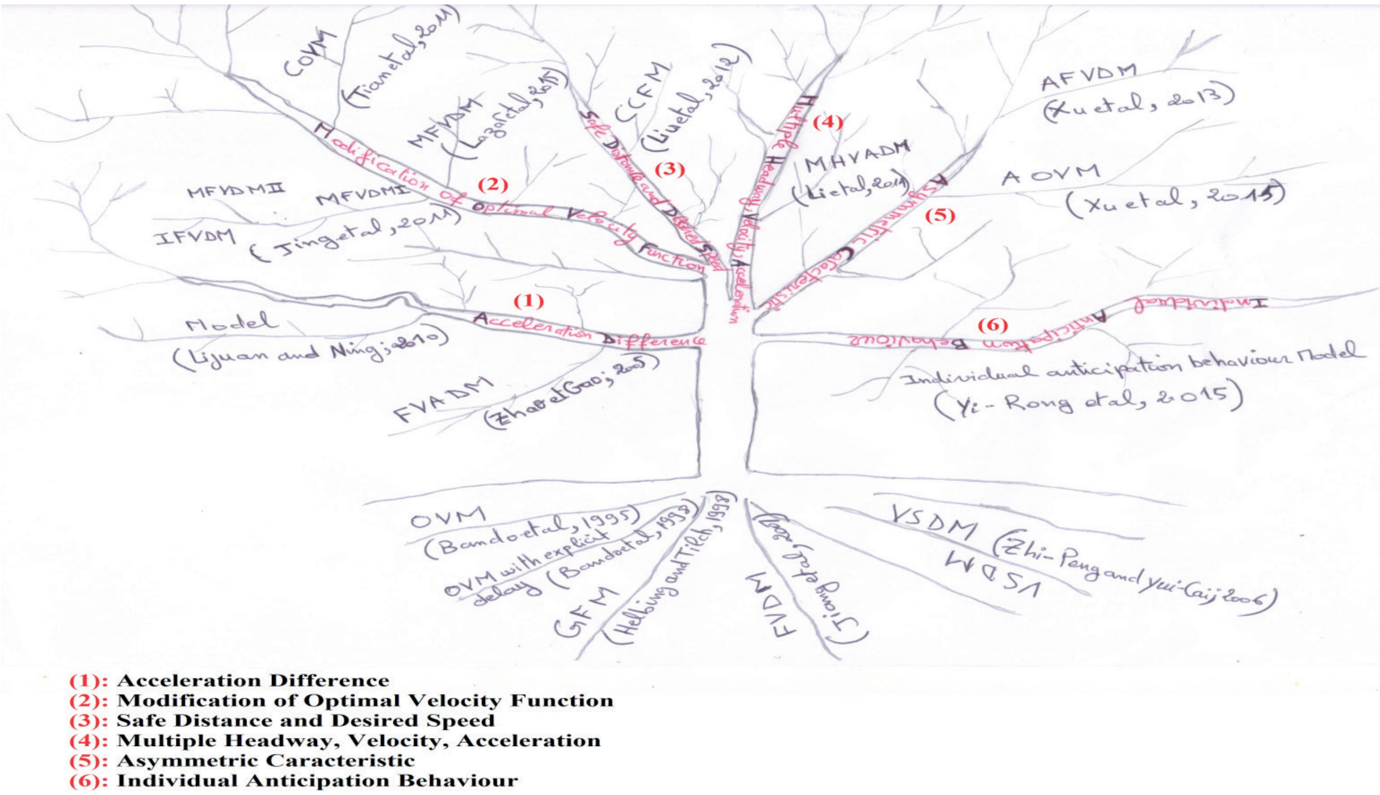

Fig. 2 A model tree of optimal velocity Models

In this survey, we aim to discuss how this model has been developed and how different types of models are related to each other. We follow a model tree to show the historical development of the optimal velocity models (Fig. 2).

2 Traffic Flow Approaches

2.1 Macroscopic approach

The macroscopic models arise from a hydrodynamic analogy of the flow of vehicles. The goal of these models is to be able to characterize the global behaviour of the traffic, in a scale of relatively important study. Their current applications cover the simulation of the traffic with the aim of the planning and of the conception of infrastructures, but also the dynamic management of the traffic and the evaluation of these measures of management. Macroscopic description is used when the state of the system is described by averaging gross quantities, namely, density k, speed v, flow q, regarded as variables dependent on time and space. Mathematical models describe the evolution of the above variables using systems of partial differential equations (Kesting et al., 2008). Since then, the research in the field of the traffic flow did not stopped attracting the scientists of any edge; so much its social, economic and environmental impacts are considerable. Traffic models answer this need by translating the application of the scientific approach in the problems posed by the transport. The first scientific studies on the traffic flow go back to the works Greenshield's model (Greenshields, 1935), Greenberg's model (Greenberg, 1959), underwood model (Underwood, 1961) .

The Greenshield's model represents how the behavior of one parameter of traffic flow changes with respect to another. The most simple relation between speed and density is proposed by green shield and scalled the fundamental relation or fundamental diagram later (van Wageningen, 2014; Jabeena, 2013). The fundamental diagram family and its most important relations is shown in Fig. 3 .

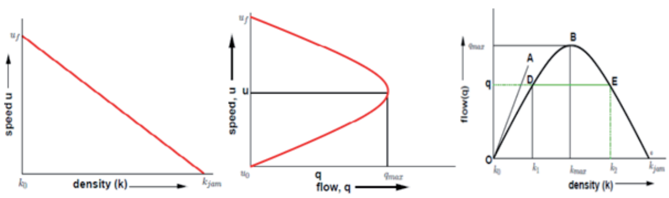

Fig. 3 Fundemental traffic flow diagram

Many other models came up, prominent among them, we found Greenberg's model. Greenberg used a fluid-flow analogy concept and proposed a logarithmic speed-density relationship. This model shows better goodness of fit compared to green shield's model. The main disadvantage of this model is its inability to predict speed at lower densities. That is due when a density approaches zero, speed tends to increase to infinity (Jabeena, 2013) (Fig. 2). In 1961, Underwood suggested an exponential speed-density relationship and derived an exponential model that attempted to overcome the limitation of the Greenberg model. The most advantage of this model is shows better than Greenshield and Greenberg Models for uncongested condition but not good in congested condition. The main drawback of the model is speed becomes zero only when density reaches infinity. Hence this cannot be used for predicting speeds at high densities (Jabeena, 2013) (Fig. 4).

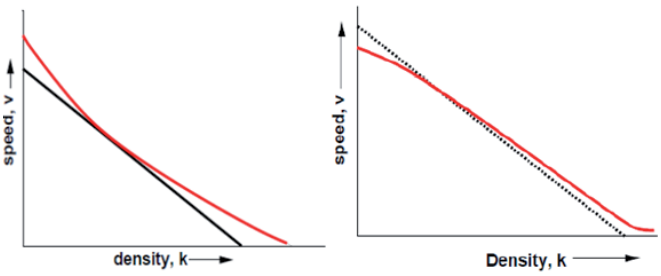

Fig. 4 Speed density relationship by Greenberg and underwood

2.2 Microscopic approach

A microscopic model of traffic describes the car following behaviour as well as the lane changing behaviour of every vehicle in the traffic. The most famous one is the Car-Following models (Bando et al., 1995; Helbing and Tilch, 1998; Jiang et al., 2001), where the driver adjusts his or her acceleration according to the conditions in front and each vehicle is governed by an ordinary differential equation (ODE) that depends on speed and distance of the car in front (Darbha et al., 2008). In microscopic models, cars are numbered to indicate their order: n is the vehicle under consideration, $n-1$ its leader, $n+1$ its follower, etc., (Fig. 5). The behaviour of each individual vehicle is modelled in terms of the position of the front of the vehicle x, velocity $v = dx/dt$, acceleration $a = d^2x/d^2t$.

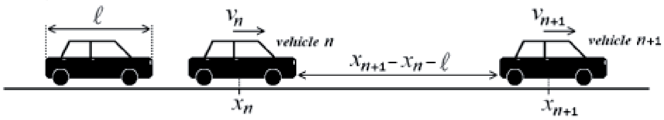

Fig. 5 Car-following notation

Several theories have been proposed to model car following behaviour, which can be divided into three classes based on behavioural assumptions, namely, Safe-distance models, stimulus-response models, optimal velocity models.

Safe-distance or collision avoidance models try to describe simply the dynamics of the only vehicle in relation with his predecessor, so as to respect a certain safe distance. One of the first models to have been developed on this idea is the simple model of Pipes (1953) Eq. (1). Then Kometani and Sasaki (1959) proposed the first model of avoidance collision. This model aims to transcribing the trajectory of a vehicle according to a minimal safe distance. A following driver keeps a safe distance to avoid a collision. The safe distance is related to vehicle velocity at time t and its leader velocity at time $t-T$ which T is the reaction time.

$$x_{n-1} = x_n + S + Tv_n + l_{n-1}^{veh} \quad (1)$$

where S the distance between two vehicles and l_{n-1}^{veh} length of the leading vehicle. Tv_n is interpreted by Pipes as the 'legal distance' between vehicle $n-1$ and n.

The works of Gipps (1981) aimed at completing this initial approach by incorporating a safe speed to keep safe distance related to distance between two successive cars and their accelerations and speeds.

$$v_n(t+\tau) = \min\left\{ \begin{array}{c} v_n(t) + 2.5a_{\max}\tau\left(1 - \frac{v_n(t)}{v_{\max}}\right)\sqrt{0.25\frac{v_n(t)}{v_{\max}}}, \\ a_{\min}\tau + \sqrt{a_{\min}^2\tau^2 - a_{\min}\left(2\left(x_{n-1}(t) - x_n(t) - s_{jam}\right) - v_n(t)\tau - \frac{v_{(n-1)}^2(t)}{b}\right)} \end{array} \right\} \quad (2)$$

with $a_{\max}$ maximum acceleration, $a_{\min}$ maximum deceleration (minimum acceleration), $v_{\max}$ the desired (maximum) velocity and s_{jam} jam spacing front-to-front distance between two vehicles at standstill.

The second class of car-following models consists of stimulus–response concept based on the assumption that the driver of the following vehicle perceives and reacts appropriately to the spacing and the speed difference between the following and the lead vehicles (Jabeena, 2013). It is assumed that drivers accelerate (or decelerate) as a reaction to three stimuli:

- Desired velocity $v_n = \frac{dx_n}{dt}$
- Relative spacing between the subject vehicle and its leader $S_n = x_{n+1} - x_n$
- Relative speed between the subject vehicle and its leader $\dot{S}_n = \frac{dS_n}{dt} = v_{n+1} - v_n$

From 1950s and early 1960s, there was a rapid development of stimulus-response models (Chandler et al., 1958; Helly, 1961) and they made their efforts to develop a famous GHR-model, named after (Gazis et al., 1961). The general formulation of this model is:

$$a_n(t) = \gamma\frac{\left(v_{n-1}(t)\right)^{c_1}}{\left(S_n(t-\tau)\right)^{c_2}}\dot{S}_n(t-\tau) \quad (3)$$

$\gamma\frac{\left(v_{n-1}(t)\right)^{c_1}}{\left(S_n(t-\tau)\right)^{c_2}}$ is the sensitivity of vehicle/driver n. γ is the sensitivity parameter and c_1 and c_2 are parameters that are used to fit the model to data. The rate $\dot{S}_n(t-\tau)$ is considered as the stimulus, the acceleration $a_n(t)$ as the response, hence the name 'stimulus–response' model.

This model allows taking into account the inter-distance between both vehicles. Numerous studies were led to determine the «optimal combination» of parameters (c_1, c_2). Among them (Gazis et al., 1961; May and Keller, 1967; Heyes and Ashworth, 1972; Ceder and May, 1976). For more details, see Table 1.

Table 1 Proposed value of (c_1, c_2) parameters for GHR model

Models	Value of	Value of
Gazis et al., 1961	[0;2]	[1;2]
Edie, 1963	1	2
May and Keller, 1967	0.8	2.8
Heyes and Ashworth, 1972	-0.8	1.2
Ceder and May, 1976	0.6	2.4

Based on the GHR model Helly (1961) proposed a linear model by adding some terms into the first GHR model to adapt the acceleration of the subject vehicle with consideration of its leading vehicle braking. A simplified version of this model is

$$a_n(t) = k_1 \dot{S}_n(t-\tau) + k_2\left[S_n(t-\tau) - D_n(t)\right] \qquad (4)$$

Where a_n is the acceleration of the nth car. k_1 and k_1 are model calibration parameters; and $D_n(t)$ is a desired following distance formulated by

$$D_n(t) = \alpha + \beta v_n(t-\tau) + \delta a_n(t-\tau) \qquad (5)$$

Optimal velocity models are another approach generally based on the difference between the driver's desired velocity and the current velocity of the vehicle as a stimulus for the driver's actions. One of the first models learning on an analysis of the trajectories of vehicles is Newell (1961) has proposed the model

$$v_n(t+\tau) = V\left(S_n(t)\right) \qquad (6)$$

With $V(S_n(t))$ is the optimal velocity under the headway $S_n(t)$. This model has directly given the speed of n-th car by the optimal velocity function. Based on this model, (Bando et al., 1995; Nugrahani, 2013) introduce an Optimal Velocity Model (OVM), is given by

$$a_n(t) = \kappa\left(V_{opt}\left(S_n(t)\right) - v_n(t)\right) \qquad (7)$$

where κ is the sensitivity. Helbing and Tilch (1998) given the function of OVM model as follows

$$V_{opt}\left(S_n(t)\right) = V_1 + V_2 \tanh\left[C_1\left(S_n(t) - l\right) - C_2\right] \qquad (8)$$

here l is the length of vehicle, and V_1, V_2, C_1, C_2 parameters calibrated. Table 2 summarizes three types of the microscopic model with his advantages and his inconvenient.

3 Review and Analysis study of microscopic models based on Optimal Velocity Models

We aim to provide a survey of car following models and our interest specially to present a review based on optimal velocity models. We proposed to present the recent models based on the model of Bando et al. (1995) and criticize them by giving the advantages and weaknesses of each model, which constitutes a strong perspective to develop a better one. We have already started to present the optimal velocity models and we introduced a basic model developed in 1995 in Section 2.2. Bando et al. (1995) proposed a dynamical model to describe many properties of real traffic flows such as the instability of traffic flow, the evolution of traffic congestion, and the formation of stop-and-go waves. For the same authors (Bando et al., 1998) analyzed the OVM with the explicit delay time. They proposed to introduce the explicit delay time in order to construct realistic models of traffic flow for that it's included in the dynamical equation of OVM (Eq. (7)) and become as follow

$$a_n(t+\tau) = \kappa\left(V_{opt}\left(S_n(t)\right) - v_n(t)\right) \qquad (9)$$

In their analysis, they found that the small explicit delay time has almost no effects. Unlike, where the large explicit delay time introduced, a new phase of the congestion pattern of OVM seems to appear. However, the OVM has encountered the problems of high acceleration and unrealistic deceleration. In order to solve that, Helbing and Tilch (1998) proposed a generalized force model GFM add new term to the right of Eq. (7).

Table 2 Summary of some existing car-following models

Type of Class	Related works	Advantages	Weakness
Safe-distance or avoidance collision models	Pipes, 1953 Kometani and Sasaki, 1959 Gipps, 1981	Takes accounts for differences between acceleration and deceleration phases of driving.	Not consider drivers' perception and any small changes may end to the reaction of the following vehicle driver
Stimulus-response models	Gazis et al., 1961 Helly, 1961 May and Keller, 1967 Heyes and Ashworth, 1972 Ceder and May, 1976	Replicates low-acceleration patterns simple to understand and use	Creates headways larger than reality when the magnitude of fluctuations of acceleration increases
Optimal velocity models	Bando et al., 1995 Helbing and Tilch, 1998	Simple to use and calibrate	Gives unrealistically large accelerations in some circumstances

This new term represents the impact of the negative difference in velocity on condition that the velocity of the front vehicle is lower than that of the follower. The GFM formula is

$$a_n(t) = \kappa\left(V_{opt}\left(S_n(t)\right) - v_n(t)\right) + \lambda\Theta\left(-\dot{S}_n(t)\right)\dot{S}_n(t) \tag{10}$$

where Θ is the Heaviside function. We compared GFM with OVM, GFM has the same form as OVM, and the difference lies in that they have different values of sensitivity κ.The main drawback of GFM doesn't take the effect of positive velocity difference $\dot{S}_n(t)$ on traffic dynamics into account and only considers the case where the velocity of the following vehicle is larger than that of the leading vehicle. In Jiang et al. (2001) pointed out that when the preceding car is much faster, the following vehicle may not break even though the spacing is smaller than the safe distance. The basis of GFM and taking the positive factor $\dot{S}_n(t)$ into account. Jiang et al. (2001) obtained a more systematic model, one whose dynamics equation is as

$$a_n(t) = \kappa\left(V_{opt}\left(S_n(t)\right) - v_n(t)\right) + \lambda\dot{S}_n(t) \tag{11}$$

The proposed model takes both positive and negative velocity differences into account, they call it a full velocity difference model (FVDM). The main advantage of FVDM is eliminating unrealistically high acceleration and predicts a correct delay time of car motion and kinematic wave speed at jam density. Then, Zhao and Gao (2005) argued that previous models OVM, GFM and FVDM does not describe the driver's behavior under an urgent case where they can be defined as:

"*A situation that the preceding car decelerates strongly, if two successive cars move forward with much small headway-distance, e.g. a freely moving car decelerates drastically for an accident in front or the red traffic light at an intersection, the following car is freely moving and the distance between the two cars is quite small*"

They found out that the velocity difference is not enough to avoid an accident under such urgent case in previous models for that, they extend the FVDM by incorporating the acceleration difference, and then get a new model called the full velocity and acceleration difference model (FVADM) as follow:

$$\begin{aligned} a_n(t) &= \kappa\left(V_{opt}\left(S_n(t)\right) - v_n(t)\right) + \lambda\dot{S}_n(t) \\ &\quad + \beta g\left(\ddot{S}_n(t-1), a_{n+1}(t)\right)\ddot{S}_n(t-1) \end{aligned} \tag{12}$$

With $\ddot{S}_n(t) = a_{n+1}(t) - a_n(t)$ is the acceleration difference between the preceding vehicle $n + 1$ and the following vehicle α. Function $g(\cdot)$ is to determine the sign of the acceleration difference term.

$$g\left(\ddot{S}_n(t-1),\ a_{n+1}(t)\right) = \begin{cases} -1, & \ddot{S}_n(t-1) > 0 \\ & \text{and } a_{n+1}(t) \le 0 \\ 1, & \text{others} \end{cases}$$

The main advantage of FVADM compared to previous models that can describe the driver's behavior under an urgent case, where no collision occurs and no unrealistic deceleration appears while vehicles determined by the previous car-following models collide after only a few seconds. In 2006, Zhi-Peng and Yui-Cai (2006) conducted a detailed analysis of FVDM and found out that second term in the right side of Eq. (11) makes no allowance of the effect of the inter-car spacing independently of the relative velocity. For that, they proposed a velocity-difference-separation model (VDSM) which takes the separation between cars into account and the dynamics equation becomes

$$\begin{aligned} a_n(t) &= \kappa\left(V_{opt}\left(S_n(t)\right) - v_n(t)\right) \\ &\quad + \lambda\Theta\left(\dot{S}_n(t)\right)\dot{S}_n(t)\left(1 + \tanh\left(C_1\left(S_n(t) - l\right) - C_2\right)\right)^3 \\ &\quad + \lambda\Theta\left(-\dot{S}_n(t)\right)\dot{S}_n(t)\left(1 - \tanh\left(C_1\left(S_n(t) - l\right) - C_2\right)\right)^3 \end{aligned} \tag{13}$$

The strong point of VSDM that the model can perform more realistically in predicting the dynamical evolution of congestion induced by a small perturbation, as well as predicting the correct delay time of car motion and kinematic wave speed at jam density. Lijuan and Ning (2010) suggested a new car following model based on FVDM with acceleration of the front car considered. With detailed study, they observed than when FVDM simulate the car motion all the vehicle accelerate until the maximal velocity and when the velocity reach maximal velocity the acceleration and deceleration appeared repeatedly. For that, they modified the Eq. (11) to take into account the influencing factor of the following car by adding up to Eq. (11) the leading acceleration. The dynamic equation of the system is obtained as

$$a_n(t) = \kappa\left(V_{opt}\left(S_n(t)\right) - v_n(t)\right) + \lambda\dot{S}_n(t) + \gamma a_{n-1}(t) \tag{14}$$

Where γ is the sensitivity, expressing the response intensity of the follow car to leading acceleration. They proved that their new model has certain enlightenment significance for traffic control, and is useful for establishment of Intelligent Transport Systems (ITS). Previous models used only one type of ITS information, either headway, velocity, or acceleration difference of other cars to stabilize the traffic flow. However, traffic flow can be more stable by introducing all the three types of ITS information. Based on this idea, Li et al. (2011) proposed a new car-following model takes into account the effects of the acceleration difference of the multiple preceding vehicles which affects to the behavior of the following vehicle just as the headway and the velocity difference, called multiple headway, velocity, and acceleration difference (MHVAD). Its mathematical description is following:

$$a_n(t)=\kappa\left(V_{opt}\left(\sum_{j=1}^{q}\beta_j S_{n+j-1}(t)\right)-v_n(t)\right)$$
$$+\lambda\sum_{j=1}^{q}\zeta_j\dot{S}_{n+j-1}(t)+\gamma\sum_{j=1}^{q}\xi_j\ddot{S}_{n+j-1}(t) \quad (15)$$

Taking q preceding vehicles and β_j, ζ_j, $\xi_j \in$ R, and $\beta_j \geq 0$, $\zeta_j \geq 0$, $\xi_j \geq 0$ are different weighting value coefficients, respectively. The β_j satisfies two conditions:

1. β_j is a monotone decreasing function with $\beta_{j\leq}$ β_{j-1}, Because the effect of the preceding vehicle to the current car reduces with the increase of the headway distance.
2. $\sum_{j=1}^{q}\beta_j=1$, $\beta_j = 1$ for q = 1, so as to ζ_j, ξ_j.

And β_j is defined as follows

$$\beta_j=\begin{cases}\dfrac{q-1}{q^j} & \text{for } j\neq q\\ \dfrac{1}{q^{j-1}} & \text{for } j=q\end{cases} \quad j=(1,2,\ldots,q)$$

The optimal velocity function $V_{opt}(.)$ used here as form:

$$V_{opt}(S_n(t))=\left[\tanh(S_n(t)-h_c)+\tanh(h_c)\right]v_{\max}/2 \quad (16)$$

Where $v_{\max}$ is the maximal speed of the vehicle, and h_c is the safe distance. The main advantage of MHVAD Compared with the other existing models is that the proposed model does not only take the headway, velocity, and acceleration difference information into account, but also considers more than one vehicle in front of the following vehicle. The model improved the stability of the traffic flow and restrains the traffic jams. Others category of car-following models inspired their idea to modify or to propose a new model via optimal velocity function Eq. (8).Among them, Jing et al. (2011) introduced a new optimal velocity function and modified the additional term of FVDM (Eq. (11)).In the first time, they proposed the modified full velocity difference model (MFVDM I) taking into account a new optimal velocity function proposed by (Helbing and Tilch, 1998) Eq. (18):

$$a_n(t)=\kappa\left(V_{opt}(S_n,v_n)-v_n(t)\right)+\lambda\dot{S}_n(t) \quad (17)$$

$$V_{opt}(S_n,v_n)=v_n^0\left(1-e^{-\frac{S_n-S(v_n)}{R_n}}\right) \quad (18)$$

where R_n is the range of the acceleration interaction and $S(v_n)$ is a certain velocity-dependent safe distance. The authors have improved that optimal velocity $V_{opt}(S_n,v_n)$ is a function of the vehicle distances and the velocity of the following vehicle which must satisfy three conditions:

1. $V_{opt}(S_n,v_n)$ is monotonically increasing to S_n and v_n
2. The larger values of $V_{opt}(S_n,v_n)$ will be beneficial to make FVDM fit with the field data better.
3. $\lim_{S_n\to+\infty}V_{opt}(S_n,v_n)\cong v_n^0$ and $\lim_{v_n\to v_n^0}V_{opt}(S_n,v_n)\cong v_n^0$ where v_n^0 is the desired velocity of the following vehicle.

For above analysis, they proposed a new optimal velocity function satisfies the above three conditions defined as forms

$$V_{opt}(S_n,v_n)=v_n^0\tanh\left(\frac{S_n-S(v_n)}{R_n}\right) \quad (19)$$

In second time, substituting the Eq. (19) into Eq. (17), and they get the second modified full velocity difference model (MFVDM II). Finally, they introduced a new optimal velocity function (Eq. (19)) and modified the additional term of Eq. (11) to get a new model called the improved full velocity difference model (IFVDM) defined as follow

$$a_n(t)=\kappa\left(V_{opt}(S_n,v_n)-v_n(t)\right)+\partial\dot{S}_n(t) \quad (20)$$

The additional term ∂ defined as a form:

$$\partial=\frac{1-\tanh\left(\frac{S_n-S(v_n)}{R_n}\right)}{\mu_n}$$

where μ_n is the reaction time of the addition term.

The author (Jing et al., 2011) pointed out that the new model can perform more realistically in predicting the correct delay time of vehicle motion and kinematic wave speed at jam density, as well as predicting the dynamical evolution of congestion induced by a small disturbance. Another car-following model proposed by Tian et al. (2011) incorporating a new optimal velocity model in Eq. (9), whose not only depends on the following distance of the preceding vehicle, but also depends on the velocity difference with preceding vehicle. As mentioned above, all of the previous models could not avoid collisions in the urgent braking situation. Based on this assumption, they proposed a new model called Comprehensive Optimal Velocity Model (COVM), its mathematical expression:

$$a_n(t)=\kappa\left(V_{opt}\left(S_n(t),\dot{S}_n(t)\right)-v_n(t)\right) \quad (21)$$

They suggested a new optimal velocity function $V_{opt}\left(S_n(t),\dot{S}_n(t)\right)$ as:

$$V_{opt}\left(S_n(t),\dot{S}_n(t)\right)=V_1\left(S_n(t)\right)+\alpha V_2\left(\dot{S}_n(t)\right) \quad (22)$$

with α is the reaction coefficient to the relative velocity and 0 < α < 1. They replaced the new function of Eq. (22) in Eq. (21), they get a new model expressed as follows:

$$a_n(t)=\kappa\left(V_1\left(S_n(t)\right)-v_n(t)V_1\left(S_n(t)\right)-v_n(t)\right)+\lambda V_2\left(\dot{S}_n(t)\right) \quad (23)$$

Taking $\lambda=\kappa\alpha$, $V_1\left(S_n(t)\right)$, is the same with that of the OVM (Eq. (8)) and $V_2\left(\dot{S}_n(t)\right)=L\tanh\left(C_3\left(\dot{S}_n(t)\right)\right)$. Where L, C_3 are constants. They improved that the unrealistically high deceleration will not appear in COVM, and the accidents in the urgent braking case can be avoid in COVM. Almost works has been reported the mechanisms of velocity difference, however, the relationship between space headway and safe distance in avoiding a collision is neglected. In real driving behaviors, keeping a safe distance reflects the drivers' driving intention and accordingly affects vehicle maneuvers. Based on this study, Liu et al. (2012) targeted at developing a new car-following model that takes the impact of a desired following speed and safe distance as part of driving behavior modeling.

According to the safe space headway theory, the safe distance $S_n^{safe}(t)$ can be defined as follows:

$$\begin{aligned} S_n^{safe}(t) &= (d_n + S_0 + L_{n+1}) - d_{n+1} \\ &= S_0 + L_{n+1} + v_n(t)T_n + \frac{\dot{S}_n(t)\left(v_n(t) + v_{n+1}(t)\right)}{2\alpha} \end{aligned} \quad (24)$$

where d_{n+1} denotes the braking distances of the leading vehicle, d_n is the braking distance of the following vehicle, S_0 is the minimum distance kept in static traffic, L_{n+1} is the length of the leading vehicle, $v_n(t)$ and $v_{n+1}(t)$ denote the speed of the two vehicles at time t, and is the acceleration.

They are modeled two effects by using force, $f_n^{(+)}(t)$ and $f_n^{(-)}(t)$ which represents the attractive force of acceleration and the retardant force of deceleration respectively. Then, they get a new car-following model as a form:

$$m_n a_n(t) = f_n^{(+)}(t) + f_n^{(-)}(t) \quad (25)$$

With $f_n^{(+)}(t) = m_n\kappa\left(V_{opt}(S_n(t)) - v_n(t)\right)$, and

$$f_n^{(-)}(t) = m_n\lambda\left(1 - S_n^{safe}(t) \Big/ S_n(t)\right)$$

Taking both the desired following speed of positive correlation and the safe distance of negative correlation, they name their model the cooperative car-following model (CCFM), replace the forces into Eq. (25) to get the CCFM expression

$$\begin{aligned} a_n(t) &= \kappa\left(V_{opt}(S_n(t)) - v_n(t)\right) \\ &+ \lambda\left(1 - S_n^{safe}(t) \Big/ S_n(t)\right) \end{aligned} \quad (26)$$

The main results of CCFM indicate that unrealistic deceleration and collisions can be prevented. Moreover, the CCFM averts negative velocity appearing in the COVM (Eq. (23)). In car-following approach, the efforts are more and more dedicated to the development of models with a high performance. In this regard, Xu et al. (2013) presented an asymmetric full velocity difference approach, in which take into account the effect of asymmetric acceleration and deceleration in a car-following. The most existing car-following models have not sufficiently taken the asymmetry of acceleration and deceleration behaviours into consideration. The authors modified the GFM (Eq. (10)) by extended to an asymmetric full velocity difference (AFVD) approach in which two sensitivity coefficients are defined to separate the model to positive and negative velocity. The AFVD model can be expressed as:

$$\begin{aligned} a_n(t) = \kappa\left(V_{opt}(S_n(t)) - v_n(t)\right) + \lambda_1 \mathrm{H}\left(-\dot{S}_n(t)\right)\dot{S}_n(t) \\ + \lambda_2 \mathrm{H}\left(\dot{S}_n(t)\right)\dot{S}_n(t) \end{aligned} \quad (27)$$

where H is the Heaviside function. They dedicated their efforts to calibrate λ_1 and λ_2 and they get the mathematical presentation.

$$\lambda_1 = \frac{1}{\tau_n'} e^{-\frac{\left(S_n - S(v_n)\right)}{R_n'}}$$

$$\lambda_2 = \frac{1}{\tau_n'''} e^{-\frac{\left(S_n - S(v_n)\right)}{R_n''}}$$

with τ_n''' and R_n' are two new parameters obtained during the mathematical derivation which need to be determined by field data. The purpose of the analysis of AFVDM pointed out that the positive velocity difference term is significantly higher than the negative velocity difference term, which agrees well with the results from studies on vehicle mechanics. In 2015, the authors (Xu et al., 2015) interested in taking the asymmetric characteristic of the velocity differences of vehicles and they proposed an asymmetric optimal velocity model for a car-following theory (AOV). They based on the assumption that the relationship between relative velocity and acceleration (deceleration) is in general nonlinear as demonstrated by actual experiments (Shamoto et al., 2011). They formulated FVDM (Eq. (11)) to get an asymmetric optimal velocity (AOV) car-following model as follows:

$$a_n(t) = \kappa\left[V_{opt}(S_n(t)) - v_n(t) + \dot{S}_n(t)\exp\left(-\mu\dot{S}_n(t)\right)\right] \quad (28)$$

The main advantages of AOV model are avoiding the unrealistically high acceleration appearing in previous models when the velocity difference becomes large, however, the asymmetry of AOV model between acceleration and deceleration depends nonlinearly on the velocity difference with the asymmetrical factor μ. Recently, Yi-Rong et al. (2015) proposed a new car-following model with consideration of individual anticipation behaviour. However, the effect of anticipation behaviour of drivers has not been explored in existing car-following models. In fact, they suggested a new model including two kinds of typical behaviour, the forecasting of the future traffic situation and the reaction-time delay of drivers in response to traffic stimulus. The main idea of this model is that a driver adjusts his driving behaviour not only according the observed velocity $v_n(t)$ but also the comprehensive anticipation information of headway and velocity difference. The dynamics equation is as follows

$$a_n(t) = \kappa\left(V_{opt}(S_n(t + p_1\tau)) - v_n(t)\right) + \lambda\dot{S}_n(t + p_2\tau) \quad (29)$$

Where $S_n(t+p_1\tau)$ denotes the driver's anticipation information of headway at time $t+p_1\tau$. $\dot{S}_n(t+p_2\tau)$ represents the anticipation information of the velocity difference at time $t+p_2\tau$. The variables $p_1\tau$ and $p_2\tau$ denote the time interval during which the headway and velocity difference information are anticipated, and variables p_1, p_2 are the anticipation coefficient corresponding to individual behavior in headway and velocity difference, respectively. Making the Taylor expansion of the variables $S_n(t+p_1\tau)$ and $\dot{S}_n(t+p_2\tau)$ and neglecting the non linear terms yields the following equation:

$$S_n(t+p_1\tau)=S_n(t)+\dot{S}_n(t)p_1\tau$$

$$\dot{S}_n(t+p_2\tau)=\dot{S}_n(t)+\ddot{S}_n(t)p_2\tau$$

Then, they calculate the optimal velocity $V_{opt}(S_n(t+p_1\tau))$ and they get it as a form:

$$V_{opt}(S_n(t+p_1\tau))=V_{opt}(S_n(t)+\dot{S}_n(t)p_1\tau)$$

The authors improved that the effect of individual anticipation behavior has an important influence on the stability of the model and this effect should be considered in the modeling of traffic flow. In fact, they suggested including other factors which have a great effect on individual anticipation behavior, such as the size of vehicles, the age, the experience, and the physical fitness level of drivers, as well as the environment of the road and so on. We also invested in car-following approach and we proposed a modified full velocity model (MFVDM). we extend the FVDM (Eq. (11)) by incorporating the new optimal velocity function obtained by the combination of optimal velocity function (Eq. (8)) with the weighting factor. We introduced the weighting factor of the optimal velocity that depends on the ratio of the relative speed to headway that is the opposite of the inverse of time to collision (TTC) expressed as form (Lazar et al., 2015)

$$TTC^{-1}=\frac{\dot{S}_n(t)}{S_n(t)}$$

The goal of the weighting factor is to obtain model more reactive on braking state. This reactivity is based on the excess of follower speed in comparison to that of the leader. The weighting factor is expressed as form

$$W(\dot{S}_n(t),S_n(t))=\left[A*\left(1+\tanh B\left(TTC^{-1}+C\right)\right)\right] \quad (30)$$

The equation of the new optimal velocity function is

$$V_{opt}^{new}(S_n(t),\dot{S}_n(t))=V_{opt}(S_n(t))*W(\dot{S}_n(t),S_n(t)) \quad (31)$$

The dynamic equation of MFVDM is described as follows

$$a_n(t)=\kappa\left(V_{opt}^{new}(S_n(t),\dot{S}_n(t))-v_n(t)\right)+\lambda\dot{S}_n(t) \quad (32)$$

We improved that the proposed MFVD model react better and demonstrate that the new optimal function introducing the weighting factor has the best effect on braking state compared as with others car following models such as OV,GFM , FVD models (Lazar et al., 2015).

4 Conclusion

The microscopic car-following model is a favorite type of traffic flow theory to describe the individual behaviour of drivers. In this paper, we presented the most car-following model well-known the optimal velocity (OV) model, which has successfully revealed the dynamical evolution process of traffic congestion in a simple way. Thereafter, inspired by the OV model, some new car-following models were successively put forward to describe the nature of traffic more realistically.

Some were extended by incorporating a new optimal velocity function or introducing multiple information of headway or velocity difference, or acceleration difference, whereas others considered the individual anticipation behaviour. We have reviewed the existing car following models and the recent one and giving their drawbacks and advantages to help the research to develop the strong car-following model which avoid the collision and interpreted the traffic flow in a real manner.

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