Abstract
SCGE models are ideal tool for modeling socioeconomic effects in a spatial manner. The aim of the research is to establish a commonly usable and expandable model application integrating the main attributes of the transport sector for promoting decision-making, or forecasting the effects of the suspected interventions. Therefore authors has established a mathematical environment that describes the effect of transport on spatial economics.

Keywords
Spatial, Computable, General, Equilibrium, SCGE, transportation, modal-shift, decision-making

1 Introduction
Transportation and mobility has become an integral part of our everyday life. Immediate, comfortable and flexible movement is a basic need of humanity. As a side effect of this growing demand for mobility and motorization the conventional transport networks and modes are unbalanced (Torok and Zoldy, 2010). The excessive spread of passenger cars instantly leads to more serious road congestions, accidents, noise and GHG emission (Szendrő and Török, 2014). Spatial analysis of transport system with SCGE models can show the socioeconomic equilibrium. This study aims to develop an SCGE model framework which considering the transport sector and its regularities as a fundamental element of economy while it can determine and forecast the optimal equilibrium point in order to define the proper investments or required changes in the modelled environment.

2 Method
The spatial computable general equilibrium methodology from a wider approach can be considered as the evolved modeling method of the regular I/O and CGE models. SCGE models are representing the socioeconomic equilibrium in a computable way, while also considering those geographical allocations and spatial distributions.

The current modelling framework is going to analyse the different interventions and developments in order to enhance the transport network for a smoother and more environmental friendly and efficient mobility, or to promote sustainability, modal rearrangement and economic growth (indirect effects included).

The methodological bases are originating in the Anas-Krugman-Fujita model (Anas, 1992; Fujita and Krugman, 1995; Fujita and Nobuaki, 2001). In line with their geographic economy model an elementary base model has been developed where spatiality differs from the earlier used Samuelson’s “iceberg” transport approach (Samuelson, 1997). In the base model several simplification has been made, which can be extracted in the later phases regarding the demands. The current characteristics of the model are as below.
The investigated geographical environment is divided into geographical regions. In each region there are consumers, whom could be the labour of any region regarding the needs, the territorial specifications and elasticity of resources are known. From the consumption side the model considers that consumers are meant to maximize their utility. Each consumer is motivated to spend the salary on every regions’ product basket and maximize their utility. There are no savings in the base system. Transport costs of goods are covered by the individuals by consumers. In the simplified system transport incomes are associated for one comprehensive economic actor who spends his/her incomes on products, regarding the share of costumer elasticity. The limit of the consumption is the income of the individuals. From the production side the firms are meant to raise their market share through the production to increase their profits. The limit of production is the resource constrain. Each product basket of the different regions is accessible for everyone.

Figure 1 shows the conceptual flowchart of the model framework operations.

After setting up the main modelling equations the beginning step of the modelling method is the initialization. In this part the model framework collecting the data from the available data sources, and setting up the main parameters by the calibration functions (see later (11), (12), (13)). Initial transport matrix of the model is also generated in this process. Preparing transport matrix is a combined methodology, as it is considering different influences in an expandable way for further extensions. Modal decisions are deduced by a multi-factorial weighting function (4), (5), taking into account the individual main decision factors (such as cost, time and comfort level) like Awad-Núñez et al. (2015). Afterwards the share of modal distribution defined it is also going to be used as a weighting factor for the matrix generating. External costs still didn’t included in this phase of model, although it is also in the aims of the future developments.

When the initialization is completed the model is eligible for processing the input information in order to investigate the effects of the different interventions and investments. Input effects are stimulating the full model (the base factors of the calibrating included). By giving an output data set with the expected values, the model makes the system proper for decision making support, and with further data processing and visualization it is also appropriate for a more integrated displaying of result matrices.

The utility maximization of the consumers is described with the classical and widely used most general CES production function Cobb-Douglas (Saito, 2011; Torok et al., 2014; Török and Török, 2014) utility function (1):

$$U_j = \prod_i \left( X_{ij}^{a_{ij}} \right)$$

where

- $i \in [1, r]$,
- $j \in [1, r]$,
- $r$ number of regions,
- $U_j$ consumer utility in the “$j$th” region,
- $X_{ij}$ consumption of the “$i$th” good in the “$j$th” region,
- $a_{ij}$ elasticity of goods $\sum_j a_{ij} = 1$.

The constraining factor of the consumer utility (Scholz et al., 2015), is the consumers’ yield can be spent on products (2) (for the $j$th region):

$$\sum_i \left( W_{ij} - T_{w_{ij}} \right) \cdot M_{ij} = \sum_j \left( P_{ij} + T_{ij} \right) \cdot X_{ij}$$

where

- $W_{ij}$ average wage of the “$i$th” region workers (“$i$th” good’s producer),
- $T_{w_{ij}}$ transport cost of labour movement from the “$i$th” region to the “$j$th” region,
- $M_{ij}$ labours of the “$i$th” region working in the “$j$th” region,
- $P_{ij}$ Product price of the “$i$th” region’s product basket,
- $T_{ij}$ transport cost of consuming “$i$th” region’s product in the “$j$th” region.

Transport related spendings are also circulated back to the system. It can be deemed as transporters are also consumers in a territorial share (3):

$$X_{tr_{ij}} = \sum_j \left( \left( T_{w_{ij}} \cdot M_{ij} \right) + \left( T_{ij} \cdot X_{ij} \right) \right) / P_{ij}$$

where

- $X_{tr_{ij}}$ stands for the transport sector’s consumption.

Working related transport costs are aggregated by an embedded function indirectly considering the modal decision determined by the unit costs, comfort level and travelling times, as shown in (4) and (5).
The theoretical conception of SCGE model for transport decision making

\[ \text{MS}_y = \frac{\{s_{ij}/CT_{U_{priv}}\} \cdot \{v_{publ_{ij}}/CV_{U}\} \cdot A_{CL_{priv}}}{\{s_{ij}/CT_{U_{publ}}\} \cdot \{v_{publ_{ij}}/CV_{U}\} \cdot A_{CL_{publ}}} \]  

(4)

and

\[ T_{W_{ij}} = MS_{publ_{ij}} \cdot CT_{U_{publ}} \cdot s_{ij} + MS_{priv_{ij}} \cdot CT_{U_{priv}} \cdot s_{ij} \]  

(5)

where

- \( MS_i \) Modal split function (private transport’s attractiveness / public transport’s attractiveness),
- \( CT_{U} \) Transport unit cost,
- \( CV_{U} \) Velocity’s unit cost (or the aggregated cost of transit time),
- \( s \) distance (integrated from the distances between connected counties weighted by the transport volumes),
- \( v \) velocity,
- \( A_{CL} \) attractiveness coefficient by comfort level.

The constrained maximization problem of (1) and (2) is treated with the Lagrange constrained extremum methodology. From the production side the firms (producers) are willing to maximize their profit (thus the market share, etc.). As model based on general equilibrium it can be presumed that we are at the market optimum, where the supply (\( Q \)) and demands (\( X \)) are equal (6).

\[ Q_x = \sum_j X_x \]  

(6)

The income from the products (in each region summarized by product types) covers the salary of the producer regions’ workers (7).

\[ \sum_j P_x \cdot X_{y_{ij}} + P_x \cdot X_{nl} = \sum_j W_i \cdot M_{ij} \]  

(7)

On the production side the available resources are the constraining factors (8).

\[ \sum_j X_{y_{ij}} = B \cdot \prod_j (M_{ij} \cdot \delta_{ij}) \]  

(8)

where

- \( B \) production coefficient,
- \( \delta_{ij} \) elasticity of resources \( \sum_{ij} \delta_{ij} = 1 \).

After the constrained Lagrange optimization (9) group of equations can be eventuated.

\[ \frac{\delta_{ij}}{M_{ij}} - \frac{\delta_{ij}}{M_{ij}} \]  

(9)

After the optimizations and calibration the model can be described with the (2), (3), (6), (8), (9) and (10) equation system.

For the calibration of the coefficients of the system (10), (11) and (12) can be used.

\[ B_i = \frac{\sum_j X_{x_{ij}}}{\prod_j (M_{ij} \cdot \delta_{ij})} \]  

(10)

\[ a_{ij} = \frac{(X_{y_{ij}} \cdot (P + T_{ij}))}{\sum_j (X_{y_{ij}} \cdot (P_i + T_{ij}))} \]  

(11)

\[ \delta_{ij} = \frac{(W_i - T_{W_{ij}}) \cdot M_{ij}}{\sum_j ((W_i - T_{W_{ij}}) \cdot M_{ij})} \]  

(12)

In the current model the number of regions is 7 (r=7 for the 7 main regions of Hungary: Central- and Northern Hungary; Northern-, and Southern Great Plains; Central-, Southern- and Western Transdanubia).

Transport related data gathered and processed in a separate sub-system where modal decisions and transport matrixes are deduced. The model is considering the national origin destination matrixes of different vehicle types in a macro-regional based territorial split (with aggregated data content).

After the declaration of this modal split (4), by the weighting of transport costs (5) an aggregated origin-destination matrix can be generated. SCGE model uses these output matrixes to determine where the planned intervention could have the most benefits.

This model system has been implemented in MATLAB environment in order to simulate the interaction between investigated parameters.

### 3 Results

By the adaptation of the classical SCGE methodology a transport related multiregional decision making support model has been developed, based on statistic and measured data. In the modeling framework transport segment is represented in a separated way and regional fragmentation can be freely extended (by the raising or reducing the number of regions an algorithm can generate the model equations).

The developed model is able to forecast the estimated socio-economic effects (such as daily traffic of workers, improvement in production, or the changes in the wages and consumption) of a relevant transport related intervention, infrastructure development, maintenance etc.

The modeling framework is also able to forecast, how could a development affect the modal attractiveness of the public or private transport modes between the regions, or globally determine the change in modal decision.

### 4 Discussion

Transport network and that’s management are the heart of our economy, while transport has become a base demand of the everyday life. Innovative and state-of-art solutions are indispensable to take successfully the occurring obstacles and the grooving needs for smooth and proper mobility.
As it has been shown, setting up an adequate modal split is essential for sustainability and also for raising the effectiveness of transportation. Further development of the system could address the more detailed modal rearrangement, to show relatively where and how could be raised the transport performance. Which solution provides the best external and additional effects or the most added value.

**List of abbreviation**

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<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>CGE</td>
<td>Computable General Equilibrium</td>
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<tr>
<td>CES</td>
<td>Constant Elasticity of Substitution</td>
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<tr>
<td>GHG</td>
<td>Green House Gas (for instance: CO$_2$, CH$_4$, etc...)</td>
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<tr>
<td>I/O</td>
<td>Input-Output</td>
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<tr>
<td>SCGE</td>
<td>Spatial Computable General Equilibrium</td>
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**References**