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RESEARCH ARTICLE

ARX Linear Modeling of Hot Mix Asphalt Compaction

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Abstract

The compaction of the asphalt layer is a fundamental operation in pavement construction but, at the same time, its control is very difficult. The consequences on costs, on execution time and on duration of the pavement are very relevant and, for that reason, we wanted to prepare a predictive model able to minimize the attempts, often unsuccessful, which the executor is forced to make in the early stages of the construction.

Due to a very large number of variables, we could not control the phenomenon with any known physical models; so, for the purpose of this research paper, we preferred to make use of a so-called black-box linear model, calibrating it with an appropriate procedure of trials and errors.

The results achieved with this model have made it possible to predict the value of the material density with a reliability of 88% that, for the number of involved variables, can be considered a satisfactory target.

With this procedure and knowing the boundary conditions (characteristics of the roller and the material), it is possible to arrange the compaction very quickly, achieving a good compromise between layer density and execution times.

Keywords

building materials, organizations, construction equipment, modeling, construction operations, estimation

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1 Introduction

Compaction is the process by which the desired density of the asphalt material as specified in the design can be obtained by the action of rollers. (Scherocman and Martenson, 1984; Scherocman, 1984; Geller, 1984; Brown, 1984; Bell et. al., 1984; Roberts et al., 1996). If the design prescriptions are properly observed, there is a better answer quality of the surface, a higher resistance to plastic deformation, fatigue, aging and cracking (Hughes, 1984; Hughes, 1989; Laurinavičius and Oginskas, 2006; Radziszewski, 2007).

In these years, advances in computation and the introduction of modern measure instruments evidenced a particular complexity of this problem (Khan et al., 1998; Dubois et al., 2010; Kavussi and Hashemian, 2011). In fact now, in a given scenario, it is possible to measure many quantities (Leng et al., 2011) that have to be controlled in order to configure the compaction operations in an optimal way as well as in accordance with economic constraints (Krishnamurthy et al., 1998; Manik et al., 2008).

In particular, the most important features that influence the density (Hildebrand et al., 2008) are attributable to environmental factors (as ground and air temperature, wind speed, solar flux), mix properties (as aggregates and asphalt characteristics) and construction factors (as rollers type and speed, number of passes, lift thickness). However, the most analyzed is always the temperature of the material (Attaelmanan et al., 2011; Sanchez-Alonso et al., 2011), because when temperature is below a particular limit (called cessation temperature), vis-à-vis other conditions, it is almost impossible to reduce further air voids (Jordan and Thomas, 1976; Hughes, 1989; Roberts et al., 1996; TRB, 2006; Airey et al., 2008). After this point, only the surface smoothness can be improved; there can be no increase of density and, therefore, there is no improvement of the pavement performance.

It is necessary, therefore, to build an analytical tool in order to be able to modify eventually the operations on site in a short time. As a matter of fact, traditional laboratory tests, performed on core samples taken during rolling, are certainly reliable but their preparation is not speedy enough to maintain optimal efficiency of the construction (Praticò and Moro, 2011).

Generally, analysts followed two consecutive paths: a theoretical method, based on models (Koneru et al., 2008; Yusoff et al., 2011) and an experimental approach, necessary to validate the correctness of the first step (Commuri and Zaman, 2008; Aflaki et al., 2011). For this purpose, pattern recognition techniques are very well suited; they were developed for the treatment of large quantities of data recorded by digital instruments and applied to sort, classify or extract data useful for understanding the problem investigated (Bosurgi et al., 2010). Among the advantages, we can also mention the removal of irrelevant dimensions and the consequential cost reduction for data acquisition. With a smaller data set, there is not only a decrease in the cost but also an improvement in the performance of the model (Duda et al., 2001; McLachlan, 2004; Pellegrino, 2011; 2012; Ripley, 2005; Theodoridis et al., 2006; Webb, 2002).

The goal of this study is to prepare an analytical model that is able to predict density of the asphalt layer based on the knowledge of some variables acquired by ordinary measuring instruments (Bosurgi and Trifirò, 2005a; 2005b; 2006; Bosurgi et al., 2011).

Since we did not know the physical law that governed the observed phenomenon, we applied a so called black-box linear model, easily configurable after collecting the initial data set and without worrying about assessing any dependencies among variables.

The procedure is quite simple and permits modification eventually, of only a few quantities during the compaction operations (Jang, 1993; Guler and Ubeyli, 2005; Mon, 2007; Gu and Oyadiji, 2008; Tahmasebi and Hezarkhani, 2010).

2 Method

During the construction phase, the target is to achieve a degree of compaction required by the designer. Unfortunately, there are no analytical equations that allow to simulate the real phenomenon and to calibrate, also with a certain approximation, the main factors involved. Knowledge of dynamic features of the roller, environmental conditions and construction characteristics does not permit having a relationship with the energy defined in the laboratory (for example, by gyratory compactor) because there are too many variables that complicate the phenomenon.

Usually the problem is solved by unloading a laying test, changing empirically the value of some factors such as roller speed, number of passes, vibration amplitude and frequency, length of paving, etc. In the material so compacted, a few cores have to be extracted and carried into the laboratory for measuring the density. The best result will give the general guidelines for the execution.

This procedure implies some issues regarding quality of the pavement and time diseconomy. First of all, the laying test should be shot down, since some sections could have densities below the prescribed limit. The second drawback concerns the time and cost required to execute the paving, to extract and to test numerous cores within a short time. In this case the executor, on the basis of laboratory results, will set out certain factors in order to achieve the desired result, but without understanding which of these is really more important.

As a solution to these problems, the present paper proposes the predisposition of a model which is able to predict the hot mix asphalt density independent of a predetermined analytical structure and, on the contrary, dependent only on the measured data.

2.1 The data set

The proposed method is of general validity since there are no constraints either on the type of compacting equipment or work organization, or on the size of the data set. In this study, we surveyed 7 input and 1 output variables in order to test the correctness of the methodology; however, it is possible to work with different quantities. As for the density of the hot mix asphalt, an electrical density gauge measured pavement compaction by means of its dielectric constant. This instrument introduces a weak current through the material, which creates an electrical sensing field. The response of this electrical sensing field depends on the pavement's complex impedance (consisting of the pavement's composite resistivity and dielectric constant) thereby gauging the pavement density.

The features surveyed in situ during the execution of lying were:

- Sp: Roller Speed in km/h.
- Le: Length of the lying in m.
- Ti: Time in seconds. It is relative to the elapsed time during a single pass, and it is, therefore, a partial measure.
- PR: Progressive in seconds. It takes into account the cumulative time during all the passes up to the time of the record and it is, therefore, a progressive measure.
- Pa: Passes number.
- Wa: Water in %. It is the water needed to avoid the pick up on the drum of the hot material.
- Te_M: Material temperature in °C.
- γ: Density of the compacted material in kN/m^3.

Other parameters, (for example roller weight, air temperature and layer thickness) although detected, were assumed constants because of their modest variability and would not be processed numerically in the later stages of the analysis.

2.2 The choice of the variables most representative

Given the nature of the problem, we opted for a model with only three input variables. This decision was, of course, subjective but sufficiently acceptable for the following reasons:

I. A model with only two variables is probably not sufficiently representative of a complex situation such as that analyzed.

II. A model with four variables can best describe the complexity of the phenomenon but is more difficult to calibrate and has more survey costs.

Therefore, assuming three as the number of input variables, we used a procedure of exhaustive search based on the construction of as many models Neuro-Fuzzy as are the possible combinations of all the variables in groups of 3.

The models were trained for an epoch and the results in terms of Root Mean Squared Error (RMSE) were sorted in ascending order and reported in a following table (section Results, Table 2). In general, the output is influenced by those results which exhibit the lowest RMSE after an epoch of training, as it is reasonable to assume that they have a tendency to have lower RMSE also with large numbers of training. The advantages of this procedure, widely known in the literature, can be summarized in the following list:

- Removal of all the useless or marginal inputs.
- Organization of a model more simple and reliable.
- Reduction of survey, model elaboration and construction costs.

2.3 The prediction model

Our aim was to build a mathematical model of real dynamic system from the knowledge of experimental measurements only. If the dynamic input-output system and the eventual error of identification can be described by a finite number of numeric parameters, as in this research, the identification is said to be parametric (Ljung, 2010).

As already said, we did not know its analytical structure and we started with the simplest structures (a linear model with few parameters) calibrated step by step with a procedure of "trial and error". The results achieved and the subsequent validation process, not reported here for the sake of brevity, led us towards the choice of an ARX model which showed a good fitness to the observed phenomenon and permitted rejection of the more complex nonlinear structures (Ljung, 2010).

In this kind of problems, the surveyed data are generally written in the following form:

$$Z^N = \{u(1), y(1), \ldots, u(N), y(N)\} \tag{1}$$

For a linear ARX model, based on the difference equations descriptions, it is possible to represent the relationship between the input u(t) and the output y(t) of a real system at time t and in discrete time with the following linear difference equation (Lyzell et al., 2011):

$$y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) = b_1 u(t-1) + \ldots + b_m u(t-m) \tag{2}$$

The output at t time is a linear combination of the input at t time and of the previous output and inputs. If the sampling interval is one second, as in our case, we can determine the next output value from the previous observations:

$$y(t) = -a_1 y(t-1) - \ldots - a_n y(t-n) + b_1 u(t-1) + \ldots + b_m u(t-m) \tag{3}$$

To simplify the equation we can introduce the following vectors:

$$\theta = [a_1, \ldots, a_n,\ b_1, \ldots, b_m]^T \tag{4}$$

$$\phi(t) = [-y(t-1) - \ldots - y(t-n) + u(t-1) + \ldots + u(t-m)]^T \tag{5}$$

And therefore:

$$y(t) = \phi^T(t)\theta \tag{6}$$

The output of the model can be inferred from the observation of the real system through the following expression (Ohlsson et al., 2010):

$$\hat{y}\left(t|\theta\right) = \phi^T(t)\theta \tag{7}$$

To calculate θ the measured outputs have to be as near as possible to the calculated values $\hat{y}(t \mid \theta)$ by means of the least squared method:

$$\min_{\theta} V_N(\theta, Z^N) \tag{8}$$

With

$$V_N(\theta, Z^N) = \frac{1}{N}\sum_{t=1}^{N}\left(y(t) - \hat{y}(t \mid \theta)\right)^2 = \frac{1}{N}\sum_{t=1}^{N}\left(y(t) - \phi^T(t)\theta\right)^2 \tag{9}$$

The value of θ that minimize V_N is:

$$\hat{\theta}_N = \arg\ \min_{\theta} V_N(\theta, Z^N) \tag{10}$$

The minimum can be easily found by setting the derivative to zero, because V_N is quadratic in θ.

$$0 = \frac{d}{d\theta} V_N(\theta,\ Z^N) = \frac{2}{N}\sum_{t=1}^{N}\phi(t)\left(y(t) - \phi^T(t)\theta\right) \tag{11}$$

Followed by:

$$\sum_{t=1}^{N}\phi(t)\ y(t) = \sum_{t=1}^{N}\phi(t)\phi^T(t)\theta \tag{12}$$

Or

$$\hat{\theta}_N = \left[\sum_{t=1}^{N}\phi(t)\ \phi^T(t)\right]^{-1}\sum_{t=1}^{N}\phi(t)y(t) \tag{13}$$

When the vectors φ(t) are defined, it is possible to find the solution quickly by numerical software.

The previous equations provide a model based on a linear regression with the regression vector φ(t). The component of φ(t) are called regressors with reference to the fact that y(t) is determined by returning to φ(t). Therefore, models where the regressor vector φ(t) contains old values of the predicted output y(t) are said auto-regressive, justifying, in such a way, the acronym ARX (Auto-Regression with eXtra inputs).

We verified that a model composed of a linear differential equation with an added error component ε(t) in terms of random noise, is sufficient to explain the dynamics hypothesized in this study.

2.4 Model calibration and validation

The first important step in the procedure involved the identification and calibration of the model parameters. To meet this requirement we estimated some orders and delays of the first attempt, in agreement with the complexity of the investigated scenario. Later it was possible to improve the previous estimates for successive approximations through a process of "trial and error", evaluating the advances with the quantification of some indexes. Among these, we can mention the fitness index between the output generated by the model and those measured and the Akaike's Final Prediction Error (FPE) which, in essence, minimizes the percentage of the ratio between the variance of the prediction error and the output unexplained variance in percent. In detail, the fitness index is defined as:

$$FIT = \left\{ 1 - \frac{J_s}{1/N \sum_{t=1}^{N} \left[y(t) - \bar{y} \right]^2} \right\} \cdot 100 \tag{14}$$

Where:

$$\bar{y} = \frac{1}{N} \sum_{t=1}^{N} y(t) \tag{15}$$

is the sample mean of the data. A FIT=100% denotes a perfect reproduction by the model and therefore corresponds to the ideal case in which the data were generated from own transfer function identified by the model G(z, $\hat{\Theta}_N$).

The Akaike's Final prediction Error (FPE) is defined by the following equation:

$$FPE = V\left(\frac{1 + d/N}{1 - d/N} \right) \tag{16}$$

Where V is the Loss Function, d is the number of estimated parameters (n+m), and N are the values in the estimation data base.

The residuals (also called prediction errors) are the differences between the output predicted by the model and the one contained in the data validation and, therefore, represent that part of validation data that was not described by the model. In short, the residuals represent those observations that the model cannot replicate and, for this reason, their statistical properties represent a good indicator about the correctness of the model. In particular, we can evaluate the autocorrelation function of residual and the cross-correlation function between the error function and the input variables:

$$\hat{R}_{\varepsilon u}^{N}(\tau) = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} \varepsilon(t+\tau) u(t) \;\; with - M \le \tau \le M \tag{17}$$

$$\hat{R}_{\varepsilon}^{N}(\tau) = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} \varepsilon(t+\tau) \varepsilon(t) \;\; with 1 \le \tau \le M \tag{18}$$

where M is an integer greater than 1 and typically M << N - τ.

The quantities expressed by (17) and (18) have to be small, since the prediction residual should not depend on the particular data set used and it should not be correlated with the input.

3 Results

3.1 Application

We applied the previous method to the maintenance of an HMA top layer in a rural road located near the town of Messina (Italy). The cross section of the road is constituted by two lanes of 4 m each, of which the first one was used by the workers and operative vehicles and the other was used for paving. The asphalt plant was near the construction site and this allowed the material to have temperatures high enough to ensure a reasonably sufficient time for compaction.

Paving occurred with a floating screed, for a width of 4.00 m, a nominal thickness of the layer of 3 cm and speed of 0.45 km/h. The floating screed was immediately followed by a roller with double metallic drum, a weight of 11.300 kN, a width of 1.950 m, which has compacted the material in static mode with a speed of 2 km/h.

The measures were recorded in 8 sections for each pass of the roller. The final number of passes is dependent on the level of compaction achieved and when this result was found satisfactory or material temperature was found too low, the number of passes was stopped. In this way, the database of 8 columns (7 input variables and one output), with 72 rows (observations) was reported in Table 1.

The output of the model is represented by the variable γ, i.e. the density of asphalt material expressed in kgN/m^3, while we chose the most representative input on the basis of the results provided by the Neuro-Fuzzy procedure, briefly illustrated in the section Method.

As already mentioned, the procedure Neuro-Fuzzy performs an exhaustive search among the most influential input, privileging the combinations with the smallest RMSE (root mean square). For this purpose, our analysis suggested the following variables:

1. TI: Elapsed time in seconds during the single pass of the roller.
2. Wa: Water in %. It is the water needed to avoid the pick up on the drum of the hot material.
3. Te_M: Material Temperature in °C.

In the Table 2 we reported the details of the combinations in order of significance:

Table 1 Data set regarding all the observations

	Speed (Km/h)	L (m)	Time (s)	Progr. (s)	Passes (n°)	H_2O %	T°C mat	γ (kg/m³)
1	0.6	60	360	360	0	4.88	152.02	1975.2
2	2	60	108	468	2	5.32	141.2	1998.4
3	2	60	108	576	4	4.96	132.98	1997.2
4	2	60	108	684	6	5.12	123.92	2013
5	0.6	50	300	300	0	5.38	122.74	1971.2
6	2	50	90	390	2	5.5	108.6	2002.8
7	2	50	90	480	4	5.16	100.32	1999.4
8	2	50	90	570	6	5.32	94.74	2026
9	0.6	65	390	390	0	5.2	149.94	1963.4
10	2	65	117	507	2	4.96	132.76	1989
11	2	65	117	624	4	4.92	127.1	1999.4
12	2	65	117	741	6	4.78	121.5	2005.2
13	0.6	60	360	360	0	3.64	147.46	1898
14	2	60	108	468	2	3.66	120.8	1965.8
15	2	60	108	576	4	3.42	107.58	1985.4
16	2	60	108	684	6	3.5	89.08	2000.4
17	2	30	54	738	7	3.38	87.02	1993.6
18	0.6	70	420	420	0	3.66	130.14	1900.6
19	2	70	126	546	2	3.72	113.42	1962.4
20	2	70	126	672	4	4.36	89.86	1995.6
21	2	70	126	798	6	4.18	79	2002.6
22	2	35	63	861	7	3.86	75.32	2004.8
23	0.6	60	360	360	0	3.96	157.3	1908.4
24	2	60	108	468	2	4.22	138.96	1928.6
25	2	60	108	576	4	5.82	124.14	1945.4
26	2	60	108	684	6	4.36	106	1972.4
27	2	30	54	738	7	3.86	89.56	1991.8
28	0.6	60	360	360	0	3.58	136.98	1901.4
29	2	60	108	468	2	5.02	102.96	1965.4
30	2	90	162	630	5	4.3	101.5	1953.8
31	2	90	162	792	8	3.78	90.92	1974.8
32	2	66	119	119	2	3.9	119.02	1946.6
33	2	66	119	238	4	3.72	103.28	1973
34	2	66	178	416	7	3.4	80.44	1960.6
35	2	33	59	475	8	16.64	40.2	2009.8
36	0.6	60	360	360	0	3.2	139.32	1911.2
37	2	60	108	468	2	3.22	113.22	1954.2
38	2	60	108	576	4	3.06	105.64	1937.4
39	2	60	108	684	6	3.62	105.64	1971.6
40	2	60	108	792	8	3	81.2	1986
41	0.6	64	384	384	0	3.52	146.26	1880.8
42	2	64	115	499	2	3.58	95.56	1950
43	2	64	115	614	4	3.16	87.82	1975.2
44	2	64	115	730	6	3.8	79.86	1993.2
45	2	64	115	845	8	2.2	70.8	1996.8
46	0.6	58	348	348	0	3.46	134.42	1893.8
47	2	58	104	452	2	3.72	115.34	1969.2
48	2	58	104	557	4	3.84	97.12	1974.6
49	2	58	104	661	6	3.78	83.4	1973.2
50	2	58	104	766	8	4.74	67.48	1999.2

	Speed (Km/h)	L (m)	Time (s)	Progr. (s)	Passes (n°)	H_2O %	T°C mat	γ (kg/m³)
51	2	58	108	874	10	4.68	67	2001.4
52	0.6	65	390	390	0	4.08	156.74	1865.4
53	2	65	117	507	2	3.82	132.3	1969.2
54	2	65	117	624	4	3.48	110.76	1977.2
55	2	65	117	741	6	4.98	101.32	2001
56	2	65	117	858	8	4.12	89.56	2007
57	0.6	75	450	450	0	3.44	119.64	1873.2
58	2	75	135	585	2	3.14	114.64	1945.2
59	2	75	135	720	4	3.74	112.86	1979.4
60	2	75	135	855	6	3.76	97.7	1990.8
61	2	75	135	990	8	4.22	99.8	2002
62	2	75	108	1098	10	4.68	67	2001.4
63	0.6	60	360	360	0	3.1	131.74	1885.6
64	2	60	108	468	2	3.16	105.08	1957
65	2	60	108	576	4	3.1	96.18	1982.2
66	2	60	108	684	6	3.12	85.38	1991
67	2	60	108	792	8	2.28	78.22	2001
68	0.6	68	408	408	0	3.68	142.48	1892
69	2	68	122	530	2	4.28	113.98	1967.4
70	2	68	122	653	4	3.76	100.64	1975.2
71	2	68	122	775	6	3.6	84.34	1977
72	2	68	122	898	8	4.46	78.68	1995.2

Table 2 Evaluation of three out of seven by means of Neuro-Fuzzy analysis. The most influent features are the group Time, Water and Material Temperature and, in order to avoid error differences, without a problem of over fitting.

Input	Training	Checking
Ti-Wa-Te_M	0.0061	0.0176
Ti-Wa-Pr	0.0072	0.0114
Ti-Wa-Pa	0.0073	0.0089
Ti-Wa-Te_A	0.0077	0.0112
Ti-Wa-Th	0.0082	0.0196
Ti-Wa-Le	0.0087	0.0257
Ti-Wa-Sp	0.0095	0.0114

At the beginning, we applied some typical filter, as mean and trend remove and our model was prepared on the basis of this modified data set. This configuration of the measured data can be evaluated in the Figs. 1, 2 and 3:

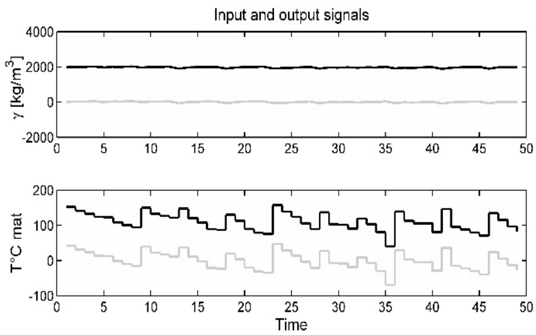

Fig. 1 Trend of the density of the asphalt materials γ (top) in relationship with the Temperature of the material.

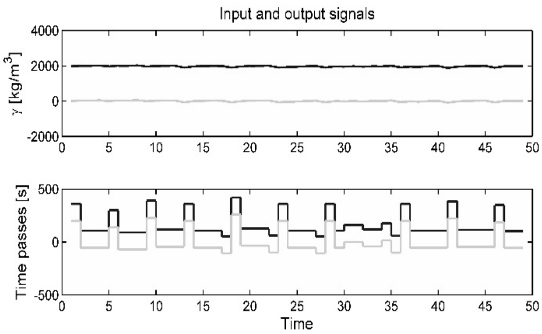

Fig. 2 Trend of the density of the asphalt materials γ (top) in relationship with the elapsed time during the single pass of the roller.

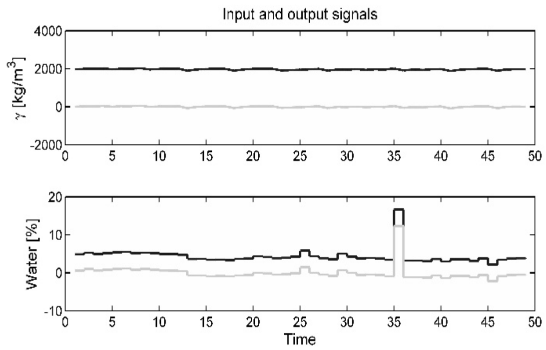

Fig. 3 Trend of the density of the asphalt materials γ (top) in relationship with the water on the layer surface.

The next step regarded the determination of the FPE and FIT indexes for different configurations of the model, in order to identify the optimal configuration. The choice would have to fall on the parameters n and m corresponding to the minimum FPE and maximum FIT values. Taking into account these indications we opted for the model called arx892.

Table 3 Quantification for different values of order and parameters. The minimum values of the FPE index and the maximum of the FIT index are the best option and are highlighted in bold.

Name	n_a	n_b	n_k	FPE	FIT
arx291	2	9 9 9	1 1 1	598.8	49.83%
arx681	6	8 8 8	1 1 1	417.8	58.48%
arx781	7	8 8 8	1 1 1	423.1	58.59%
arx881	8	8 8 8	1 1 1	406.9	59.76%
arx981	9	8 8 8	1 1 1	255.6	68.38%
arx1082	10	8 8 8	2 2 2	175.7	74.10%
arx892	8	9 9 9	2 2 2	90.46	88.55%

The resulting ARX model has the following features:

Equation of the model: A(q) y (t) = B(q) u (t) + e(t)

Where:

$A(z) = 1 + 0.7711\ (+/-\ 0.2765)\ z^{-1} - 0.3918\ (+/-\ 0.3804)\ z^{-2} - 0.08218\ (+/-\ 0.3983)\ z^{-3} + 1.436\ (+/-\ 0.4541)\ z^{-4} + 0.2676\ (+/-\ 0.6774)\ z^{-5} - 0.6786\ (+/-\ 0.6459)\ z^{-6} + 0.9554\ (+/-\ 0.6332)\ z^{-7} - 0.755\ (+/-\ 0.4306)\ z^{-8}$

$B_1(z) = -0.2378\ (+/-\ 0.4685)\ z^{-2} + 1.131\ (+/-\ 0.6561)\ z^{-3} + 0.5955\ (+/-\ 0.7922)\ z^{-4} - 1.497\ (+/-\ 0.7943)\ z^{-5} + 0.7824\ (+/-\ 0.8644)\ z^{-6} + 0.7806\ (+/-\ 0.6105)\ z^{-7} - 0.03448\ (+/-\ 0.6296)\ z^{-8} - 0.4331\ (+/-\ 0.6041)\ z^{-9} + 1.727\ (+/-\ 0.513)\ z^{-10}$

$B_2(z) = 0.4811\ (+/-\ 0.1692)\ z^{-2} + 0.5578\ (+/-\ 0.1796)\ z^{-3} + 0.06427\ (+/-\ 0.1395)\ z^{-4} + 0.2994\ (+/-\ 0.1878)\ z^{-5} + 0.5363\ (+/-\ 0.1965)\ z^{-6} + 0.08952\ (+/-\ 0.1736)\ z^{-7} + 0.2441\ (+/-\ 0.1562)\ z^{-8} + 0.2531\ (+/-\ 0.128)\ z^{-9} + 0.06522\ (+/-\ 0.106)\ z^{-10}$

$B_3(z) = 4.549\ (+/-\ 3.52)\ z^{-2} + 4.344\ (+/-\ 3.685)\ z^{-3} - 4.396\ (+/-\ 3.45)\ z^{-4} - 1.194\ (+/-\ 3.538)\ z^{-5} + 6.683\ (+/-\ 3.938)\ z^{-6} - 3.958\ (+/-\ 3.062)\ z^{-7} - 5.055\ (+/-\ 3.313)\ z^{-8} + 2.523\ (+/-\ 3.03)\ z^{-9} + 2.785\ (+/-\ 3.576)\ z^{-10}$

This model has a very good adaptation to the system detected, with an index of fit of about 88.55% (Fig. 4).

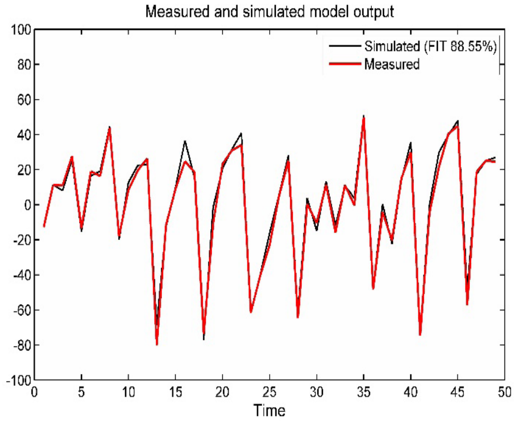

Fig. 4 Capability of the model to simulate the output γ with respect to the observed data.

The determination of the residual was useful to verify if the prediction errors are white and uncorrelated with respect to the input data. For our model, the residuals were all inside the 99% confidence limits.

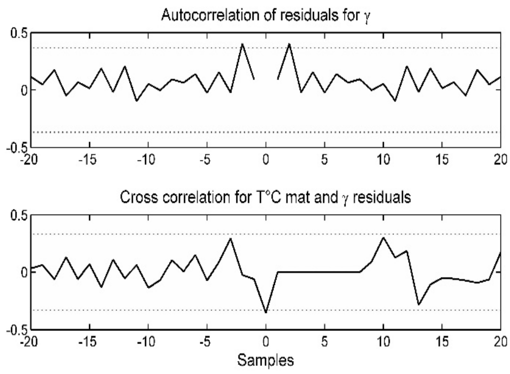

Fig. 5 Autocorrelation and cross-correlation regarding the output density (γ) and the input Temperature of the asphalt material (Te_M). The residuals are widely inside the confidence interval region, indicating that all the dynamics have been captured by the model.

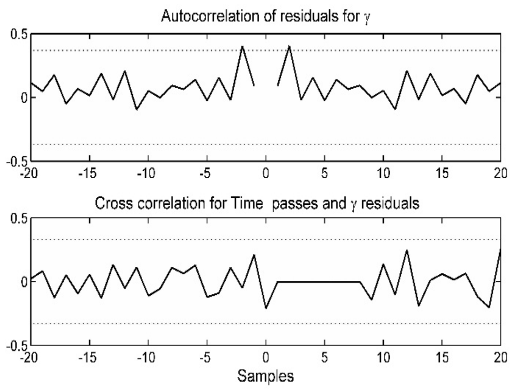

Fig. 6 Autocorrelation and cross-correlation regarding the output density (γ) and the input Elapsed time during the single pass of the roller (TI). Also in this case, the residuals are widely inside the confidence interval region, indicating that all the dynamics have been captured by the model.

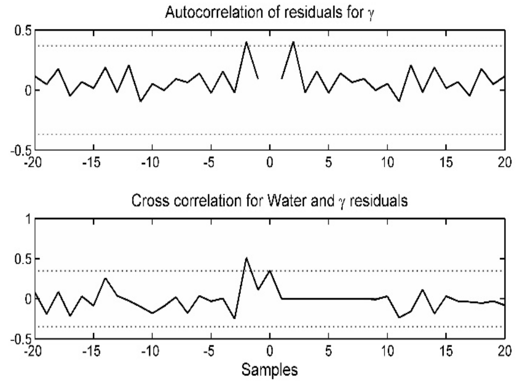

Fig. 7 Autocorrelation and cross-correlation regarding the output density (γ) and the input Water contained in the surface layer of the pavement (Wa). Also in this case, the residuals are widely inside the confidence interval region, indicating that all the dynamics have been captured by the model.

4 Discussion

The simple survey of some data such as temperature of the material, passes, number of rollers, its dynamic characteristics, geometry of layers and so on, permits knowledge of only the density in progress but does not consent to organize a reliable strategy during the execution phase. Consequently, it is necessary to prepare in advance an analytical tool that is simple, but takes into consideration the influence of all the environmental variables that can affect the observed process.

To perform this, we proposed a procedure with a black-box model entirely based on the collected data set. The first step was to identify these variables that had the most influence on the output, by means of the Neuro-Fuzzy technique (Table 5), on which, however, it is necessary to make a clarification. The differences between the RMSE values are not so evident: this circumstance depends on a very short data set unable to identify with evidence sharp bonds among all the different combinations. However, those reported in Table 5 are only the first rows of the total number of 84, which do not allow highlighting the deviations really existing with respect to the subsequent combinations.

The second aspect to be highlighted concerns the calibration mode of the parameters and the selection of the final configuration of the model. We opted for a linear ARX type simplifying so probable nonlinear behavior that could be neglected as confirmed by the performed validation measurements. The choice of the orders and parameters, given the nature of the model („black -box", that is without recognizable physical law), took place with a procedure based on trials and errors. For the sake of brevity, we did not write about the various tested models but only reported in Table 6 the obtained results regarding the FPE and FIT indexes. The final decision was, therefore, based on these findings and allowed us to assert that the Fit value of 88.55% is the best choice. Further improvements towards more complex models (such as nonlinear type) would have resulted in higher computational and survey costs.

The residuals evaluation represented that part of the data that our model was not able to reproduce. The Figures 5, 6 and 7 attest a good correctness of the system identification, as it is known; since the residues are computed on the basis of the identified data and model, they have to be white and independent from the input. In the present case, the trend of the autocorrelation and cross correlation functions are all inside the confidence interval and this demonstrates that our model is sufficiently reliable.

Therefore, the advantages of the proposed procedure mainly regard:

- The speed in determining the variables that most affect the compaction.
- Minor computational and survey costs since the number of input variables are reduced.
- Improvements regarding economic and time aspects, since it is not necessary to build and then demolish relevant sections of pavement.

5 Conclusions

As it is known, during the construction of an asphalt pavement, some important quantities continuously change. Compaction operators are always interested to know the consequences of such changes on the material density and, thus, on the final quality of the layer.

However, there are several difficulties in the real cases. First of all, achieving on site the same compaction energy used in the laboratory with a Marshall or gyratory compactor is highly unlikely because the variables involved are not completely independent among themselves and the phenomenon is often of a nonlinear type. Some of these quantities are related to the roller characteristics, but others, more complex to control, are linked to the environment and are continuously modified. For these reasons, it is easier (but more expensive) to prepare some laying tests, characterized by different starting conditions, from which the operator extracts some cores and sends them to the laboratory. The density test on cores provides a measure of the correctness of the methodology followed. However, these activities are carried out over a couple of days and if the boundary conditions change (air temperature and material, paving surfaces, humidity, etc.) it is possible that real results are different from the previous hypothesis.

For all these reasons, in this research we presented a procedure, perhaps less refined from the analytical point of view, than the complex rheological models already existing in literature, but one that would provide realistic answers and in a very short time.

In particular, we prepared an ARX linear model based on some variables measured on that particular site where the pavement had to be constructed. In addition to performing a prediction action, this methodology can prevent laying tests that, in the sections where density does not achieve a minimal value, must be demolished. Moreover, costs are more contained because it is no longer necessary to use laboratory tests with the same frequency. And, above all, for that particular scenario, it is possible to recognize the most sensitive variables.

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